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PI Front Steering and PI Rear Steering Control with Tire Workload Analysis

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Abstract— **The main contribution of this paper it to show that a proportional-integral active front steering control and a proportional-integral active rear steering control from the yaw rate tracking error can set arbitrary steady state values for lateral speed and yaw rate at any longitudinal speed.**
The proposed control system can: (i) assign real stable eigenvalues, without lateral speed measurements, for any value of longitudinal speed; (ii) set steady state responses to driver constant inputs to zero lateral speed for any value of longitudinal speed without additional steady state tire workload if the yaw rate reference for the controlled vehicle is equal to the uncontrolled one.
The controlled system shows the advantages of both active front and rear steering control: higher controllability, enlarged bandwidth for the yaw rate dynamics, suppressed resonances, new stable cornering manoeuvres, enlarged stability regions and improved manoeuvrability; in addition comfort is improved since the phase lag between lateral acceleration and yaw rate is reduced. Several simulations are carried out on a standard small SUV CarSim® car model to confirm the analysis and to explore the robustness with respect to unmodelled dynamics such as pitch, roll and nonlinear combined lateral and longitudinal tire forces according to combined slip theory.

I. INTRODUCTION

Several cars on the market use 4WS technologies [1]; in the first generation (Honda) of rear steering vehicles [2], the front wheels steering angle δ_f is transmitted to the rear wheels mechanically by a shaft; in this case the control law is given by $\delta_r = K(\delta_f)\delta_f$ with δ_r the rear steering angle. In active rear steering systems a feedback control $\delta_r = K(v)r$ from the yaw rate measurement r is also used with K depending on the vehicle speed. More recently, some patented feedback control laws on the rear axle are based on the yaw stability management using the yaw tracking error; in [4] a static lookup table determines the desired closed loop yaw rate and a PID controller on the yaw rate tracking error is proposed while in [5] a feed-forward control algorithm includes a proper transfer function between driver and control signals to reduce the lateral velocity and to increase stability. Also on the front axle many control laws are proposed [6], [7], [8] and implemented on steer by wire prototypes in which the conventional steering elements are replaced by two electrical actuators positioned in the front corners of the vehicle and turn the front wheels. In [6] a PI active steering control on the yaw rate tracking error with different gains for braked and unbraked driving condition is used; in [7] it is shown that the lateral acceleration of the front axle may be robustly triangularly decoupled from the yaw rate dynamics, using only the front wheel steering angle as a control input, by feeding back the yaw rate error through an integrator while in [8] for 4WS vehicle yaw damping can be adjusted so that it becomes velocity independent. Recently many papers are focused on the design of integrated global chassis control system [9] to optimize tire workload [10], [11].

In [12] we proposed a four wheel steering control strategy consisting in an additional proportional front controller and a proportional integral rear controller depending on the yaw rate error: resonances suppression are always obtained while lateral speed can only be reduced at low speed where the uncontrolled (U) vehicle static gain between the driver input and the vehicle yaw rate can be changed.

In this paper we propose a PI active steering control employed on the rear steering angle and a PI active steering control on the front steering angle both from the yaw rate tracking error generated on the basis of a first order nonlinear reference model driven by the driver input. This control structure allows us to set arbitrary steady state values for lateral speed and yaw rate at any longitudinal speed. In this work we set the steady state lateral speed equal to zero and the steady state yaw rate equal to the uncontrolled value so that no additional steady state tire workload is required.

The proposed control brings several benefits to the controlled (C) vehicle steering dynamics: negative real eigenvalues for every longitudinal speed, even though lateral speed measurements are not available, higher controllability, enlarged bandwidth for the yaw rate dynamics, suppressed resonances, new stable cornering manoeuvres, enlarged stability regions and improved manoeuvrability.

