

# A switched optimized approach for road-departure avoidance: implementation results

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**Abstract**—We present in this contribution the design and implementation of a steering assistance control. The main goal is to avoid the lane departure in case the driver loses attention. This control has been developed to keep the vehicle's trajectory within certain bounds during the assistance intervention, while maintaining a limited torque control input. In order to achieve this goal we have employed both Lyapunov theory and LMI optimization methods.

**Index Terms**—Automatic steering assistance, Lyapunov function, LMI.

## I. INTRODUCTION

Lateral control of the vehicle has two main applications: autonomous highway driving and driver steering assistance. Both of these applications require automated steering, which is mainly designed as steering torque control or as steering angle control [1], [2]. The present contribution implements a steering torque control.

In order to find a control law, which is optimal with respect to the stability and the transient dynamics specifications, LMI methods (Linear Matrix Inequalities) can be implemented. [3] has proposed a steering angle controller based on a LMI approach, that ensures bounded state variables and bounded control input in spite of perturbations like lateral wind and road curvature. In [4] a linear controller with a static output feedback has been designed using LMI methods.

Among the practical implementations of steering control laws we mention a comparative study between a lead-lag control law, a full-state linear controller and an input-output linearizing control law given by [5]. Concerning full lateral control, [6] proposes and implements in a prototype vehicle a control strategy that optimizes information given by the vision system and works for strong curvatures.

In the present paper the main goal is to help the driver during diminished driving capability due to inattention, tiredness or illness. For these time intervals the steering assistance should provide path-tracking, keeping the vehicle all the time on the lane during correction, and ensure bounded system dynamics. To control the vehicle trajectory we consider not only the lateral offset at the center of gravity of the vehicle or at a look-ahead distance but also the vehicle geometry, i.e. the positions of the vehicle front wheels on the lane. For instance, for a lateral offset of  $0.3m$  at the center of gravity

of the vehicle, a yaw angle of  $2^\circ$  and a vehicle width of  $1.5m$ , the front left wheel is located at  $1.10m$  from the center of the lane (see Fig. 1). Assuming that the lane width is  $3.5m$  we see that the steering control should guarantee during its action less than  $0.65m$  lateral displacement for the front left wheel to avoid the bump into the lane edges.

We have considered the above goal and constraints in the design of the steering torque control (performed by means of LMI methods) and of the switching strategy between the driver and the assistance control in [9]. Activated by the front wheels exceeding a predefined central lane strip, the developed control guarantees to maintain the vehicle within an extended strip in the middle of the lane. Moreover, the assistance torque and the values of the state variables are bounded during the transient response.

The contribution of this paper can be structured in three axes: first, a robust approach for a varying speed in a given interval is proposed to complete our controller from [9]. Second, a comparison is provided between the controller that makes use of look-ahead and look-down lateral displacement. Finally, we show the results of the practical implementation of our controller.

The remainder of this paper is divided as follows. The next section contains the model description for the vehicle and for the steering column. The assistance requirements are given in Section III. We further introduce the switching conditions between the driver and the assistance control in Section IV. The development of the control law is summarized in Section V. Section VI presents some robustness issues concerning the vehicle speed. The driving test results are given in Section VII. Conclusions in Section VIII wrap up the paper.

## II. VEHICLE MODEL WITH ELECTRICALLY POWERED STEERING

As this study concerns the lateral control of a vehicle, the classical fourth order linear model (“bicycle model”) has been used [10]. We assume in the present paper a very low road curvature ( $\rho_{ref} = 0$ ), which is realistic for highways. The steering assistance is provided by a DC motor mounted on the steering column. The model of the vehicle with electrical steering assistance is given in the following:

$$\dot{x} = A \cdot x + B \cdot (T_a + T_d), \quad (1)$$

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$$A = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & b_1 & 0 \\ a_{21} & a_{22} & 0 & 0 & b_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ v & l_S & v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{T_{S\beta}}{I_S R_S} & \frac{T_{S_r}}{I_S R_S} & 0 & 0 & -\frac{2K_p c_f \eta_t}{I_S R_S^2} & -\frac{B_S}{I_S} \end{pmatrix},$$

$$B = \left( 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{R_S I_S} \right)^T, \text{ where}$$

$$\begin{aligned} a_{11} &= -\frac{2(c_r + c_f)}{mv}, & a_{12} &= -1 + \frac{2(l_r c_r - l_f c_f)}{mv^2}, \\ a_{21} &= \frac{2(l_r c_r - l_f c_f)}{J}, & a_{22} &= -\frac{2(l_r^2 c_r + l_f^2 c_f)}{Jv}, \\ c_r &= c_r 0 V, & c_f &= c_f 0 V, \\ b_1 &= \frac{2c_f}{mv}, & b_2 &= \frac{2c_f l_f}{J}, \\ T_{S\beta} &= \frac{2K_p c_f \eta_t}{R_S}, & T_{S_r} &= \frac{2K_p c_f l_f \eta_t}{R_S v}. \end{aligned} \quad (2)$$

