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Performance comparison of collision avoidance controller designs

Geraint P. Bevan, Simon J. O'Neill, Henrik Gollee and John O'Reilly

Centre for Systems and Control, University of Glasgow
Glasgow G12 8QQ, Scotland

{g.bevan, s.oneill, h.gollee, j.oreilly}@eng.gla.ac.uk

***Abstract*— A comparison is made between two vehicle control strategies for two different manoeuvres: a gentle and aggressive lane-change. Simulation results demonstrate that the choice of control objectives and selection of appropriate design approximations have a significant impact on the performance of the controller under these different manoeuvre conditions. A lateral control design trade-off between passenger comfort and collision avoidance capability is evident.**

I. Introduction

Control of vehicle dynamics has been a research area of great interest in recent years. The availability of brake-by-wire technology has been a significant factor leading to deployment on high- and mid-range vehicles of driver assistance systems, such as anti-lock braking, traction control and electronic stability programmes [1].

As the introduction of steer-by-wire systems becomes more practical, new possibilities arise for vehicle control engineers. Four-wheel steering can be used to achieve simultaneous control of vehicle sideslip, yaw rate and lateral velocity by means of system decoupling, e.g. [2], [3]. Vehicle yaw rate is controlled in [4] by generating a control moment using the vehicle steer-by-wire system, while a disturbance observer is used to take account of any disturbances acting on the front wheels.

The integration of steering and braking systems is of increasing importance as engineers seek to extend the limits of vehicle performance beyond that which can be accomplished by considering subsystems in isolation. In particular, work by Burgio et al. [5] uses a non-linear tyre model and a two degree of freedom vehicle model with feedback linearisation techniques to control vehicle yaw rate, while Cherouat et al. [6] use a simplified, linear approximated tyre model to design a feedback controller to control yaw rate and longitudinal velocity. A highly nonlinear vehicle model which includes saturations and nonlinear tyre model is used to derive a sliding mode controller in [7], demonstrating a controller that enables vehicle yaw rate to be controlled for a given longitudinal velocity and radius of curvature.

In this paper, two approaches to controlling the lateral dynamics of a vehicle equipped with brake-by-wire and steer-by-wire systems are undertaken: the first to investigate the potential for using automatic braking for lateral control during gentle lane change manoeuvres; the second as an emergency lateral collision avoidance controller. The differing nature of the tasks to be performed by the controllers leads to differences in the assumptions made during the design process and different choices of control objectives. However, the design objectives are sufficiently similar that it is reasonable to compare the performance of both controllers for each task. The comparison demonstrates that the choice of variables to control plays a significant role in the suitability of a controller for its intended purpose.

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Section II describes the controller objectives for each controller, after which the design architecture and methods of the controllers are explained in Section III. Simulation results are then presented in Section IV, followed by discussion and conclusions.

II. Controller objectives

A. A gentle lane-change controller

The design objective for controller A is to simultaneously control the yaw rate and sideslip of a vehicle during a gentle lane-change manoeuvre. The controller is intended to apply small steering inputs to control direction and individual wheel braking to maintain stability of a vehicle that is operating close to equilibrium conditions. Passenger comfort should not be jeopardised by the automatic control inputs. Yaw rate is controlled so that the vehicle can be made to follow an intended path.

At the same time, it is important to regulate vehicle sideslip as this could be increasing despite controlled yaw rate, indicating that the vehicle is sliding and potentially unstable.

B. An aggressive lane-change controller

The design objective for controller B is to perform an emergency lane-change collision avoidance manoeuvre, for use when there is insufficient space for a longitudinal collision avoidance system (i.e. automatic braking) to prevent an impending crash with an obstacle ahead. In order that the system is not activated unnecessarily, it is desirable that it should not be operated until the last possible moment, thus imposing a requirement that the controller should cause the vehicle to manoeuvre at its physical limits. In an emergency scenario, passenger comfort is necessarily only a minor consideration compared to safety. Thus it is to be expected that an emergency lateral collision avoidance system will apply large aggressive inputs to a vehicle operating at the extremes of its dynamic envelope, far from equilibrium.

