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Estimating Queues at Signalized Intersections: Value of Location and Time Data from Instrumented Vehicles

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Abstract—**Vehicles instrumented with location tracking and wireless communication technologies (i.e., the so called probe vehicles) can serve as sensors for monitoring traffic conditions on transportation links. This paper is focused on estimating queue lengths in real-time at a signalized intersection approach based on the location and time data from probe vehicles that may constitute a given percentage of the total traffic. The paper also addresses the evaluation of the accuracy of such estimates. Using a virtual queuing model and conditional probability distributions, new expressions are derived for the variance of the estimates to understand how accuracy is affected by the percentage of probes in the traffic stream and by the type of information collected, which include (i) location of probes in the queue and (ii) both the location of probes and the times/instances at which they join the back-of-the queue. Numerical examples are presented to compare and contrast the accuracies of these two cases. The findings and the formulation presented in this paper could be used in evaluating and designing a traffic monitoring system that relies on probe vehicle data for queue length estimation at signalized intersections.**

I. INTRODUCTION

THIS paper investigates the estimation of queue length at signalized intersections based on data from vehicles instrumented with wireless communication and location tracking technologies (e.g., GPS). Estimating queue length in real-time enables optimal control through efficiently allocating the available capacity (i.e., green time) such that a defined performance metric is optimized (e.g., minimize total delays or minimize the maximum queue length). To estimate these performance measures in real time, various surveillance technologies are being employed today (e.g., inductive loops, video) to measure traffic flow parameters (e.g., volume, density) which are subsequently utilized in models for delay estimation/prediction. Such signal systems are called real-time traffic-responsive or traffic-adaptive control systems [1]. However, these detection technologies are not effective in estimating queue lengths. It is hoped that as the vehicle-based information collection technologies gain momentum, there will be great interest on capitalizing on the probe data for traffic monitoring. In order to support the development of these applications, research is needed to understand how probe vehicle technology could potentially improve the estimation of desired parameters. One key issue that pertains to the vehicle-based data collection systems is understanding the relationship between the market penetration (or the percentage of probe vehicle population) and the accuracy of the estimated parameters. The relationship between the accuracy of the queue length estimates and the probe percentage is explored in this paper.

It is assumed that probe vehicles can communicate (send information) to a roadside unit (e.g., signal controller) that uses the data for queue length estimation. The paper does not discuss the technologies for wireless communication neither the details of the information flow and network. It rather focuses on the impacts of such technologies on traffic data collection and system state estimation. Two important data elements that are assumed to be collected include the relative location of probe vehicles in the queue and the time instances when they join the back of the queue. The use of these data in queue length estimation is explained in the subsequent sections.

Earlier studies on vehicle probes and their application to traffic engineering deal with understanding the relationships between the market penetration and the reliability of the travel time estimates [2]-[4]. Network coverage is also an important issue that is addressed in the literature [5]-[7]. Due to the complexity of the problem, none of these studies develop analytical models or closed form solutions that relate the number of probes to the reliability of the estimates. Instead, empirical analyses are performed in these studies that require data to be generated for numerous scenarios with different probe vehicle percentages. Typically, data from microscopic traffic simulation models are used for that purpose since real-world data with a large number of probes to support such analyses are not available.

In this paper, analytical models are developed to assess how queue length estimation is influenced by the percentage of probe vehicles in the traffic stream. These models require the marginal probability distribution of queue length to be known. Even though this distribution may not be readily

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available, the proposed analytic approach is better than the simple empirical approaches cited above, which require data to be generated for numerous scenarios. The application of the proposed approach is illustrated through numerical examples for an isolated intersection with fixed signal timing. The arrivals are assumed to follow a Poisson distribution whereas the vehicles are assumed to queue vertically for simplicity.

It should be noted that there is a vast body of literature on queues at signalized intersections. Since fixed cycle traffic light allows a detailed analytical analysis, it has been studied by many researchers. One of the earliest studies is by Webster [8] who generated relationships for the number of stops and delays by simulating traffic flow on a one-lane approach to an isolated signalized intersection. In particular, the curve he fitted to the simulation results has been fundamental to traffic signal setting procedures since its development. Miller [9] found an approximation to the average overflow queue for any arrival and departure distributions. Later, Newell [10] derived an analytical approximation to the mean queue length for general arrivals. McNeill [11] derived a formula for the expected delay and approximate mean of the overflow queue length for general arrival process and constant departure rate.