II. NON LINEAR SINGLE TRACK CAR MODEL

A detailed non linear full car model with nonlinear tire characteristics, combined slip and differential load transfer for each wheels is considered for simulations. However, to design the controller, a simplified single track car model (Fig. 1) with nonlinear tire characteristics is considered which captures the essential vehicle steering dynamics. The nonlinear car model [13] is described by the following equations:

$$f_{si}(\alpha_i) = D\sin[C\arctan(1-E)B\alpha_i + E\arctan(B\alpha_i)] \qquad (1)$$

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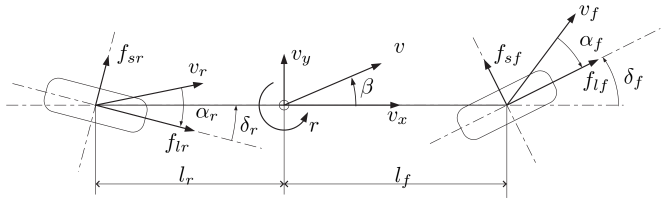

Fig. 1. Single track car model.

$$
\begin{aligned}
m(\dot{v}_x - r v_y) &= f_{lf}\cos\delta_f - f_{sf}\sin\delta_f + \\
&+ f_{lr}\cos\delta_r - f_{sr}\sin\delta_r - c_a v_x^2 \\
m(\dot{v}_y + r v_x) &= f_{lf}\sin\delta_f + f_{sf}\cos\delta_f + \\
&+ f_{lr}\sin\delta_r + f_{sr}\cos\delta_r \\
J\dot{r} &= l_f(f_{lf}\sin\delta_f + f_{sf}\cos\delta_f) - \\
&- l_r(f_{lr}\sin\delta_r + f_{sr}\cos\delta_r) \\
a_y &= \dot{v}_y + r v_x
\end{aligned}
\tag{2}
$$

$$
\alpha_f = \delta_f - \frac{v_y + l_f r}{v_x} \qquad \alpha_r = \delta_r - \frac{v_y - l_r r}{v_x} \tag{3}
$$

where: v_x and v_y ($v_y = v\sin\beta$) are the longitudinal and lateral vehicle speed, r is the vehicle yaw rate, δ_r is the rear steering angle, δ_f is the front steering angle, $f_s\,(f_l)$ are the lateral (longitudinal) forces, given by Pacejka tire model [14] (1), $c_f\,(c_r)$ is the front (rear) cornering stiffness, $\alpha_f\,(\alpha_r)$ is the front (rear) sideslip angle, $l_f\,(l_r)$ is the distance from the front (rear) axle to the centre of gravity, c_a is the aerodynamics drag coefficient, m is vehicle mass, J is the vehicle inertia respect to the vertical axle through the CG, a_y is the lateral acceleration and v is the vehicle velocity.

The system (2) is linearised around uniform rectilinear motion ($v_x = v$ =constant, $r = 0$, $v_y = 0$, $\delta_f = 0$, $\delta_r = 0$); the longitudinal dynamic is decoupled from the lateral dynamic and can be neglected as far as the steering dynamics are concerned. The reduced linear system, $\dot{X} = AX + BU$, is given by:

$$
\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}. \tag{4}
$$

The coefficients, which may depend on v are:

$$
\begin{aligned}
a_{11} &= -\frac{(c_f + c_r)}{mv}; \quad a_{12} = -1 - \frac{(c_f l_f - c_r l_r)}{mv^2}; \\
a_{21} &= -\frac{(c_f l_f - c_r l_r)}{J}; \quad a_{22} = -\frac{(c_f l_f^2 + c_r l_r^2)}{Jv};
\end{aligned}
\tag{5}
$$

$$
b_{11} = \frac{c_f}{mv}; \quad b_{12} = \frac{c_r}{mv}; \quad b_{21} = \frac{c_f l_f}{J}; \quad b_{22} = -\frac{c_r l_r}{J}. \tag{6}
$$

III. NON LINEAR PI CONTROL DESIGN

The designed proportional-integral control law on the front axle and the proportional-integral control law on the rear axle on the yaw rate tracking error is presented in this section. With yaw rate and longitudinal speed measurements the target reference yaw rate signal can be asymptotically tracked with the steady state value of $\beta = 0$. The functional scheme for the proposed nonlinear decentralized control system is described in fig. 2; it is composed of a nonlinear reference model and a PI controller for a time varying system.

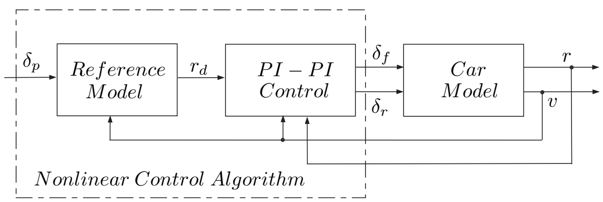

Fig. 2. Functional scheme for the controlled system.