III. Controller design

A. A linear design method for gentle lane-changing

Two controls inputs are used for controller A: feedforward steering to generate the desired yaw rate $\dot{\psi}$ and feedback braking to correct any yaw rate errors and to control vehicle sideslip angle β.

Table I: List of symbols

Symbol	*Description*	*Units*
	Vehicle symbols	
α	tyre slip angle	rad
β	vehicle sideslip angle	rad
δ	wheel steering angle	rad
ψ	vehicle yaw angle	rad
μ	road/tyre friction coefficient	
b_l, b_r	lat'l distance from CG to wheel (left, right)	m
c	tyre cornering stiffness	N/rad
f_x	longitudinal (brake) tyre force	N
f_y	lateral (cornering) tyre force	N
g	gravitational acceleration	m/s^2
l_f, l_r	long'l distance from CG to axle (front, rear)	m
	Controller A symbols	
δ_0	feedforward steering angle	rad
λ	desired closed-loop pole	
A	state matrix	
$\tilde{A}$	diagonalised state matrix	
B	input matrix	
G	augmented plant	
K	state feedback gain matrix	
T	transformation matrix	
u	input vector	
x	state vector	
	Controller B symbols	
Δ	feedback steering angle	rad
δ_0	feedforward steering angle	rad
ϕ	velocity feedback control signal	
B_f	linearised input matrix	
e_{pos}	position error	m, rad
e_{vel}	velocity error	m/s, rad/s
K_b	velocity feedback gain matrix	
K_Δ	position feedback gain matrix	

The feedforward steering control is derived from a linear relationship involving yaw rate and steering angle, which is described in Section III-A.5, while the feedback controller is designed using pole-placement. The design is based on a linear vehicle model which is derived below. Cross-state feedback is used on the state matrix to obtain a diagonal feedback controller.

1) Linear vehicle model: A linear two-track model is used to design the linear feedback controller. Several linearising assumptions are made with respect to the tyre forces at the tyre-road interface. Longitudinal and lateral forces are included in the model, together with the two vehicle states $\dot{\psi}$ and β and the front wheel steering angle δ. The vehicle geometry is shown in Fig. 1 and listed in table I. During normal driving situations (i.e not at the car's physical limits) the lateral force f_y acting on any tyre can be assumed to depend linearly on the tyre slip angle α, [8]

$$f_y = c\,\alpha \tag{1}$$

where c is the cornering stiffness. If the lateral velocity v_y is small compared to the forward velocity v_x, from Fig. 1, vehicle sideslip angle β can be approximated as

$$\beta = \frac{v_y}{v_x} \tag{2}$$

The tyre slip angle α and steering angle δ are both assumed to be small. Consequently, the steering effect on the longitudinal wheel forces can be neglected, enabling a linear model to be formed with the four longitudinal braking forces and the wheel steering angle as the inputs. Only front wheel steering is considered in this work, so

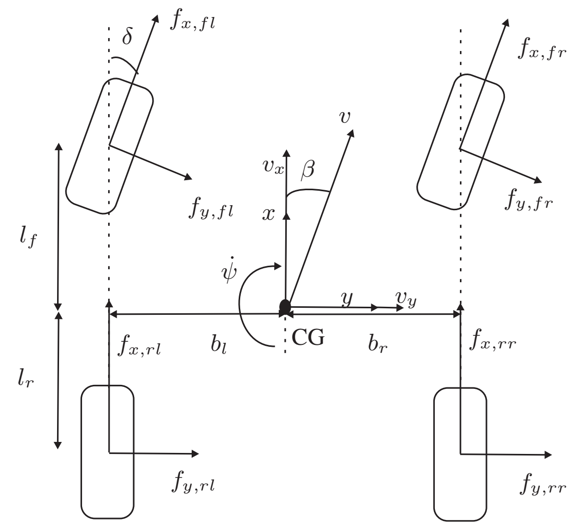


Figure 1: Two track vehicle model

the rear steering angle is set to zero. Further assumptions simplify the controller design: the vehicle lateral velocity v_y is assumed to be constant and positive; the vehicle longitudinal velocity v_x is assumed to be greater than zero; and vertical dynamics are not considered in the model.