Other than the studies on the estimation of mean, there have been some attempts to obtain the probability function for the queue length. Some researchers obtained the conditional probability distribution of the overflow queue at the end of one period given the queue length at the preceding period assuming homogenous Poisson arrival process [12]-[14]. Yet some others derived the probability generating function (p.g.f.) of the stationary overflow queue [15], [16] in the hope of obtaining the probability functions for delays and queue lengths. Obtaining a probability function from a p.g.f. involves inverting the p.g.f. function [17], and this inversion process is quite complicated and entails finding complex roots and numerical evaluations of parameters [18]. Therefore, the applicability of this procedure is very limited and complicated. Following similar methods presented in [12], Olszewski [19] used Markov chains to obtain the probability distribution of overflow queue, and developed a computer program that estimates the mean queue length and its variance under different conditions such as stationary and non-stationary arrival processes, and variable service rates.

II. Problem Statement

A. Definition

As mentioned above, this paper focuses on queue length estimation based on data from probe vehicles; where data include location/position of vehicles in the queue as well as the time stamps at which they join the queue. The authors in a recent study have investigated the use of only location information in queue length estimation [20]. This paper extends the models developed in [20] to account for the time stamp information. Hence, the problem is to determine, if any, the contribution of this new data in queue length estimation.

Figure 1 illustrates a snapshot of a signalized intersection approach at the end of a red phase. Solid rectangles represent probe vehicles. The main objective is to estimate the queue length, N, if the locations of probe vehicles in the queue and the times they joined the queue are known. To be more precise, N is a random variable to be predicted that represents the total number of vehicles accumulated in the queue at the end of red interval. It is assumed that the time and the location or position of probe vehicles in the queue can be measured. In Figure 1, there are three probe vehicles and the location of the last probe, L_p, is 8. The value of N is estimated in real-time at the end of each cycle (end of red period) based on time and the location of probe vehicles. Even though the models are developed for vertical queues, this figure is included to clarify the notation. In vertical queuing models, it is assumed that vehicles do not occupy space and can accelerate and decelerate instantaneously.

The models developed in this paper are based on the assumption that only time stamps and locations of probes are observable. As it is demonstrated in [20], L_p is sufficient for estimating the queue length (when the estimate is based only on the location data); neither locations of other probe vehicles nor the total number of probes vehicles in the queue (N_p) are needed. This result was obtained when it is assumed that the percentage of probe vehicles in the traffic stream (p) and the marginal distribution of N are known.

In order to estimate N in real-time, the conditional expectation of N given the probe information is needed. The next section presents the formulation whereas Section IV provides examples and analysis.

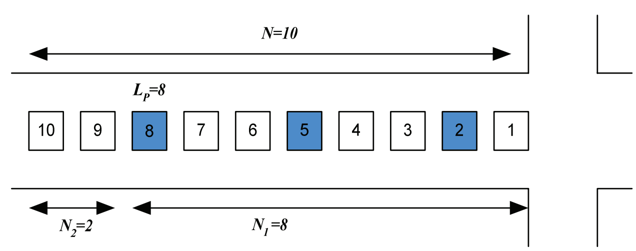

Figure 1. Snapshot of an intersection right before the red interval terminates. The positions of vehicles in the queue are measured from the stop bar.

B. Notation

The following notation will be used in the development of the models.

R = red period in seconds.

p = the proportion of probe vehicles in the entire vehicle population.

λ = arrival rate per second– number of arrivals per cycle. Arrivals are independent identically distributed (i.i.d.) with rate λ. Probe vehicles arrive with rate λp, while other vehicles arrive with $\lambda(1-p)$.

β = the mean inter-arrival time (seconds), $1/\lambda$.

N = total queue length at the end of the red period.

N_1 = queue length measured from the stop bar until the location of last probe (including the last probe).

N_2 = queue after the location of last probe that is the number of arrivals from the time of last probe until the end of red period.

L_P = location of last probe (equal to N_1).

N_P = total number of probes in the queue.

T_P = the clock time last probe joins the back of the queue measured relative to the beginning of red.

X_i = independent identically distributed inter-arrival times.

III. ANALYTICAL FORMULATION

In this section, queue length estimation under two cases is investigated. In the first case, the only data available/collected from probe vehicles is the location of probes in the queue. For the first case, only a short summary is provided below since the details can be found in [20]. In the second case, both location data and the time instances the probes join the queue are utilized for queue length estimation.