The functional subsystems, shown in (Fig. 2), are described follow. The steering wheel angle given by the driver is the input δ_p to a non linear first order reference model which, according to the velocity v, generates the yaw rate references signal r_d or equivalently, for a given velocity, the lateral acceleration reference a_{yd}. The reference model is defined in general as:

$$
\begin{aligned}
\dot{a}_{yd} &= -\lambda_{ref}(v)\left(a_{yd} - \underset{a_{y\max}}{sat}\left[G(\delta_p, v)\,\delta_p v\right]\right) \\
r_d &= \frac{a_{yd}}{v}
\end{aligned}
\tag{7}
$$

where $\lambda_{ref}(v)$ is a design parameter for the non linear reference model and $G(\delta_p, v)$ is the imposed static gain between δ_p to r which may depend on δ_p making the reference model nonlinear.

In (7) the saturation with respect to the lateral acceleration a_{ymax} is used increase safety avoiding unstable car behaviours due to inadmissible driver steering inputs. The values of a_{ymax} can be set greater than the maximum uncontrolled vehicle lateral acceleration to improve performance.

The designed control law is a PI - PI on the front and rear axles and is defined as:

$$
\begin{cases}
\delta_f = -K_{pf}\tilde{r} - K_{if}\alpha_0 = -K_{pf}\tilde{r} - K_{if}\displaystyle\int_0^t \tilde{r}(\tau)\,\mathrm{d}\tau \\
\delta_r = -K_{pr}\tilde{r} - K_{ir}\alpha_0 = -K_{pr}\tilde{r} - K_{ir}\displaystyle\int_0^t \tilde{r}(\tau)\,\mathrm{d}\tau \\
K_{ir} = f(a_{ij}, b_{ij})K_{if} \\
\dot{\alpha}_0 = \tilde{r}
\end{cases}
\tag{8}
$$

where $\tilde{r} = r - r_d$ and $f(a_{ij}, b_{ij})$ is a function depending on the model parameters given in (5) and (6).

The used car models are: a simple single track car model for the linear analysis and a full car model exported from CarSim® to confirm, by simulations, the results with respect to unmodelled dynamics. The properties of the proposed nonlinear control system are described in the following paragraph.

IV. LINEAR ANALYSIS

In many four wheel steering control systems the steady state value of the lateral velocity is set to zero to improve comfort and manoeuvrability since the yaw rate and lateral acceleration are in-phase [1], [5]. Computing the equilibrium point for a four wheel steering vehicle we obtain:

$$\begin{bmatrix} \beta_e \\ r_e \end{bmatrix} = -\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{-a_{22}b_{11}+a_{12}b_{21}}{a_{11}a_{22}-a_{12}a_{21}} & \frac{-a_{22}b_{12}+a_{12}b_{22}}{a_{11}a_{22}-a_{12}a_{21}} \\ \\ \frac{a_{21}b_{11}-a_{11}b_{21}}{a_{11}a_{22}-a_{12}a_{21}} & \frac{a_{21}b_{12}-a_{11}b_{22}}{a_{11}a_{22}-a_{12}a_{21}} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}.$$

Setting $\beta_e = 0$, a structural relation is obtained:

$$\delta_r = -\frac{a_{22}b_{11}-a_{12}b_{21}}{a_{22}b_{12}-a_{12}b_{22}}\delta_f = \frac{-l_r + \frac{ml_f}{c_r(l_f+l_r)}v^2}{+l_f + \frac{ml_r}{c_f(l_f+l_r)}v^2}\delta_f \tag{9}$$

which implies:

$$G_n = \frac{r_e}{\delta_f} = \frac{b_{22}b_{11}-b_{21}b_{12}}{a_{22}b_{12}-a_{12}b_{22}}. \tag{10}$$