The values of the above parameters are described in Table III at the end of the paper. The state vector is  $x \triangleq [\beta, r, \psi_L, y_L, \delta_f, \dot{\delta}_f]^T$ , where  $\beta$  denotes the side slip angle,  $r$  the yaw rate,  $\psi_L$  the relative yaw angle,  $y_L$  the lateral offset,  $\delta_f$  the steering angle and  $\dot{\delta}_f$  the derivative of the steering angle. The inputs of the system (1) are the driver torque  $T_d$  and the assistance torque  $T_a$ . We consider as output the whole state vector  $z = x$ .

**Remark** It can be easily shown that the system (1) is controllable except for a longitudinal speed  $v$  equal to zero. Having two poles at the origin the system is not stable.

At this point we shortly discuss the difference between the measure of the lateral offset  $y_L$  at the center of gravity of the vehicle and at a look-ahead distance  $l_S$  (characteristic of vision systems).

[7] has shown that the look-ahead distance directly influences damping of the zeros of the transfer function from the steering angle to the lateral acceleration of the vehicle, but does not alter the system poles. Considering also the vehicle speed, the authors have concluded that increasing the look-ahead distance proportionally to the velocity improves the stability margins of the above transfer function.

In [8] the influence of the look-ahead distance on the system stability has been analyzed. The authors have stated that the look-ahead distance can always be chosen large enough to guarantee the closed loop stability considering some limits for the longitudinal speed. Nevertheless one is aware that an increase of the look-ahead distance is not always possible due to the vision and weather restrictions.

As a complement to [7] and [8] we compare in this paper a steering control law using the lateral offset taken either at the center of gravity of the vehicle or at a look-ahead distance.

### III. REQUIREMENTS CONCERNING THE ASSISTANCE SYSTEM

We recall in this section the qualitative control goals related to our assistance system. The main aim of the presented steering assistance is to avoid the lane departure in case the driver has a lack of attention during “normal

driving”. By “normal driving” we mean a driving situation such that the front wheels of the vehicle are in a predefined strip in the middle of the lane and the state variables are in a bounded region defined by minimum and maximum values.

To accomplish the above control goal we have to provide:

- 1) A switching strategy that activates and deactivates the steering assistance depending on the driver attention and on the danger of lane departure.
- 2) A steering control law that drives the vehicle during driver diminished attention satisfying the requirements below: (a) The closed loop system assistance-vehicle has to be asymptotically stable to zero steady state. (b) During assistance intervention the vehicle shall not leave the lane. Moreover, the overshoot of the front wheels with respect to a fixed predefined central lane strip has to be as small as possible. (c) The state and the assistance torque have to be bounded to guarantee safety and comfort.

### IV. SWITCHED SYSTEM

#### A. Mathematical definition of the “normal driving” zone

We have started to develop the required switching strategy between the driver and the steering assistance by fitting the qualitative description of the “normal driving” in a mathematical form. Therefore we have considered that the two front wheels remain during “normal driving” in a central lane strip of width  $2d$  (total lane width is  $L$ , see Fig. 1).

For reasons of better understanding we have first considered the case where  $y_L$  is taken at the center of gravity of the vehicle ( $l_S = 0m$ ). In a second approach, the positions of the front wheels on the lane are calculated for a look-ahead distance  $l_S$ .

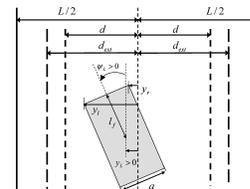


Fig. 1. Vehicle on the lane.