A. Estimating Queue Length Based on Location Data

In order to estimate the total queue length from location data, one needs to formulate the conditional distribution of the queue length given the locations of probes. For any given arrival (or queue length) distribution, the derivation is illustrated below.

Assuming that the percentage of probe vehicles (p) is known, the relationship between the total number of probe vehicles (N_p) and the total vehicles in the queue (N) at the end of red period can be written as follows.

$$N_p = \sum_{i=0}^{N} y_i \quad where\ y_i \in \{0,1\},\ y_0 = 0, P(y_i = 1) = p \quad (1)$$

The equation above asserts that every vehicle has an equal probability of being a probe vehicle. Both N and N_p are two discrete random variables. The formulation is for a general probability mass function (p.m.f.) for N, which is denoted by $P(N{=}n)$. Given $N = n$, it follows that the number of probe vehicles in the queue has the following binomial distribution.

$$P(N_p = n_p | n, p) = \binom{n}{n_p} p^{n_p} (1-p)^{n-n_p} \quad (2)$$

$$for\ 0 \le p \le 1,\ 0 \le n_p \le n$$

Given $N{=}n$ and $N_p = n_p$, the probability distribution for the location of last probe (L_p) can be derived by considering the number of possible combinations. It can be verified that the conditional probability of L_p given n and n_p is as follows.

$$P(L_p = l_p | n, n_p) = \frac{\binom{l_p - 1}{n_p - 1}}{\binom{n}{n_p}}, \quad (3)$$

$$for\ l_p = n_p, n_p + 1, n_p + 2, \ldots, n$$

As mentioned before, the main purpose is to estimate the conditional distribution of the queue length given the location of last probe and the total number of probes, i.e., $P(N{=}n|l_p, n_p)$. This conditional distribution can be written by using the Bayes' rule:

$$P(N = n | l_p, n_p) = \frac{P(L_p = l_p | n_p, n) P(N_p = n_p | n) P(N = n)}{P(L_p = l_p, N_p = n_p)} \quad (4)$$

$$P(N = n | l_p, n_p) = \frac{P(L_p = l_p | n_p, n) P(N_p = n_p | n) P(N = n)}{\sum_{n=l_p}^{\infty} P(L_p = l_p | n_p, n) P(N_p = n_p | n) P(N = n)} \quad (5)$$

Substituting the respective equations and performing some simplifications, the following conditional distribution function for the queue length is obtained.

$$P(N = n | l_p, n_p) = \frac{(1-p)^n P(N = n)}{\sum_{n=l_p}^{\infty} (1-p)^n P(N = n)} \quad (6)$$

$$for\ n \ge l_p$$

As it turns out, this probability only depends on the location of last probe (l_p) and does not depend on the total number of probes in the queue (n_p). It should be noted that equation (6) is valid for any probability distribution of N, i.e. for any arrival distribution.

Based on the above conditional probability distribution, the expected queue length can be easily computed as follows.

$$E(N = n | l_p) = \sum_{n=l_p}^{\infty} n P(N = n \mid l_p) = \sum_{n=l_p}^{\infty} n \frac{[(1-p)]^n P(N = n)}{\sum_{k=l_p}^{\infty} [(1-p)]^k P(N = k)} \quad (7)$$

The equation above gives the best prediction of the queue length given the probe location information. For real-time applications, this equation can be used to estimate the queue length, provided that $P(N{=}n)$ is known.

In order to assess how accuracy of this estimated queue length changes by the proportion of probe vehicles (p), some additional results are obtained. First, the conditional variance of queue length (given l_p) can be written as follows.

$$VAR(N = n|l_p) = \sum_{n=l_p}^{\infty} n^2 \frac{[(1-p)]^n P(N=n)}{\sum_{k=l_p}^{\infty}[(1-p)]^k P(N=k)} - [E(N=n|l_p)]^2 \tag{8}$$

Since the above variance depends on l_p, a more general measure that does not depend on l_p is needed to determine the effects of p on accuracy. The error in the estimates, denoted by D, can be treated as a random variable that is the difference between the actual queue length (N) and its estimate:

$$D = N - E(N = n|l_p) \tag{9}$$

If the expectation of both sides of this equation is taken, it can be seen that the expected error is zero (Since E[E($N|l_p$)] = E(N)). On the other hand, the variance of error, shown in (10), is obtained by the law of iterated expectations:

$$Var(D) = Var[N - E(N = n|l_p)] = E[Var(N = n \mid l_p)] \tag{10}$$

The expected value on the right hand of this equation can be calculated as follows.