Equation (9), (see also [1] and [5]), guarantees, at steady state, zero lateral speed. From the viewpoint of vehicle dynamics and control, it is impossible to achieve zero lateral speed by using only the front wheel steering angle as control input.
Equation (10) gives the ratio between yaw rate and the front steering angle when $\beta = 0$. We may observe that the static gain (10) which is compatible with zero lateral speed is different from the uncontrolled vehicle ($\delta_r = 0$) static gain (G_u) between δ_f and r (see (Fig. 3)) which is given by:

$$\begin{aligned} G_u &= \lim_{s\to 0} C(sI-A)^{-1}B = \\ &= \frac{a_{21}b_{11}-b_{21}a_{11}}{a_{11}a_{22}-a_{12}a_{21}} \end{aligned} \tag{11}$$

where:

$$C = [0 \quad 1].$$

Analysing the controlled non linear system (4,7,8) with both the active front and rear steering control, the controlled linearised system can be written in the state space form $\dot{X}_c = A_cX_c + B_c\delta_p$, obtaining:

$$A_c = \begin{bmatrix} a_{11} & \bar{a}_{12} & \bar{a}_{13} & \bar{a}_{14} \\ a_{21} & \bar{a}_{22} & \bar{a}_{23} & \bar{a}_{24} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & \lambda_{ref} \end{bmatrix},$$

$$X_c = \begin{bmatrix} \beta \\ r \\ \alpha_0 \\ r_d \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\lambda_{ref}G \end{bmatrix} \tag{12}$$

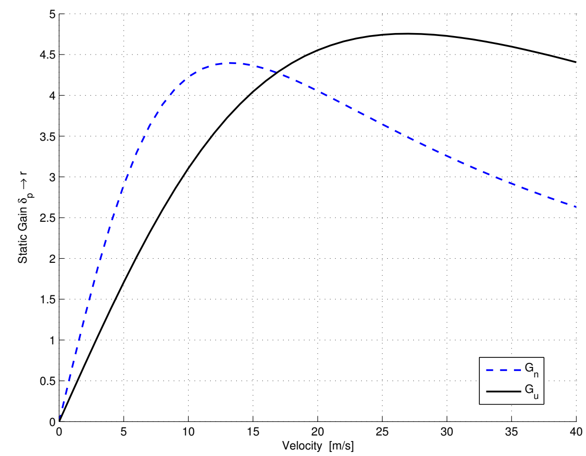

Fig. 3. Static Gain from $\delta_f \to r$.

where $G = G(0,v)$ from (7) and:

$\bar{a}_{12} = a_{12} - b_{11}K_{pf} - b_{12}K_{pr}$, $\quad \bar{a}_{13} = -b_{11}K_{if} - b_{12}K_{ir}$,
$\bar{a}_{14} = b_{11}K_{pf} + b_{12}K_{pr}$, $\quad \bar{a}_{22} = a_{22} - b_{21}K_{pf} - b_{22}K_{pr}$,
$\bar{a}_{23} = -b_{21}K_{if} - b_{22}K_{ir}$, $\quad \bar{a}_{24} = b_{21}K_{pf} + b_{22}K_{pr}$.

Computing the equilibrium point X_{ce} for the closed loop system we obtain.

$$X_{ce} = G\begin{bmatrix} -\frac{(-a_{12}b_{21}K_{if}-a_{12}b_{22}K_{ir}+a_{22}b_{11}K_{if}+a_{22}b_{12}K_{ir})}{-a_{11}b_{21}K_{if}-a_{11}b_{22}K_{ir}+a_{21}b_{11}K_{if}+a_{21}b_{12}K_{ir}} \\ 1 \\ \frac{(-a_{11}a_{22}+a_{21}a_{12})}{-a_{11}b_{21}K_{if}-a_{11}b_{22}K_{ir}+a_{21}b_{11}K_{if}+a_{21}b_{12}K_{ir}} \\ 1 \end{bmatrix}\delta_P \tag{13}$$

Setting $X_{ce}[1] = \beta_e = 0$ in (13) we obtain the following constraint:

$$K_{ir} = -\frac{K_{if}(-a_{12}b_{21}+a_{22}b_{11})}{-a_{12}b_{22}+a_{22}b_{12}}. \tag{14}$$

Solving (13) with respect to K_{ir} the closed loop equilibrium point is:

$$X_{ce} = \begin{bmatrix} 0 \\ G \\ \frac{(-a_{12}b_{22}+a_{22}b_{12})G}{(b_{21}b_{12}-b_{22}b_{11})K_{if}} \\ G \end{bmatrix}\delta_p. \tag{15}$$

From (15) we may observe that G in (15) (and consequently $G(\delta_p,v)$ in (7)) can be arbitrarily chosen as an additional design degree of freedom.
Note that (14) is zero when $v = \sqrt{c_rl_r(l_f+l_r)/(ml_f)}$ i.e. when $G_u = G_n$ as shown in (Fig. 3).