The linear vehicle model is represented in state space form:

$$\dot{x} = Ax + Bu \tag{3}$$

where both states are measurable system outputs and

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} \qquad u = \begin{bmatrix} \delta \\ f_{x,fl} \\ f_{x,fr} \\ f_{x,rl} \\ f_{x,rr} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{-(2c_f+2c_r)}{m\,v_x} & \frac{-2c_f\,l_f+2c_r\,l_r}{m\,v_x^2}+1 \\ \frac{2c_f\,l_f-2c_r\,l_r}{Jz} & \frac{-2c_f\,l_f^2-2c_r\,l_r^2}{v_x\,Jz} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{-c_f}{m\,v_x} & \frac{-v_y}{m\,v_x^2} & \frac{-v_y}{m\,v_x^2} & \frac{-v_y}{m\,v_x^2} & \frac{-v_y}{m\,v_x^2} \\ \frac{l_f\,c_f}{J_z} & \frac{-b_l}{J_z} & \frac{b_r}{J_z} & \frac{-b_l}{J_z} & \frac{b_r}{J_z} \end{bmatrix}$$

The state matrix A and the input matrix B are parametrised by the vehicle mass m, moment of inertia J_z about its yaw axis, the longitudinal distances, l_f and l_r, from the centre of gravity (CG) to the forward and rear axles, respectively, and the lateral distances from the CG to left and right wheels, b_l and b_r.

2) Input Transformation: It is desired to control both of the state variables $\dot{\psi}$ and β, while five actuators are to be used for the task (one steering angle and four braking forces). It is therefore evident that a control allocation problem needs to be solved. An input transformation matrix $T \in \mathbb{R}^{2\times 4}$ works effectively, where T is a constant-unity gain matrix, the signs of the elements of which depend on the wheel configuration. The matrix may be considered to comprise two parts: one part $T_\beta \in \mathbb{R}^{1\times 4}$ to control sideslip and the other $T_{\dot{\psi}} \in \mathbb{R}^{1\times 4}$ to control yaw rate.

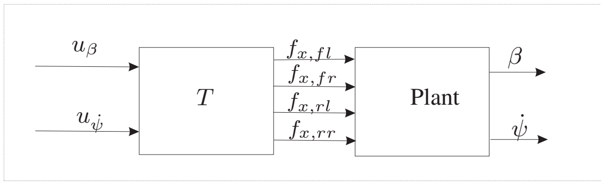


Figure 2: Concept of input transformation

In order to relate the braking forces on the four wheels to the two state variables, differential braking is used to induce a yawing moment while braking both sides equally will change the longitudinal velocity of the vehicle, resulting in a change in sideslip angle. Thus T is configured as

$$T = \begin{bmatrix} T_\beta \\ T_{\dot\psi} \end{bmatrix}' = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & +1 & -1 & +1 \end{bmatrix}' \tag{4}$$

and placed before the plant, as illustrated in Fig. 2.

3) Cross-State Feedback: The two states are highly coupled, cf. eq. (3). In order to reduce this system coupling, cross state feedback is used to reduce the state matrix A to the diagonal matrix

$$\tilde{A} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$$

This is achieved by subtracting from the control inputs, the product

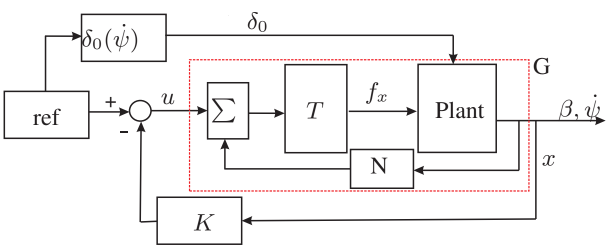


Figure 3: Complete control design problem for Controller A

of the two states and the expression $N = (A - \tilde{A})\,(B_f\,T)^{-1}$, thus creating a new diagonalised plant G, which is controlled by the controller K, as shown in Fig. 3, where B_f is that part of the input matrix relating to the four brake forces.

4) Feedback Control: Full state feedback control is used for braking. The feedback gain matrix K is found using a pole-placement technique, driving the open-loop poles to desired closed-loop locations. Note that, for the design, the modified plant G is used, which includes the transformed input matrix BT and the diagonalised state matrix $\tilde{A}$.