1) “Normal driving” zone, the look-down case ( $l_S = 0m$ ): For a vehicle position on the lane with a relative yaw angle  $\psi_L$  and a lateral offset from the centerline  $y_L$  at the vehicle center of gravity we have deduced the coordinates of the two front wheels  $y_l$  and  $y_r$  relative to the centerline<sup>1</sup> (see Fig. 1). Assuming a small relative yaw angle  $\psi_L$  we obtain:

$$y_l = y_L + l_f \psi_L + \frac{a}{2}, \quad y_r = y_L + l_f \psi_L - \frac{a}{2}, \quad (3)$$

where  $l_f$  is the distance from the center of gravity of the vehicle to the front axle and  $a$  is the vehicle width<sup>2</sup>. The

<sup>1</sup>We have considered the lateral offset  $y_L$  positive on the left side of the lane and the relative yaw angle  $\psi_L$  positive for trigonometric rotation with the origin in the centerline.

<sup>2</sup>Values for the parameters  $l_f$  and  $a$  are described in Table III at the end of the paper.

front wheels of the vehicle are inside the central lane strip  $\pm d$  if:

$$-\frac{2d-a}{2} \leq y_L + l_f \psi_L \leq \frac{2d-a}{2}. \quad (4)$$

Hence, a system state  $x$  that accomplishes the above inequalities (4) belongs to the set

$$L(\bar{F}) \triangleq \{x \in \mathbb{R}^6 : |\bar{F}x| \leq 1\}, \quad (5)$$

where  $\bar{F} \in \mathbb{R}^{1 \times 6}$ ,  $\bar{F} = (0, 0, \frac{2l_f}{2d-a}, \frac{2}{2d-a}, 0, 0)$ .

Moreover, we consider that there is a danger to leave the lane when at least one of the two front wheels crosses one of the edges of the central lane strip  $\pm d$ , which means  $|\bar{F}x| = 1$ .

We have further expressed a bounded space region for the “normal driving” for the state  $x$ . If we suppose that  $|x_i| \leq x_i^N$  for  $i = 1, \dots, 6$ , where  $x_i$  denotes the  $i$ -th component of the state vector  $x$ , then the state vector  $x$  belongs for a “normal driving” to the set

$$L(F^N) \triangleq \{x \in \mathbb{R}^6 : |f_i^N x| \leq 1, i = 1, \dots, 6\}, \quad (6)$$

where  $F^N \in \mathbb{R}^{6 \times 6}$ ,  $f_i^N$  represent the rows of  $F^N$ ,  $f_{i,i}^N = (x_i^N)^{-1}$  and  $f_{i,j}^N = 0$  for  $i \neq j$ ,  $i, j = 1, \dots, 6$ .

We have concluded that a driver provides a “normal driving” if  $x \in (L(\bar{F}) \cap L(F^N))$ .

2) “Normal driving” zone, the look-ahead case ( $l_s > 0$ ): We have considered in the following a look-ahead measure of the lateral offset  $y_L$ , taken at the look-ahead distance  $l_s > 0$ . In Fig. 2 let  $y_L^{CG}$  be the lateral offset measured at the center of gravity of the vehicle. We have next expressed the front wheels coordinates in function of the look-ahead lateral offset  $y_L$  and of the relative yaw angle  $\psi_L$ .

We can see in Fig. 2 that for a straight road and small  $\psi_L$  we have  $y_L \cong y_L^{CG} + \psi_L \cdot l_s$  and hence we have obtained  $y_L^{CG} \cong y_L - \psi_L \cdot l_s$ . Rewriting the eqs. (3) we have obtained:

$$y_l = y_L + (l_f - l_s)\psi_L + \frac{a}{2}, \quad y_r = y_L + (l_f - l_s)\psi_L - \frac{a}{2}. \quad (7)$$

The coordinates of the front wheels are located inside the fixed central lane strip  $\pm d$  if:

$$-\frac{2d-a}{2} \leq y_L + (l_f - l_s)\psi_L \leq \frac{2d-a}{2}. \quad (8)$$

Hence a system state  $x$  that accomplishes the above inequalities (8) belongs to the set

$$L(\bar{F}_{l_s}) \triangleq \{x \in \mathbb{R}^6 : |\bar{F}_{l_s}x| \leq 1\}, \quad (9)$$

where  $\bar{F}_{l_s} \in \mathbb{R}^{1 \times 6}$ ,  $\bar{F}_{l_s} = (0, 0, \frac{2(l_f - l_s)}{2d-a}, \frac{2}{2d-a}, 0, 0)$ .