$$E(Var(N = n|l_p)) = \sum_{l_p=0}^{\infty} P(L_p = l_p) * VAR(N = n|l_p) \tag{11}$$

This expected value in essence represents the weighted variance over all possible values of l_p. To calculate this weighted variance, the marginal probability distribution of L_p is needed. This marginal distribution can be readily written in terms of the conditional distributions obtained thus far.

$$P(L_p = l_p) = \sum_{n_p=1}^{l_p}\sum_{n=l_p}^{\infty} P(L_p = l_p|n, n_p) P(N_p = n_p|n) P(N = n) \tag{12}$$

Then, the marginal distribution of L_p becomes (for $L_p>0$),

$$P(L_p = l_p) = \sum_{n_p=1}^{l_p}\sum_{n=l_p}^{\infty} \binom{l_p - 1}{n_p - 1} p^{n_p}(1-p)^{n-n_p} P(N = n) \tag{13}$$

The formulation presented above can be utilized to assess the relationship between the percentage of probes (p) and the accuracy of the estimated queue lengths. The main input required is a probability function for the total number of vehicles in the queue, $P(N)$.

B. Estimating Queue Length Based on Location and Time Data

In the previous section it is shown that the location of the last probe (L_p) is sufficient in estimating queue length. In this section, the time instance at which the last probe joined the back of the queue (T_p) is also utilized in the estimation. Inclusion of time data adds to the complexity of the formulation since the probability distribution of time T_p also needs to be considered. To make the derivation manageable, it is assumed that the arrivals follow a Poisson distribution. In addition, the formulation is carried out for a simplified situation where the overflow queue (the leftover queue from a previous cycle) is assumed to be zero. A more general case where the overflow queue is also modeled will be considered in the future.

The total queue length, N, can be written as the sum of two queue lengths, N_1 and N_2, by the assumption of iid Poisson arrivals. Thus, the total queue length given the location and time of the last probe can be expressed as follows.

$$N|L_p, T_p = N_1|L_p, T_p + N_2|L_p, T_p \tag{14}$$

The expected conditional total queue length becomes,

$$E(N|L_p, T_p) = E(N_1|L_p, T_p) + E(N_2|L_p, T_p) \tag{15}$$

The first conditional expectation is constant since L_P is given, and the expected value of N_2 corresponds to the number of arrivals in the given time interval that is equal to R-Tp, the time period during which no probe vehicle has arrived. Therefore, the arrival rate during this period is $\lambda(1-p)$.Then, N2 is distributed with *POI* $(\lambda(1-p)(R- T_P))$. Thus the overall expected value given in (15) becomes,

$$E(N|L_p, T_p) = l_p + (1-p)\lambda(R - t_p) \tag{16}$$

Similar to the expression in (7), the above expectation is the best predictor of the queue length given both location and time information of the last probe vehicle. In order to assess how accuracy of this estimate changes by the proportion of probe vehicles (p), some additional results need to be obtained as well. First, the conditional variance of the queue length (given T_p and L_p) can be written as follows.

$$VAR(N|L_p, T_p) = (1-p)\lambda(R - t_p) \tag{17}$$

Since the first term in (14) is constant ($N_1= l_p$) its variance is zero. The second term in (14) specifies a Poisson distribution with a parameter $\lambda(1-p)(R-t_p)$, which gives the conditional variance in (17). Similar to the analysis performed in the previous section, the variance of the difference D shown below is needed to assess how error is changing with respect to the percentage of probes, p.

$$D = N - E(N|L_p ,T_p) \tag{18}$$

The variance of D, for $L_p>0$, is then as follows.

$$VAR(D) = VAR[N - E(N|L_p ,T_p)] \\ = E[VAR(N|L_p ,T_p)] \tag{19}$$

After substituting the conditional variance by the expression in (17),

$$VAR(D) = E[(1 - p)\lambda(R - T_p)] \\ = (1 - p)\lambda(R - E(T_p)) \tag{20}$$

The expected value of T_p, $E(T_P)$, is needed to compute the variance given above. Even though the marginal probability distribution of T_p is not known to compute this expected value, it is not difficult to define the conditional probability distribution of T_P given L_P, $f(T_p|L_p)$. Since the arrivals are Poisson the interarrival times (X_i) are i.i.d. Exponential with parameter $\beta = 1/\lambda$. Then, this conditional distribution is the sum of L_P Exponentials, which is equivalent to a Gamma distribution with parameters (L_P, β).