5) Feedforward Control: Feedforward control is used for the steering, to obtain quick response times while avoiding the 'delay' associated with feedback control. The feedforward steering effort drastically reduces the control effort of braking in the feedback path. From [6] a relationship between yaw rate and steering angle is given as

$$\dot\psi_{ref} = \frac{a\,v_x}{1 + b\,{v_x}^2}\,\delta \tag{5}$$

where a and b are constants,

$$a = \frac{1}{l_f + l_r} \qquad b = \frac{m}{(l_f + l_r)^2}\left(\frac{l_r}{c_f} - \frac{l_f}{c_r}\right)$$

Eq. (5) can be rearranged to obtain the feedforward steering command δ_0 from the desired yaw rate $\dot\psi_{ref}$,

$$\delta_0 = \dot\psi_{ref}\,\frac{1 + b\,{v_x}^2}{a\,v_x} \tag{6}$$

The overall controller architecture is shown in Fig. 3.

B. A nonlinear method for aggressive lane-changing

Classical approaches to linearising vehicle dynamics, as used for controller A, are not particularly well suited to creating models that are valid when operating far from equilibrium conditions. Controller B was therefore designed using nonlinear models as part of a simulation-based design. The controller architecture shown in Fig. 4 has four main elements: a trajectory generator; a feedforward steering loop; a feedback braking loop; and a feedback steering loop.

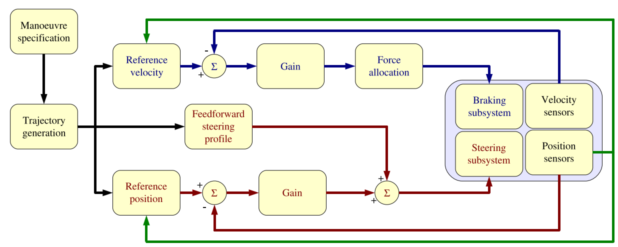


Figure 4: Controller architecture for Controller B: an emergency lateral collision avoidance system

The trajectory generation routine calculates a feasible trajectory that causes the vehicle to avoid specified obstacles, providing reference positions and velocities (longitudinal, lateral and yaw) to the rest of the controller.

As part of an emergency collision avoidance system, the trajectory generation routine attempts to find a path that moves the vehicle out of danger as soon as possible. The maximum force that may be exerted between a vehicle tyre and the road is a complex nonlinear function of tyre slip that is highly dependent on the particular characteristics of the tyre and the road and tyre conditions. However, if it is desired to eliminate the parametric uncertainties associated with detailed tyre models, it may be assumed that the maximum acceleration that a vehicle is capable of generating is approximately $\mu g[\mathrm{m/s^2}]$ where μ is the local friction coefficient and g is the acceleration due to gravity. This traction saturation limits the maximum achievable centripetal acceleration and hence the minimum radius of curvature for a turn by the vehicle. The trajectory generator creates a trajectory consisting of minimum radius turns connected by straight sections. Having defined a trajectory, a reference yaw rate profile is calculated by demanding that the vehicle remain tangential to the reference trajectory, i.e. $\psi = \arctan\frac{dy}{dx}$ throughout the manoeuvre.

The main control effort results from the feedforward steering loop which calculates a nominal steering angle δ_0 from the reference trajectory using an inverse of a simple linear bicycle model (Ackerman steering).

Of the two feedback loops, the braking loop is the most important during the transient part of the manoeuvre. The errors in the vehicle lateral and yaw velocities are fed to a proportional controller to produce a control effort $\phi = K_b e_{vel}$ where K_b is a gain matrix and e_{vel} is the vector of velocity errors. Lateral velocity is chosen instead of sideslip (*c.f.* III-A.1) to avoid introducing an unnecessary nonlinearity, i.e. the trigonometric function, $\arctan$, which cannot be approximated away if its argument is not small.

Allocation of the control effort among the four brake actuators is accomplished using a pseudo-inversion of a velocity-based linearisation [9], [10] of the vehicle dynamics, $f_x = B_f^{\dagger}\phi$.

