Proceedings of the
2007 IEEE Intelligent Vehicles Symposium
Istanbul, Turkey, June 13-15, 2007

Assessing the maneuverability of tractor trailer systems in heavy goods transport

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Abstract— **In recent years strong efforts have been made for assistance and automation of heavy goods vehicles. The major efforts focus on driver and driving assistance systems for a broad scope of use cases. Minor efforts have been spent for autonomous driving of heavy goods vehicles. Here the scope of application is limited to non-public traffic, e.g. logistics centers and factory grounds. Also the degree of autonomy is limited, typically to the same level as for AGV's following predefined trajectories.**

However, extending autonomous driving to logistics centers and factory grounds comes along with a variety of new challenges. An important one is discussed here and regards the maneuverability of vehicles with a high degree of nonholonomy on narrow grounds. In doing this a quantifiable measure of maneuverability has to be defined and applied to comparable heavy goods vehicles.

I. INTRODUCTION

Nonholonomic systems, particularly nonholonomous vehicles, attract attention for more than a century now (introduction of term and concept by [11] and [10]). A profound theory on the kinematic behavior of vehicles and on the planning and control of motion has been established, particularly in the last two decades (see among others [6], [7] and [1]). Even though there are a lot of results available only a low rate of practical applications have been developed and run under commercial conditions. Besides the legal barriers this mainly due to the fact that practical applications involve a variety of scientific disciplines and require a sophisticated technical infrastructure. So, formerly nonholomic systems have been in the domain of mechanical engineering. Nowadays they are investigated by control engineers, e.g. with respect to the stability of steering controls, by computer scientists, e.g. with respect to versatile software frameworks, by electrical engineers, e.g. for sensor devices for measuring the angle between different parts of the articulated vehicle, and several other researchers.

The nonholonomic property states that motion not only depends on the geometry of the vehicle, but also on its driving and steering velocities. The most simple system investigated in this context is the unicycle, nothing else than a running coin. Slightly more complicated are Hilare-like robots and car-like robots (see among others [8] and [3]). The next step in complexity comes by using vehicles as tractors to tow trailers. A systematic characterization is given by categorizing vehicles as *general-n-trailers* (see [1]), were n stands for the number of trailers following a steering axle.

Heavy goods traffic on the road is executed by trucks (general-1-trailer), by trucks with one-axle trailer (general-2-trailer) and by trucks with a two-axle trailer which are – in this categorization – two trailers (general-3-trailer). Today more than 85% of the long distance goods transport is performed by general-2-trailers. However, this category of vehicles which is in the focus of this paper is not homogenous. So, there are on one hand semi-trailers mounted by fifth wheel kingpin coupling devices to semi-trailer trucks (t1). On the other hand there are trucks with one-axle trailers hitched together by drawbars (t2). There is also a difference with respect to the load: type t1 carries load in the trailer only, whereas type t2 carries load on both truck and trailer.

These differences become essential when using general-2-trailers for driverless charging and discharging purposes. In the scope of harbor areas special car-like robots with two steerable axles are used. In contrast AGV's mostly are built following the Hilare-like principle. First applications using standard trucks in automation are on factory grounds, executing goods shuttle services following predefined trajectories (see image 1). As far as we know the degree of autonomy is still very low in these applications. The vehicles are supervised by a central control station and are autonomous in that they are allowed to execute control rules in following the predefined paths and pragmatic rules to detect and to react in emergency situations. Hence, the next step in autonomy is to delegate the motion planning to the vehicle. This is particularly reasonable for goods transport where series vehicles both are used by man in public traffic and are maneuvered autonomously on certain non-public grounds. From the logistical perspective geometric and steering properties are in the proprietary of the vehicle and therefore the computation of motion planning and control naturally belongs to the vehicle.

For the assessment vehicles two questions have to answered first:

- There are so many general-2-trailer type vehicles. What is the convenient criterion for comparability? In the sequel we adopt the overall geometric length, what is to say the distance from the steering axle to the axle of the trailer.
- There is an big variety of maneuvers that have to be executed by vehicles on logistics centers and factory grounds. Which maneuver is both relevant in this scope of application and output simple one-dimensional results. Here we prototypically use the maneuver of parallel parking applied to tractor trailer systems and as for the minimal parking distance s_x give a depth s_y

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Fig. 1. A driverless truck passes slowly through the washing station of an experimental logistics center.