$$f(T_p|L_p) = f(X_1 + X_2 + ... + X_{l_p}) \\ = Ga(l_p ,\beta)$$

Thus,

$$E(T_p) = E(E(T_p|L_p)) = E(\beta L_p) = \beta E(L_p) \tag{21}$$

The expected value of L_p in (21) can be obtained from the marginal probability distribution of L_p given in (13).

It should be noted that the formulation above is relevant when $L_p > 0$, i.e., when there is at least one probe vehicle in the queue. When there is no probe at all, then the estimated queue length will be equal to $(1-p)\lambda R$, which is equal to the expected number of arrivals other than probe vehicles. In that case, the error of the estimate will also be equal to $(1-p)\lambda R$ because of the property of Poisson distribution. Therefore, the overall variance will be the weighted average given below.

$$VAR(D) = (1 - p)\lambda(R - \beta E(L_p))(1 - P(L_p = 0)) \\ + (1 - p)\lambda R(P(L_p = 0)) \tag{22}$$

Where, $P(L_p=0) = \exp(-\lambda(p)R)$.

IV. Numerical Experiments

In order to show how the overall error behaves with the percentage of probe vehicles (p) an example case is illustrated. The queuing at an intersection approach where the overflow queue is assumed to be zero is considered. The red duration (R) is assumed to be 45 seconds whereas the average interarrival time is 4.5 seconds. Therefore, the arrival rate during the red interval, λR, is 10 vehicles. Using the formulation developed in the previous section, the overall variance of D (the difference between the estimated and the actual queue length) is computed for both cases; see (11) and (22). The results for various probe percentages are plotted in Figure 2 for both cases. As anticipated, the error decreases with increasing p and becomes zero when p approaches to 100%. In addition, when there is no probe vehicle the variance is equal to the variance of the Poisson process as one would expect. The more important result is the fact that error is always smaller when both location and time information is utilized in the estimation. The absolute difference between the two is largest at about 25-30% and then diminishes when p changes in both directions. Even though this example ignores the overflow queue, the overall conclusions would not be affected in a more realistic case where overflow queue is also represented [20].

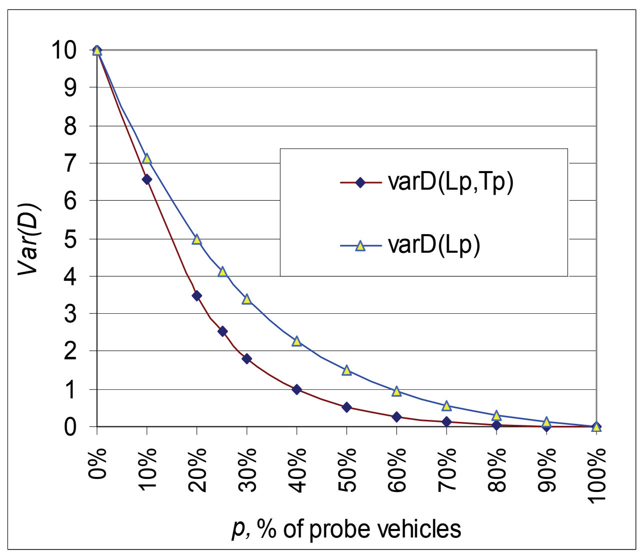

Figure 2. Variance of the difference between the estimated and actual queue length. The curve labeled varD(Lp) is for the scenario where only location of the last probe vehicle is utilized in queue length estimation. The second curve, varD(Lp, Tp), represents the variance when both location and time of the last probe vehicle in the queue are utilized in the estimation model.

V. Conclusion

This paper presents a statistical formulation for real-time estimation of queue length at a signalized intersection approach from probe vehicle data. The presented formulation allows evaluation of the accuracy of estimates analytically for Poisson arrivals. The models are developed to estimate queue length from the location of probe vehicles as well as from data both on location and time of arrivals. It is found that using both time and location information provides relatively more accurate estimates. The formulation presented could be used in evaluating and designing a traffic monitoring system that relies on probe vehicle data for queue length estimation at signalized intersections. Some future research is needed to generalize the formulation to more realistic settings (e.g., including overflow queue, unknown prior knowledge of $P(N)$, oversaturated and time dependent cases).