In (Fig. 4) it is shown the steady state value of δ_r/δ_p and the static gain between δ_p and v_y for the uncontrolled system for different velocities. From (Fig. 4) we may observe that the steady state values for v_y (for the uncontrolled vehicle) and δ_r have the same sign: in particular at low longitudinal speed δ_r and δ_p are in phase while they are out of phase at high longitudinal speed as expected. Substituting (14) in

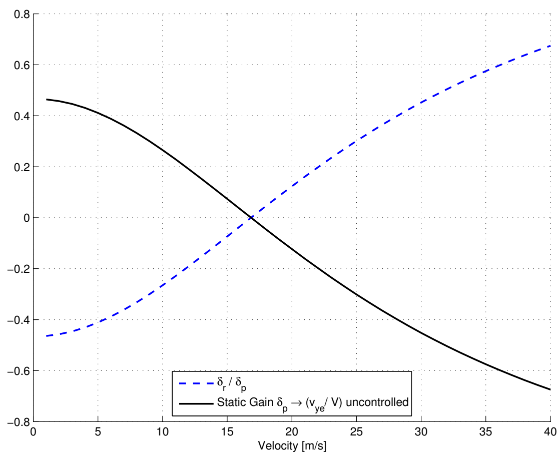

Fig. 4. Static gain $\delta_p \to v_y$ for the uncontrolled system and K_{ir} for different velocities.

(12) the eigenvalues belong to the negative real axis for the used control parameters as shown in (Fig. 5).

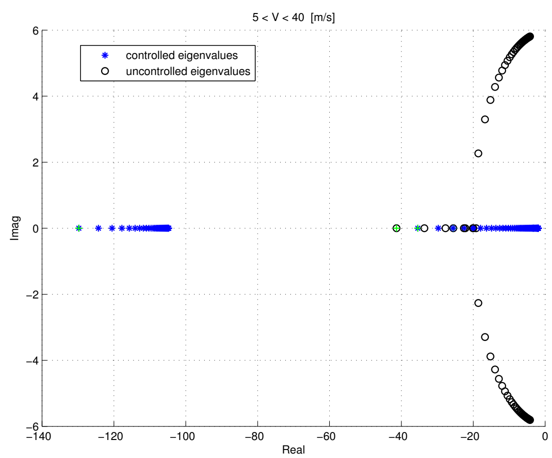

Fig. 5. Eigenvalues for the uncontrolled and controlled vehicle for different velocities.

A. TIRE WORKLOAD ANALYSIS

Many recent papers propose several methods to optimally distribute the vehicle stabilizing forces among wheels through longitudinal [10] and lateral slip regulation [11]. The analysis of the tire slip angle is very important for vehicle stability and for the evaluation of the tire workload. Computing the steady state tire sideslip angle (3) for the uncontrolled vehicle $(\alpha_{fu}, \alpha_{ru})$ and for a vehicle with steady state rear steering angle δ_{rn} equal to (9) so that the steady state value of lateral speed is zero $(\alpha_{fn}, \alpha_{rn})$ we obtain:

$$\alpha_{fu} = \delta_{fu} - \frac{v_{yu} + l_f r_u}{v_x}; \qquad \alpha_{ru} = -\frac{v_{yu} - l_r r_u}{v_x}; \qquad (16)$$

$$\alpha_{fn} = \delta_{fn} - \frac{l_f r_n}{v_x}; \qquad \alpha_{rn} = \delta_{rn} - \frac{-l_r r_n}{v_x}. \qquad (17)$$