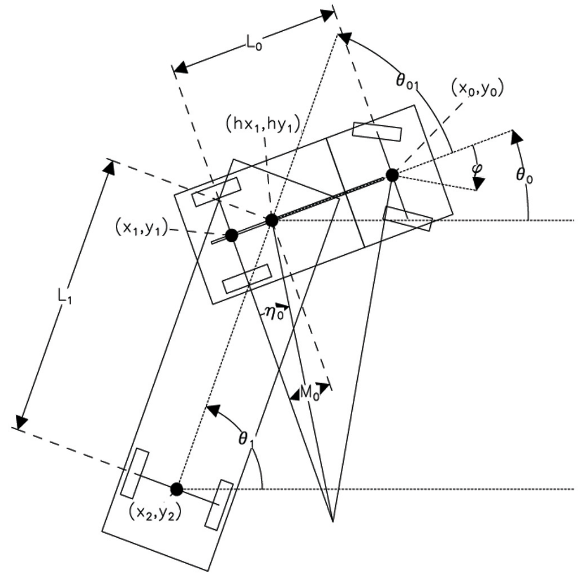

Fig. 2. Geometric parameters of a semi-trailer truck (type t2)

of the parking box.

The rest of the paper is structured as follows: in the basic notation for tractor trailer systems as well as the relevant formulas their geometric and kinematic behavior are introduced. There are several ways to park vehicles. For reasons of comparability we derive in section 3 the minimum parking maneuver which is not trivial for the general-2-trailer. Different types t1 and t2 of general-2-trailers are evaluated with respect to their parallel parking properties in section 4.

II. Parallel parking with tractor trailer systems

The problem of parking has been considered in literature predominantly for vehicles without trailer. The articles focus around the software architecture for the human-machine interface (e.g. [12]), the trajectory planning and driving control (e.g. [5] and [9]) and finally the optimality of parking maneuvers (e.g. [4]).

As far as we know parallel parking for articulated vehicles has been a topic for a small number of scientific articles. They predominantly center on kinematic modelling (in [3]), the investigation practical maneuvering strategies (in [14] and the principal strategies for planning the parking trajectories in [3]).

Over the last decade our institute has gathered experiences with trajectory planning, motion control and safety concepts with respect to autonomous and assisted driving. These experiences are based on the theory of nonholonomous vehicles, but also on experiments with different model trucks and trailers and recently also with real series vehicles. In the context of a project[1] for driver assistance for backing up articulated vehicles several test drivers already appraised and criticized our newest driving assistance systems (e.g. [2]).

A parking assistant is one part of a general driver assistance system for articulated vehicles. Principally, the parking assistant works by applying the following steps:

- A radar system scans the lateral environment for parking boxes.
- The system sends a signal to the driver when an adequate parking box has been detected.
- The driver decides to use this parking box.
- The vehicle stops at a convenient position.
- Under the throttle control of the driver, but with his hands off the steering wheel, the articulated vehicle drives backward into the parking box.
- Driving forward for a small distance centers the vehicle in the parking box.

An experimental system which is capable to execute these steps is available since a few years and has been steadily improved to integrate handling and safety issues.

The maneuver is based on the well known dependencies for a *general-2-trailer* (see [1]):

$$\begin{pmatrix} \dot{x_0} \\ \dot{y_0} \\ \dot{\theta_0} \\ \dot{\theta_1} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} cos(\theta_0) & 0 \\ sin(\theta_0) & 0 \\ tan(\phi)/L_0 & 0 \\ \dot{\theta_1}/v_1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad (1)$$

The parameters on the left side are the temporal derivatives of the geometric description of an articulated vehicle with a one-axle trailer given in figure 2. The input parameters are the velocity v_1 of the fixed axle of the tractor and the steering velocity v_2. The most decisive value in the matrix above is the change of the angle of the trailer. Here the dependency is:

$$\frac{\dot{\theta_1}}{v_1} = \left(\frac{sin(\theta_1 - \theta_0)\, tan(\phi)}{L_1} + \frac{M_0\, cos(\theta_1 - \theta_0)}{L_0\, L_1} \right) \qquad (2)$$

Driving into a parking box requires taking into account a variety of constraints which restrict the degree of freedom

[1]The project *Fahrassistenz beim Rückwärtsfahren* is funded by *Stiftung Rheinland-Pfalz für Innovation*, Mainz, Germany.