Although close control of the vehicle velocity may cause the vehicle to exhibit the required transient behaviour, disturbances, sensor noise and unmodelled dynamics will prevent it from finding and keeping the target lane unless there is some position feedback. This is the purpose of the steering feedback loop, which adds a steering angle $\Delta = K_\delta e_{pos}$, where K_δ is a simple gain and e_{pos} is the lateral position error, to the feedforward steering angle to generate a total front-wheel steering angle $\delta = \delta_0 + \Delta$.

C. Controller comparison

Both controllers use feedforward steering to cause the car to follow a desired trajectory, but the reference trajectory calculated by each differs significantly. Controller A creates a gentle trajectory by passing a step function through a low pass filter, whereas Controller B calculates an aggressive trajectory that is designed to operate the vehicle close to its physical limits.

Both controllers use feedback control of the brakes, however the signals controlled differ significantly; Controller A uses the brakes to control both vehicle yaw rate $\dot{\psi}$ and vehicle sideslip β. In contrast, Controller B uses the brakes to achieve fine control of the vehicle velocity (lateral and yaw), without regulating the sideslip. An additional feedback steering loop exists in Controller B for the purposes of acquiring and keeping the centre of a new lane.

IV. Results

Simulations were performed to evaluate the performance of each controller, gentle Controller A and aggressive Controller B. Each controller attempted two lateral manoeuvres: a gentle single-lane change, representative of normal driving conditions; and a severe double lane-change, representative of an emergency collision avoidance scenario. In each case, the same highly complex and non-linear proprietary model of the vehicle dynamics was used to simulate the plant.

The first manoeuvre requires a single lateral shift of approximately 3.5 m to be performed within a distance of 45 m while travelling at a relatively sedate speed of 40 km/h. The second manoeuvre requires a severe double lane change to be performed at the higher speed of 80 km/h in a tightly constrained area defined by ISO 3888: a test track for a severe lane change manoeuvre, Part 2: obstacle avoidance [11]. The lane-width throughout the manoeuvre is specified in terms of the vehicle width; for the car under consideration, which has a width of 1.57 m, this translates into an initial lateral shift from a lane of width 1.98 m to a lane of width 2.57 m centred 3.27 m to the side, within a longitudinal distance of 12.0 m. After a straight section of length 11.0 m, a further lane-change must be performed to a lane of width 3.0 m centred 3.79 m from the new lane, within a longitudinal distance of 12.5 m.

In each case, the forward speed of the vehicle is allowed to vary freely once the manoeuvre has been initiated.

A. Gentle manoeuvre

Fig. 5a and Fig. 5b show the output trajectories during the gentle lane change for Controller A and B, respectively. The result obtained with Controller A demonstrate that the vehicle remains well within the track bounds and the entire manoeuvre is conducted very smoothly, indicating that passenger comfort is not compromised. Controller B is also capable of performing the manoeuvre but the turns into and out of the manoeuvre are far sharper and there is some oscillation as the vehicle acquires its new lane.

The lateral vehicle accelerations during this manoeuvre are compared in Fig. 5d, showing that the peak lateral acceleration caused by Controller B is five times greater than that of Controller A. Passengers in this vehicle operated by Controller B would encounter rather higher and more oscillatory lateral accelerations than those produced by Controller A. Throughout the manoeuvre, Controller A attempts to track reference signals for the vehicle sideslip β and yaw rate $\dot{\psi}$. The reference and output values are shown in Fig. 5c. The reference value of β is chosen to be zero to minimise vehicle sideslip, while the reference profile for $\dot{\psi}$ is chosen to enable the vehicle to complete the manoeuvre satisfactorily.

The reference trajectory for Controller B is shown as the chain line in Fig. 5b and the reference yaw rate is defined to keep the vehicle tangential to this trajectory.

B. Aggressive manoeuvre

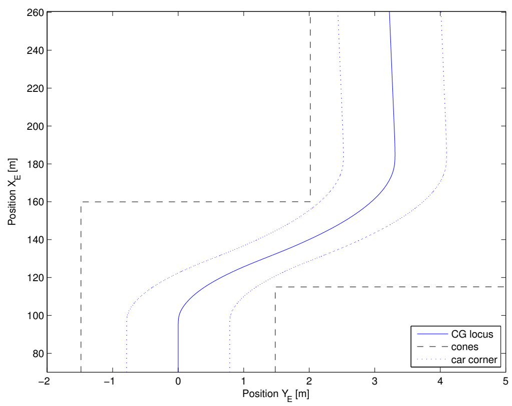
The double lane-change manoeuvre at 80 km/h places higher demands on the vehicle acceleration if the manoeuvre is to be accomplished within the very tightly constrained area defined by the specification. Fig. 6d shows that both controllers cause the car to accelerate with a lateral acceleration of close to 1 g. However, this is beyond the range of accelerations which Controller A is designed to handle.