Subtracting (17) from (16) we compute $\Delta\alpha_f = \alpha_{fu} - \alpha_{fn}$ and $\Delta\alpha_r = \alpha_{ru} - \alpha_{rn}$ as follows:

$$\begin{aligned} \Delta\alpha_f &= \delta_{fu} - \delta_{fn} - \frac{v_{yu}}{v_x} - \frac{(r_u - r_n) l_f}{v_x}; \\ \Delta\alpha_r &= -\delta_{rn} - \frac{v_{yu}}{v_x} + \frac{(r_u - r_n) l_r}{v_x}. \end{aligned} \qquad (18)$$

Computing the driver steering angles as follows:

$$\delta_{fu} = r_u / G_u; \qquad \delta_{fn} = r_n / G_n, \qquad (19)$$

computing v_{yu} as the static gain from δ_{fu} to v_{yu} times δ_{fu}, substituting to δ_{rn} the structural relation found in (9) and substituting (19) in (18) we obtain the following relations:

$$\begin{aligned} \Delta\alpha_f &= \frac{(r_u - r_n) l_r m v_x}{c_f (l_f + l_r)}; \\ \Delta\alpha_r &= \frac{(r_u - r_n) l_f m v_x}{c_r (l_f + l_r)}. \end{aligned} \qquad (20)$$

If $r_u = r_n$ than $\Delta\alpha_f = \Delta\alpha_r = 0$. Note that no additional tire workload is added to set zero lateral speed for a given yaw rate reference.

Solving the front and the rear steady state tire sideslip angle (3) for the proposed linearised control system (4,7,8) we obtain:

$$\alpha_{fc} = \delta_{fc} - \frac{l_f r_c}{v_x}; \qquad \alpha_{rc} = \delta_{rc} - \frac{-l_r r_c}{v_x}. \qquad (21)$$

Computing $\Delta\alpha_f = \alpha_{fu} - \alpha_{fc}$ and $\Delta\alpha_r = \alpha_{ru} - \alpha_{rc}$ we obtain:

$$\begin{aligned} \Delta\alpha_f &= \delta_{fu} - \delta_{fc} - \frac{v_{yu}}{v_x} - \frac{(r_u - r_c) l_f}{v_x}; \\ \Delta\alpha_r &= -\delta_{rc} - \frac{v_{yu}}{v_x} + \frac{(r_u - r_c) l_r}{v_x}. \end{aligned} \qquad (22)$$

Substituting to δ_{fc} and δ_{rc} the following steady state control law values according to (8):

$$\delta_{fc} = -K_{if}\alpha_0; \qquad \delta_{rc} = -K_{ir}\alpha_0; \qquad (23)$$

and substituting K_{ir} (14) in (23) we obtain $\Delta\alpha_f = \Delta\alpha_r$ and they coincide with (20) in which r_n is replaced by r_c. For the same yaw rate reference $(r_u = r_c)$ no additional steady state tire workload is required to set zero lateral speed moreover, the steady state value of $v_y = 0$ is obtained without using (10) in (7); this improvement gives an additional degree of freedom for the designer who can set $v_y = 0$ and different static gain from $\delta_p \to r$.

During the transient responses the tire workload has been optimized tuning optimally the control law parameters to achieve a compromise between performance and stability. To implement this strategy the Simulink® Response Optimization toolbox was used; this module can tune the design parameters through numerical optimization; the design requirements, expressed in terms of rise time, settling time and overshoot, optimize the closed loop performance of controlled systems.

The uncontrolled and controlled vehicle responses for two increasing steering step angle are shown in (Fig. 6). We may observe, for the greater steering step input, a new stable cornering manoeuvre, while, for the open loop stable steering manoeuvre, we may observe in (Fig. 7) the steady state value of the tire sideslip angle and the optimized transient response.

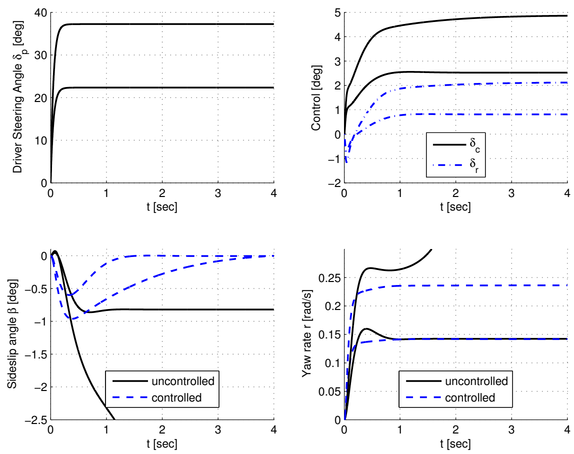

Fig. 6. Sudden direction change for the uncontrolled and controlled vehicle for $v = 30$ [m/s].