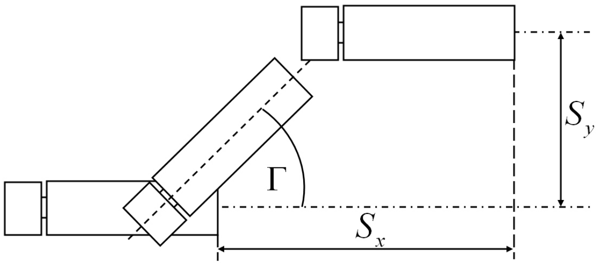

Fig. 3. Definition of the parameters for the parallel parking process right backward

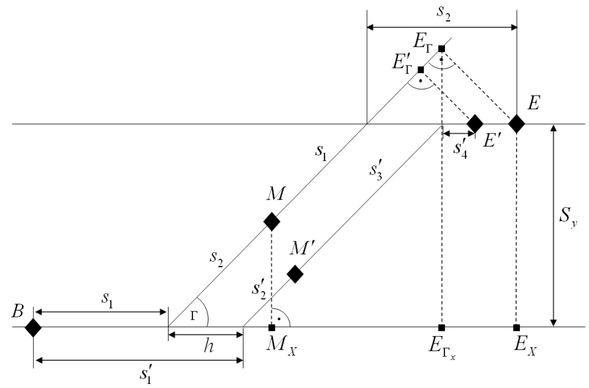

Fig. 4. The distinguished coordinates right backward. The diamond symbol refers to some position of the articulated vehicle.

that is given by the formula above. Two major constraints are:

- Limitations to the steering angle given by the mechanics of the tractor: $-\phi_{max} \leq \phi \leq \phi_{max}$
- Jack-knifing limitations to the angle between tractor and trailer: $-jk_{max} \leq \theta_2 - \theta_1 \leq jk_{max}$

Further constraints which consider practical aspects of parking a real vehicle in real traffic situations will be considered in a later section.

Without loss of generality the description of the parallel parking maneuver is given for parking right backward. Typically the depth of the parking box is at about $1\frac{1}{2}$ of the breadth of the vehicle. Reducing the vehicle to a bicycle model we ask for the minimum longitudinal distance s_x for a given lateral depth s_y (see figure 3).

However, there are further questions which are important when thinking about parallel parking in the scope of the development of a respective parking assistant. They will be discussed in section IV.

III. Optimal path into the parking box

The existence of a minimum parking distance s_x and a strategy to get it is considered only for reasonable and efficient maneuvers for parallel parking, they have to be restricted so:

- The unique sense of driving is backward into the parking box.
- The angular maximum deviation of tractor and trailer from their initial and final direction must not exceed $90°$.

With this restrictions and for a given, "reasonable" $s_y > 0$ it can be concluded that also $s_x > 0$. We can further conclude, that between the initial configuration and the final configuration there is an intermediate configuration, where truck and trailer are in a straight line again (see figure 3).

We call this the inflecting configuration, which has an angle Γ with respect to the initial or final configuration. The angle Γ is also called *gain*. During the exercise of particular parallel parking maneuver the configuration forming a straight line can reappear several times. In the case of repeated inflecting configurations we only consider the biggest of the occurring angles as Γ.

By a sequence of refinement steps the minimum parking maneuver is constructed, [13]. Given some reference point of the vehicle (e.g. the center of the trailer's axle) there are three distinguished coordinates: the initial coordinate B, the coordinate of inflection M, and the final coordinate E (see figure 4). At all three coordinates the tractor and trailer are in a straight line.