REFERENCES

[1] Gartner N. H., F. J. Pooran, and C. M. Andrews. Optimized Policies for Adaptive Control Systems: Implementation and Field Testing. *Transportation Research Record*, No. 1811, 2002, pp. 148-156.

[2] Cetin, M., G .F. List, and Y. Zhou. Factors Affecting the Minimum Number of Probes Required for Reliable Travel Time Estimation. *Transportation Research Record*, No. 1917, 2005, pp. 37-44.

[3] Cheu, R.L., C. Xie, and D. Lee. Probe Vehicle Population and Sample Size for Arterial Speed Estimation. Computed-Aided Civil and Infrastructure Engineering, 17(1), 2002, pp. 53-60.

[4] Chen, M., and S.I.J. Chien. Determining the number of probe vehicles for freeway travel time estimation by microscopic simulation. *Transportation Research Record*, No. 1719, 2000, pp. 61-68.

[5] Srinivasan, K.K., and P.P. Jovanis. Determination of Number of Probe Vehicles Required for Reliable Travel Time Measurement in Urban Network. *Transportation Research Record*, No.1537, 1996, pp. 5-22.

[6] Turner, S., and D. Holdener. Probe Vehicle Sample Sizes for Real-Time Information: The Houston Experience. *Proceedings of the Vehicle Navigation & Information Systems Conference*, Seattle, WA, United States, 1995, pp. 3-10.

[7] Boyce, D. E., J. Hicks, and A. Sen. In-Vehicle Navigation Requirements for Monitoring Link Travel Times in a Dynamic Route Guidance System. *Operations Review*. 8 (1), 1991, pp. 17-24.

[8] Webster, F. V. Traffic Signal Settings. Road Research Laboratory Technical Paper, No. 39, HMSO, London, 1958.

[9] Miller, A. J. Settings for Fixed-Cycle Traffic Signals. *Operations Research Quarterly*, Vol. 14, 1963, pp. 373-386.

[10] Newell, G. F. Approximation Methods for Queues with Application to the Ficed-Cycle Traffic Light. *SIAM Review*, Vol. 7, 1965.

[11] McNeil, D.R. A Solution to the Fixed Cycle Traffic Light Problem for Compound Poisson Arrivals. *Journal of Applied Probability*, 5, 1968, pp. 624-635.

[12] Haight, F. A. Overflow At A Traffic Flow. *Biometrika*. Vol. 46, Nos. 3 and 4, 1959, pp. 420-424.

[13] Heidemann, D. Queue Length and Delay Distributions at Signalized Intersections, *Transportation Research Part B*, 28 B, No. 5, 1994, pp. 624-635.

[14] Mung K.S.G., A.C.K. Poon, and W.H.K. Lam. Distributions of Queue Lengths at Fixed Time Traffic Signals, *Transportation Research Part B*, Vol. 30, No. 6, 1996, pp. 421-439.

[15] Meissl, P. Zufallsmodell einer Lichtsignalanlage mit Mehrspurigem Stauraum. *Mathematik-Technik-Wirtschaft*, 10 (1), 1-4 amd (2), 1963, pp. 63-68.

[16] Darroch, J. N. On the Traffic-Light Queue. Ann. Math. Statistics, 35, 1964, pp. 380-388.

[17] Van Leeuwaarden, V.J.S.H, Delay Analysis for the Fixed-Cycle Traffic-Light Queue, *Transportation Science*, 40 (2), 2006, pp. 189-199.

[18] Abate, J., and W. Whitt. Numerical Inversion of Probability Generating Functions, *Operations Research Lette*rs, 12, 1992, pp. 245-251.

[19] Olszewski, P. Modeling of Queue Probability Distribution at Traffic Signals. Transportation and Traffic Theory, ed M.Koshi, Elsevier, Amsterdam, 1990, pp. 569-588.

[20] Comert, G., and M. Cetin, "Queue Length Estimation from Probe Vehicle Location: Unsaturated Conditions," *Transportation Research Record*, submitted for publication, 2007 (Also in the *proceedings of 2007 Annual TRB Meeting, Jan 21-25, 2007, Washington DC.*).