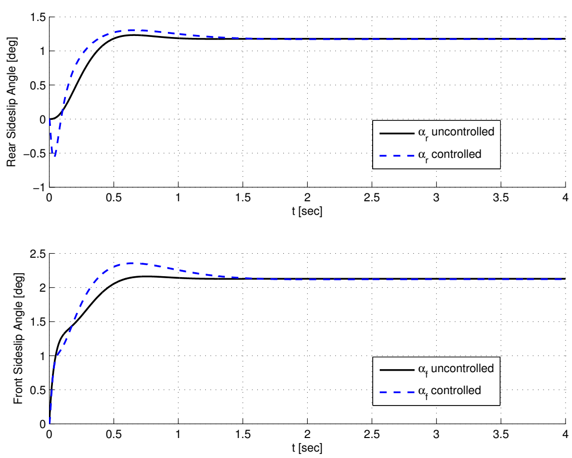

Fig. 7. Tire sideslip angles for the uncontrolled and controlled vehicle for $v = 30$ [m/s].

B. Four Wheel steering improvement

The controlled system shows the classical advantages of both active front and rear steering control: higher controllability, enlarged bandwidth for the yaw rate dynamics, suppressed resonances, new stable cornering manoeuvres, enlarged stability regions. The control law (7,8) can be tuned in order to prevent the uncontrolled vehicle oscillations for the speed range of interest (Fig. 5). The bandwidth (Fig. 8, 9) of the closed loop car is increased with respect to the open loop system in the range of interest. All these mathematical advantages are confirmed by the Simulation Results paragraph for the canonical manoeuvres such as sudden direction change or overtaking; moreover other standard benchmark manoeuvres are tested to confirm the results (i.e. understeering diagram and moose test).

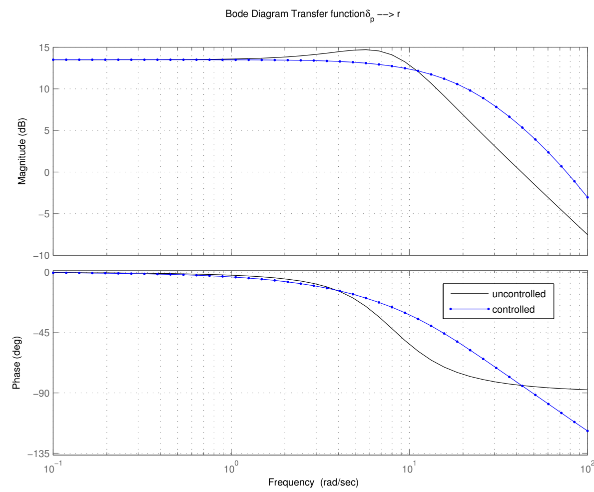

Fig. 8. Bode $\delta_p \rightarrow r$ for the uncontrolled and controlled vehicle.

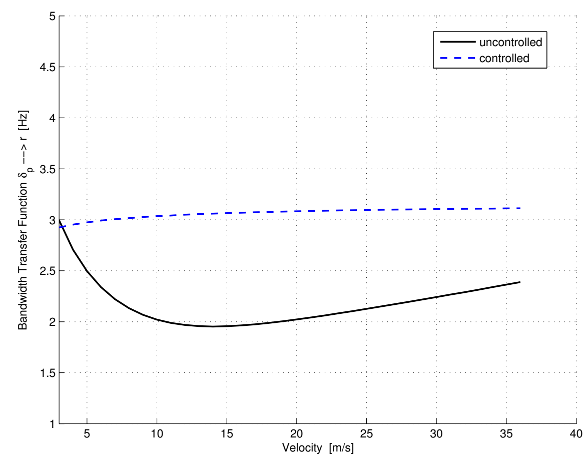

Fig. 9. Bandwidth $\delta_p \rightarrow r$ for the uncontrolled and controlled vehicle.