First, we will consider only the first phase of the parking process, that is the progression from the initial to the inflecting configuration. $BM_x = s_{x_1} = s_1 + s_2 \cdot \cos(\Gamma)$ is the distance we want to minimize for parking on a straight line, which under angle Γ to initial line is. (see figure 4).

To realize the minimum parking process for parking on a straight line above mentioned, one has to go left with steering-angle $-\phi_{max}$ by to stipulated moment and than go to right with steering-angle ϕ_{max} to achieve a straight line, which is under angle Γ to initial line.

The proof for the parking process distance s_{x_1} is based on the fact that the integral of the curvature of the vehicle's trajectory is equal to Γ (see [13]). The function of curvature for this minimum parking processes is increasing or decreasing faster as the function of curvature for another parking processes. That meant, for any value K_1 from small neighborhood increased a curvature, the curvature of the minimum parking in a neighborhood from K_1 increase faster. Analog is for decreasing neighborhoods. It hold maximum of the minimum parking curvature is greater as maximum any parking curvature. Therefore the condition for integral forced longer s_{x_1} for any parking process as for minimum parking.

Second, we will consider the second phase of the parking process. This is a repetition of the first phase. Obviously the inflecting configuration and the final configuration build the angle Γ again. If the depth of the parking box is $s_y = (s_1 + s_2) \cdot \sin(\Gamma)$ we can suggest the minimizing maneuvers from the first part and next the repetition the first maneuvers - but with opposite sign of the steering angle - minimize the distance s_x. The maneuvers of the second phase begin exactly at the point, where the maneuvers of the first phase stopped, this means $s_3 = s_1$ and $s_4 = s_2$.

These maneuvers are shown on the figure 4. S_{x_1} is minimal for all the parking processes (with given restriction) from one straight line to another (under angle Γ). So for any another parking process (all indications for different parking process haven indicator $'$) the coordinate M' is in excess of M on this straight line or M' is on another parallel straight line, right from the interval $M^{\bar{\prime}}M_x'$.

If our claim concerning minimum s_x was not true, then exists another parking process for first - to inflecting configuration - and second parts - from inflecting configuration - whose distance $BE_x' = s_x'$ is smaller than $BE_x s_x$. In this case the inflecting position M' is e.g. like on the figure 4. Then exists a $h > 0$ so that $s_1' = s_1 + h$, and

$$s_2' \quad > \quad \frac{s_2 \cdot \cos(\Gamma) - h}{\cos(\Gamma)} \tag{3}$$

This means that there exists $h_2 > 0$ and $h_2 < h$, that

$$s_2' \quad = \quad \frac{s_2 \cdot \cos(\Gamma) - h_2}{\cos(\Gamma)} \tag{4}$$

Then

$$\begin{aligned} BM_x' &= s_{x_1}' = s_1' + s_2' \cdot \cos(\Gamma) && (5)\\ &= s_1 + h + \frac{s_2 \cdot \cos(\Gamma) - h_2}{\cos(\Gamma)} \cdot \cos(\Gamma) && (6)\\ s_{x_1}' &= s_{x_1} + (h - h_2) > s_{x_1} && (7) \end{aligned}$$

It follows

$$s_3' = (s_1 + s_2) - s_2' = s_1 + \frac{h_2}{\cos(\Gamma)} \tag{8}$$

And it is necessary that $s_4' < s_2 - h$, then exists $h_1 > h$ that $s_4' = s_2 - h_1$. However $M'E_\Gamma$ for the second part of parallel parking is that, what s_{x_1} for the first part is and

$$M'E_\Gamma \quad = \quad s_{x_2}' = s_3' + s_4' \cdot \cos(\Gamma) \tag{9}$$

and

$$\begin{aligned} s_3' + s_4' \cdot \cos(\Gamma) &= s_1 + s_2 \cdot \cos(\Gamma) && (10)\\ &\quad + \left(\frac{h_2}{\cos(\Gamma)} - h_1 \cdot \cos(\Gamma)\right) && (11) \end{aligned}$$