It can be seen in Fig. 6a that Controller A, the controller designed to perform gentle lane-changes, is not able to keep the car within the specified bounds, with the vehicle exceeding the manoeuvre limits upon entry to the first turn and failing to remain within the bounds when acquiring the next lane. Figure 6c shows that the yaw rate and sideslip angle cannot be controlled to follow the reference values. Note that the maximal demanded yaw rate is 10 times larger than the peak yaw rate in the gentle manoeuvre.

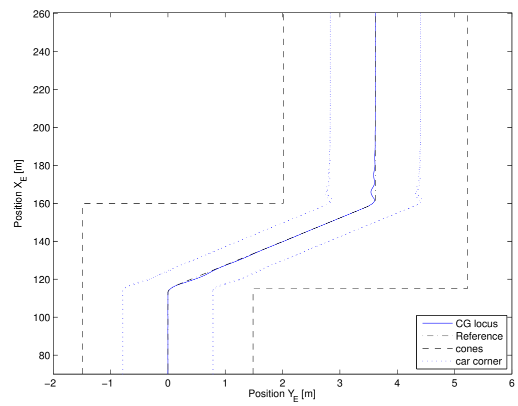
In contrast, Fig. 6b demonstrates that Controller B, which is designed to perform aggressive manoeuvres at the vehicle's physical limits, is able to navigate the car successfully throughout the entire severe double lane-change manoeuvre, although there is some minor departure from the reference trajectory.

V. Discussion

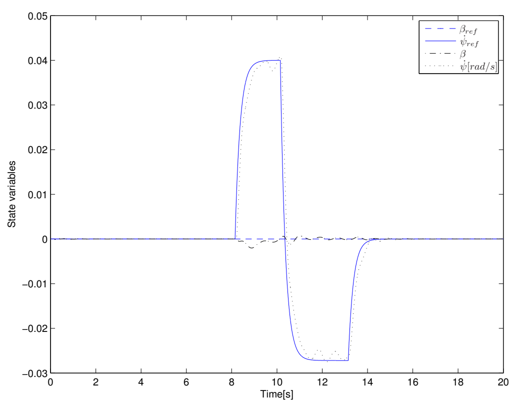
As would be expected of two controllers designed to cause the same plant to perform similar operations, there are several similarities between them. Both controllers use feedforward steering control together with feedback braking. However, there are also important differences.



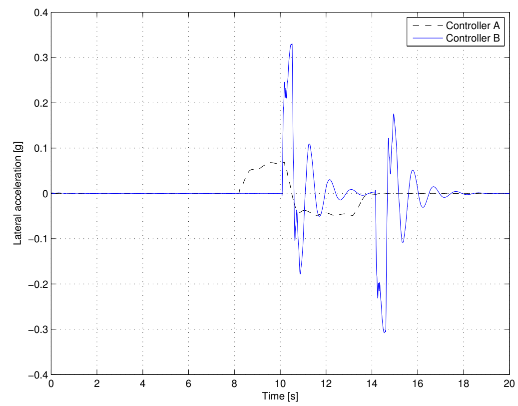
(a) Controller A: output trajectory



(b) Controller B: output trajectory



(c) Controller A: reference and measured outputs



(d) Lateral acceleration

Figure 5: Simulation results for the gentle single lane change manoeuvre with an initial speed of 40 km/hr.

The reference trajectory generation routine for the aggressive controller B determines the maximum turning rate which the vehicle is capable of attaining and uses this to calculate a very demanding trajectory; one that requires very tight control of the vehicle yaw rate if it is to be achieved. In contrast, the gentle controller A does not seek to achieve demanding yaw rates with such a high level of precision. The assumption that the trajectory may be characterised by a step filtered with a low-pass filter of modest time constant leads to a controller which is more suited to smooth transitions over greater distances.