V. Simulation Results on Carsim® Vehicle

A full car model of a standard Carsim® small SUV is used to analyse the actual response of the controlled car and to check robustness with respect to unmodelled dynamics. CarSim® vehicle takes into account the major kinematics and compliance effects of the suspensions (nonlinear spring models) and steering systems and uses detailed nonlinear tire models obtained from experimental measurements. The simulation are performed using in (7) the open loop static gain $G(\delta_p, v)$ obtained from real data by storing the steady state yaw rate value in a lookup table for different steering angles and car velocities. For canonical manoeuvres, such as sudden direction change, for three increasing steering angle, the results are shown in (Fig. 10). In (Fig. 10) are shown the three increasing driver steering angles and the corresponding: three computed front and rear steering angles, three uncontrolled and controlled body sideslip angles and three yaw rate controlled and uncontrolled signals. For the sudden direction change corresponding to $\delta_p = 52$ [deg] it is shown (Fig. 11), for the uncontrolled and controlled vehicle, the values of the tire sideslip angle to confirm the analysis

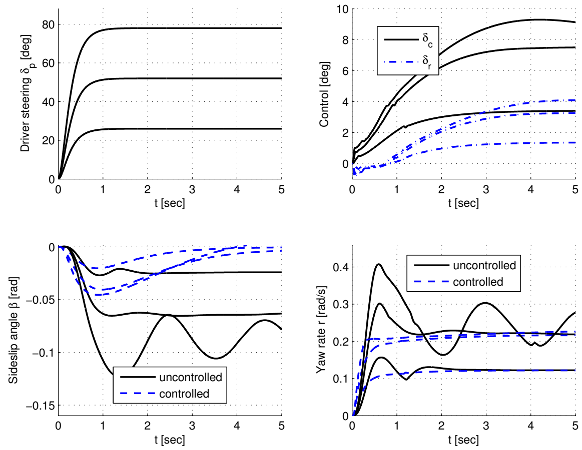

Fig. 10. Sudden direction change for the uncontrolled and controlled Carsim® car model.

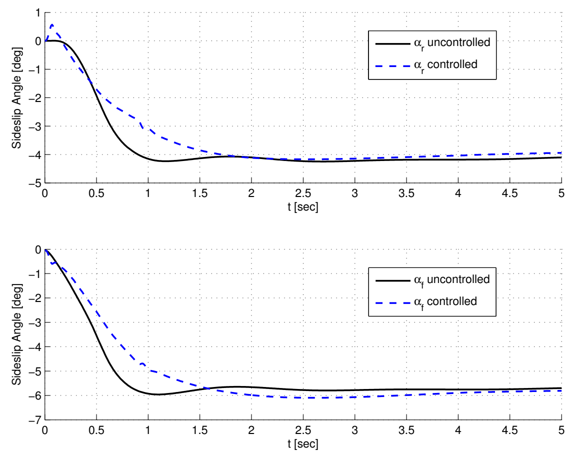

Fig. 11. Tire sideslip angles for the uncontrolled and controlled Carsim® car model.

on the simplified model. The proposed control counteracts yaw rate tracking errors; the dynamics show no oscillations and this behaviour produces a better direction change.

VI. Conclusion

A PI steering control on the front wheels and a PI steering control on the rear wheels are proposed and designed in order to obtain new advantages and maintain the classical benefit of both front and rear active steering. The controller feeds back the yaw rate tracking error which is generated on the basis of a first order nonlinear reference model driven by the driver steering wheel input. It is shown that a PI controller on the front steering angle and a PI controller on the rear one can set arbitrary steady state values for lateral speed and yaw rate at any longitudinal speed. If the lateral speed is set to zero for any value of longitudinal speed without setting a different static gain for the transfer function between the yaw rate and the driver steering angle no additional tire workload is required. The transfer function between the driver input and the yaw rate, for the controlled vehicle, has larger bandwidth than the uncontrolled one and shows exponential modes for every speed of interest. A robustness analysis with respect to unmodelled dynamics is carried out to confirm the theoretical improvements for the controlled system on a CarSim® vehicle. A drawback for the proposed control system, which is common to the most automotive control systems, is the dependence of the reference model from the coefficient of adherence: to overcome it on line estimation schemes should be incorporated in the controller. This will be the object of future works.

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