That means

$$s_{x_2}' \quad = \quad s_{x_1} + \left(\frac{h_2}{\cos(\Gamma)} - h_1 \cdot \cos(\Gamma)\right) \tag{12}$$

For h_1, h_2 so that $\left(\frac{h_2}{\cos(\Gamma)} - h_1 \cdot \cos(\Gamma)\right) < 0$ it is impossible since s_{x_1} is minimal. In another case, $\left(\frac{h_2}{\cos(\Gamma)} - h_1 \cdot \cos(\Gamma)\right) > 0$ follows

$$\begin{aligned} & s_{x_1}' + s_{x_2}' \cdot \cos(\Gamma) && (13)\\ = \quad & s_{x_1} + (h - h_2) + && (14)\\ & \left(s_{x_1} + \frac{h_2}{\cos(\Gamma)} - h_1 \cdot \cos(\Gamma)\right) \cos(\Gamma) && (15)\\ > \quad & s_{x_1} + s_{x_1} \cdot \cos(\Gamma) + (h - h_2) && (16)\\ > \quad & s_{x_1} + s_{x_1} \cdot \cos(\Gamma) && (17) \end{aligned}$$

This is impossible, see figure 4.

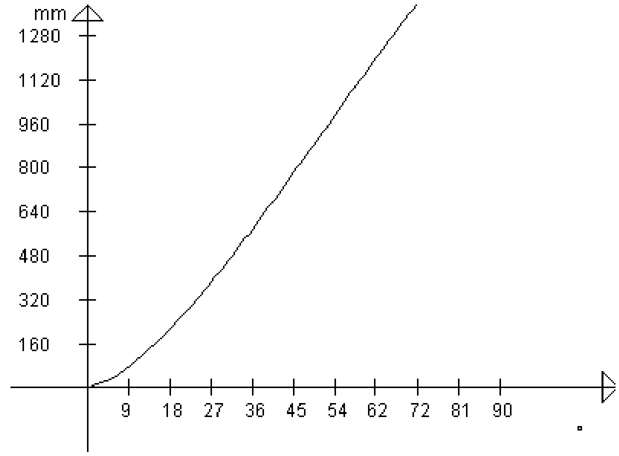

Fig. 5. Dependence of the depth s_y on gain Γ.

Similarly we get a contradiction, if M' is in another allowed position.

The minimization from both phases of the parking process together minimizes distance s_x

$$s_x = (s_1 + s_2) \cdot (1 + \cos(\Gamma)) \tag{18}$$

among all maneuvers, for which the inflecting configuration has gain Γ. Lengths of s_1 and s_2 are obviously dependent on Γ. Gain Γ is the maximal angle, that can be achieved for the depth s_y without breaking accepted restrictions, see [11].

For any s_y (with the aforementioned restrictions) the gain Γ is uniquely defined. The function

$$s_y(\Gamma) = (s_1(\Gamma) + s_2(\Gamma)) \cdot \sin(\Gamma) \tag{19}$$

that mirrors the dependence between s_y and Γ is $one-to-one$ as shown on figure 5.

So, we have proved the existence of a minimum distance s_x for parallel parking and the principal strategy of steering to get there by reasonable parking box.

IV. Assessing different tractor trailer systems

Since the section above introduced a defined maneuver we can vary the geometric parameters of general-2-trailers. As seen in figure 6 the type t1- and t2-vehicles do not overlap. While the overall length $L_0 + L_1$ is kept constant three kinds of variations are applied. The first (A) refers to the coupling point, which is moved with respect to the tractors rear axle. The second variation (B) concerns the length of the tractor. The final variation (C) regarded in this paper is on the maximum steering angle of the tractor's front axle.