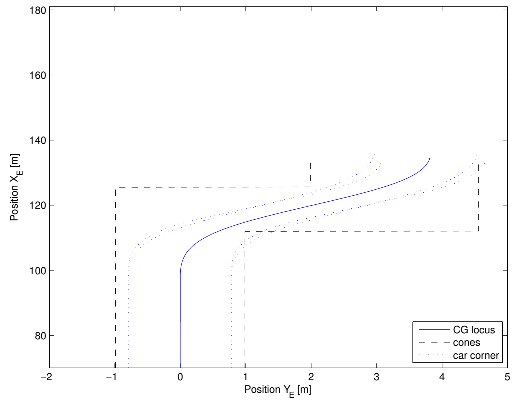
Of the two controllers presented, only the aggressive controller includes a feedback loop in the steering control, to assist in accurately acquiring the final lane and heading. A lane-tracking loop could be added to Controller A but it is not considered important for this design. The addition of such a feedback loop could aid the lane-acquiring performance of the gentle controller, but perhaps at some expense in terms of simplicity and smoothness of action.

The most significant difference between the two controllers is the implementation of feedback braking control. Both controllers use the brakes to control the vehicle lateral dynamics, but with different control objectives. Controller A uses the brakes primarily to alter the longitudinal velocity of the vehicle as a means of controlling the vehicle sideslip β, whereas Controller B uses the brakes to control the vehicle's lateral velocity and yaw rate. While sideslip can vary only slowly, yaw rate can be controlled far more rapidly, and it this fact that enables Controller B to more tightly control the transient behaviour of the vehicle. This is enhanced further by the tuning of the feedback gain matrix. The gain matrix K used by Controller A is designed for gentle manoeuvres and thus is less sensitive to error compared to the gain matrix K_b of Controller B which acts to eliminate deviations from the reference velocity profile as quickly as possible.

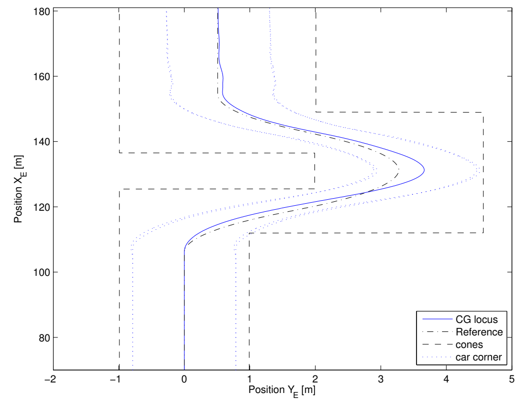
Controller A is designed to perform gentle manoeuvres while maintaining passenger comfort, and achieves this by controlling the vehicle yaw rate and sideslip angle. The controller is designed by assuming that the vehicle will operate close to equilibrium conditions and by making several other simplifying assumptions, detailed earlier. Controller B is designed to perform emergency collision avoidance manoeuvres and achieves this by paying closer attention to the vehicle velocity throughout the manoeuvre, but neglecting vehicle sideslip and disregarding the higher lateral accelerations that such manoeuvres entail. The design does not assume that the vehicle operates near any equilibrium points; nor is it assumed that inputs are small or smooth.

VI. Conclusions

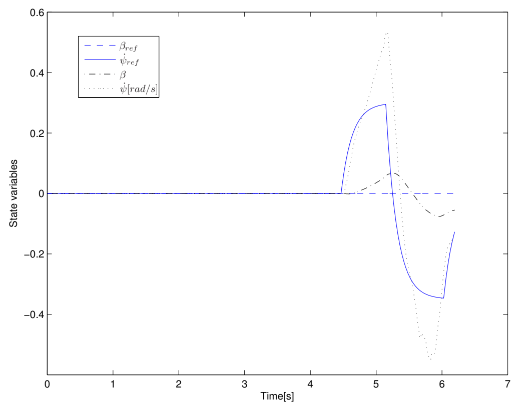
Simulation results are presented for two vehicle lateral controllers, each performing two types of manoeuvre: a gentle lane change and a severe double lane change. It is observed that