A. Protrusion M_0

M_0 is equal to zero, if the coupling point is above the middle of the tractors rear axle. A decreasing value M_0 moves the coupling point towards the tractors steering axle. This is a characteristic for semi-trailer tractors (t1) with fifth wheel kingpin. In contrast trucks with trailers (t2) have their coupling device behind the tractors rear axle. This is also true for cars with their different types of trailers. The

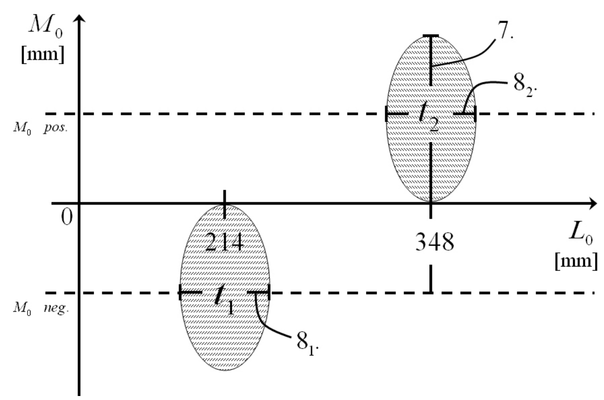

Fig. 6. Type t1 and type t2 vehicles have different characteristics with respect to the parameters L_0 and M_0. The variations of M_0 are depicted in figure 7 and that of L_0 in figure 8.

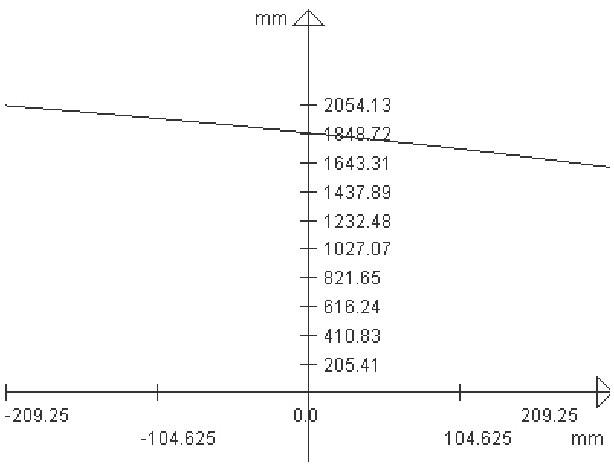

Fig. 7. Dependence of M_0 on parking distance s_x. The depth s_y of this parking process will be fixed.

diagram (figure 7) shows that the parking distance s_x slightly decreases when moving from t1 to t2.

B. The length of the tractor

Second, we analyze the impact of length L_0. For total length is kept constant, the length of the trailer L_1 decreases with increase of L_0. With respect to the parking distance we notice that firstly the increase of L_0 also increases the s_x and reduces the maneuverability (see figure 8). But after some maximum point where maybe $L_0 \approx L_1$ the parking distance s_x decreases again. So we can say, that there is some maximum parking distance s_x.
The diagram (figure 8) shows that these effects are similar for both types t1 and t2. Furthermore it can be seen that the principle maneuverability of a semi-trailer is better than of type t1 tractor trailer systems which results from to the fact that $L_0(t1)$ is typically smaller than $L_0(t2)$.
The abruption (a1) on the left side of the diagram (figure 8) occures, because there don't exists a stable circle drive.
The abruption (a2) on the right side however occures, because we break one of our conditions before. The trailer is so short and so mobile, that we will exceed the 90° deviation

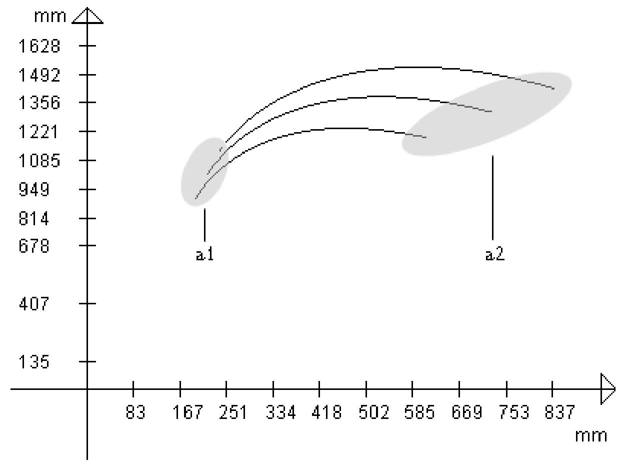

Fig. 8. Dependence of L_0 on parking distance s_x. The curve with the intermediate height corresponds to a value $M_0 = 0$. The upper curve is for type t1 and corresponds to the variation 8_1 in figure 6. The lower curve is for type t2 and corresponds to the variation 8_2 in figure 6.