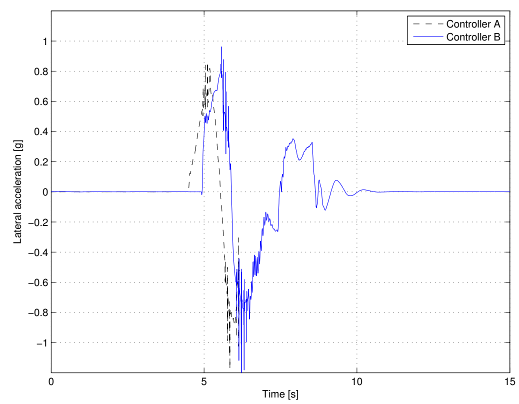
(a) Controller A: output trajectory



(b) Controller B: output trajectory



(c) Controller A: reference and measured outputs



(d) Lateral acceleration

Figure 6: Simulation results for the severe collision avoidance manoeuvre (ISO 3888:2) with an initial speed of 80 km/hr.

Controller A, which is designed for gentle vehicle control, is able to complete the gentle lane change competently, but cannot perform the aggressive double lane change. On the other hand, Controller B, designed as part of an emergency lateral collision avoidance system, is able to meet the specifications of both manoeuvres, but the response is more satisfactory for the more aggressive of the two manoeuvres, while passenger comfort may be compromised during the gentle manoeuvre.

The choice of control variables for the feedback braking system in each case is pivotal to controller performance. Controller B uses high gain to tightly control the vehicle velocity (lateral and yaw) throughout the transient part of the manoeuvre, to ensure that the vehicle can follow a very demanding trajectory. Meanwhile, Controller A uses the brakes to control an additional variable: vehicle sideslip angle. Control of sideslip is appropriate for improving vehicle performance during gentle manoeuvres but it is seen that it cannot force the vehicle to achieve high yaw rates while performing aggressive manoeuvres.

REFERENCES

[1] L. Austin and D. Morrey, Recent advances in antilock braking systems and traction control systems, *Proceedings of the IMechE, Part D: Journal of Automobile Engineering*, vol. 214, 2000, pp 625-638.

[2] J. Ackermann and T. Buente, Yaw disturbance attenuation by robust decoupling of car steering, *Control Engineering Practice*, vol. 5, 1997, pp 1131-1136.

[3] M. A. Vilaplana, O. Mason, D. J. Leith, W. E. Leithead and J. Kalkkuhl, Non-Linear Control Of Four-Wheel Steering Cars With Actuator Constraints, *Proceedings of the IFAC World Congress*, Barcelona, Spain, 2002.

[4] M. Hosaka and T. Murakami, Yaw rate control of electric vehicle using steer-by-wire system, *Proceedings - 8th IEEE International Workshop on Advanced Motion Control, AMC'04*, Kawasaki, Japan, 2004, pp 31-34.

[5] G. Burgio and P. Zegelaar, Integrated vehicle control using steering and brakes, *International Journal of Control*, vol. 79, 2006, pp 534-541.

[6] H. Cherouat and S. Diop, An observer and an integrated braking/traction and steering control for a cornering vehicle, *Proceedings of 2005 American Control Conference*, Portland, OR, 2005, pp 2212-2217.

[7] M. Lakehal-ayat, S. Diop and F. Lamnabhi-Lagarrigue, Yaw rate control for cornering 4WD vehicle, *Proceedings of the 14th International Symposium of Mathematical Theory of Networks and Systems (MTNS 2000)*, Perpignan, France, 2000.

[8] T. D. Gillespie, "Fundamentals of Vehicle Dynamics", Society of Automotive Engineers, 1992.

[9] D. J. Leith and W. E. Leithead, Gain-scheduled and nonlinear systems: dynamic analysis by velocity-based linearization families, *International Journal of Control*, vol. 70, 1998, pp 289-317.

[10] D. J. Leith, A. Tsourdos, B. A. White and W. E. Leithead, Application of velocity-based gain-scheduling to lateral auto-pilot design for an agile missile, *Journal of Control Engineering Practice*, vol. 9, 2001, pp 1079-1093.

[11] International Organization for Standardization, ISO 3888 Passenger cars - Test track for a severe lane-change manoeuvre, 1999, 2002.