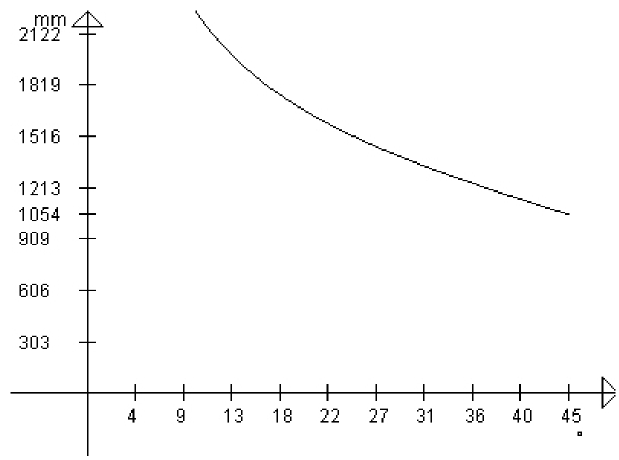

Fig. 9. Tradeoff between maximum steering angle and parking distance s_x.

too early.

C. Variation of maximum steering angle

Finally, we analyze how much influence comes from the maximum steering angle of the tractor. This angle is very relevant, because in the minimum parallel parking maneuver all paths are driven with maximum steering wheel at the stop. In contrast to the other relatively small impacts by variation (A) and (B) we have high potential here to increase maneuverability (see figure 9).

V. Applicative Results

In our laboratory we have got two experimental general-2-trailer models at a scale of $1:16$. They are autonomous in the sense that they plan their path and control their path following operations. As they have a difference in length of $25mm$ at a lengths of more than $800mm$ either we can consider them as comparable heavy goods vehicles.

The semi-trailer truck (t1, see figure 10) and the truck with the one-axle trailer (t2, see figure 11) have the

Fig. 10. Semi-trailer truck model (t1) executing a parallel parking maneuver.

Fig. 11. Truck with one-axle trailer model (t2) executing a parallel parking maneuver.

following longitudes:

	semi-trailer truck	truck with one-axle trailer
L_0	214 mm	348 mm
M_0	-12 mm	124 mm
L_1	610 mm	365 mm

They need for several parking depth s_y the following parking distance s_x:

s_y / **mm**	s_x / **mm**	
	semi-trailer truck	truck with one-axle trailer
200	1315.07	1365.54
300	1486.15	1562.62
400	1612.51	1719.26
500	1709.96	1848.28

The total length of both models differs only in 25 mm. So we can say, that our truck with one-axle trailer needs more parking distance s_x to park parallel than the semi-trailer truck model and so short tractors can be called more mobile than the long ones.

So we compare articulated vehicles under the aspect of parallel parking and we can adhere:

- the smaller M_0 the longer the parking distance s_x.
- for the variation upon L_0 there exists a maximum of parking distance and on both ends this distance decreses.
- the more maximum steering angle the merrier.

But a small value L_0 is more important than a positive M_0. Independent of these longitudes the maximum steering angle reduces the parking distance s_x significantly. But the three comparisons, we have made, have to be seen as qualitative results and don't include all sufficient conditions. We have to take notice of the safety hull of the articulated vehicle, where the border of our truck and trailer moves during the parallel parking process, because the overhang of this vehicle overshoots.

VI. Conclusion

On one hand this paper discloses some important tradeoffs between the vehicle geometry and the maneuver distances for general-2-trailers. On the other hand still more questions arise. So, we would like to know what are the right maneuvering strategies for the cases where due to the limitations of the steering angle it becomes impossible to follow the optimal maneuvering strategies. Furthermore, there is a vivid interest for a similar assessment of general-3-trailers which have a higher versatility with respect to their geometry.

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