The driver provides in this case a “normal driving” if  $x \in (L(\bar{F}_{l_s}) \cap L(F^N))$ .

### B. Continuous and discrete dynamics

For the measure of the driver attention level the readers are referred to the concept of driver monitoring [11], [12]. In the present paper we have assumed that only the driver torque on the steering wheel is accessible to detect the driver attention level. We have considered that the driver is inattentive for a driver torque below a threshold  $\sigma_1$ :  $|T_d| < \sigma_1$ . However,

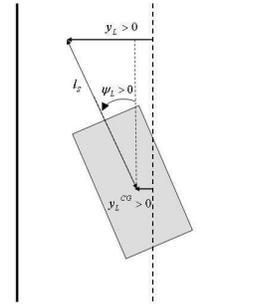


Fig. 2. Vehicle on the lane, look-ahead measured lateral offset.

the analysis presented here remains valid also if we choose another variable to detect the driver attention level.

The fact that the steering assistance switches on only in particular situations establishes two distinct time continuous systems. One system describes the vehicle controlled by the human driver:  $\Sigma_1 : \dot{x} = A \cdot x + B \cdot T_d$ . The other system reflects the vehicle lateral motion under automatic steering assistance, perhaps influenced by the inattentive driver:  $\Sigma_2 : \dot{x} = A \cdot x + B \cdot (T_a + T_d)$ .

At any time the model of the vehicle corresponds to one and only one of the above descriptions. The transitions between  $\Sigma_1$  and  $\Sigma_2$  are considered instantaneous and depend on the driver attention and on the danger of lane departure. Thus, a discrete time dynamic completes the continuous dynamics of the system (1).

### C. Switching conditions

The steering assistance has to be switched on for a driver lack of attention during “normal driving” in case of danger of unintended lane departure<sup>3</sup>:

$$T_r^{12} : (|T_d| < \sigma_1) \wedge (x \in (L(\bar{F}_{l_s}) \cap L(F^N))) \wedge (|\bar{F}_{l_s}x| = 1). \quad (10)$$

The steering control has to be switched off whenever the driver recovers attention but for safety reasons only if the vehicle is in the “normal driving” zone:

$$T_r^{21} : [(\sigma_1 \leq |T_d| < \sigma_2) \wedge (x \in (L(\bar{F}_{l_s}) \cap L(F^N)))] \vee (|T_d| \geq \sigma_2). \quad (11)$$

However, for safety reasons, the assistance shall be removed whenever the driver considers it necessary and applies a torque  $|T_d| \geq \sigma_2$  on the steering wheel.

### V. OPTIMAL CHOICE OF A TORQUE CONTROL LAW

To ensure closed loop stability, a minimal overshoot with respect to the fixed central lane strip  $\pm d$  and passengers’ comfort we have chosen a linear feedback control law with a compensation of the driver torque:  $T_a = Kx - T_d$ . An appropriate Lyapunov function  $V(x) = x^T Px$  ensures bounded vehicle trajectory during control activation (for more details see [9]).

In [9] we have expressed the above control requirements in LMI inequalities that contain  $Y = KQ$  and  $Q = P^{-1}$  as

<sup>3</sup>  $\wedge$  denotes the logical “and” and  $\vee$  denotes the logical “or”.

matrix variables. The end has been to formulate the control design as a LMI linear cost optimization problem<sup>4</sup>:

$$\min \quad -\alpha$$

$$QA^T + AQ + BY + Y^T B^T < 0, \quad (12)$$

$$\begin{pmatrix} 1 & f_i^N Q \\ (f_i^N Q)^T & Q \end{pmatrix} \geq 0, i = 1, \dots, 6, \quad (13)$$

$$\alpha \leq (\bar{F}_{l_s})^T Q \bar{F}_{l_s}, \quad (14)$$

$$(\bar{F}_{l_s})^T Q \bar{F}_{l_s} < 1, \quad (15)$$

$$\begin{pmatrix} 1 & \frac{1}{TM} Y \\ \frac{1}{TM} Y^T & Q \end{pmatrix} \geq 0. \quad (16)$$

Eq. (12) ensures asymptotic stability in closed loop. Eq. (13) confines the ellipsoid  $\varepsilon = \{x \in \mathbb{R}^6 : x^T P x \leq 1\}$  to the normal driving hypercube  $L(F^N)$ . Eqs. (14) and (15) force such a choice of the matrix  $Q$  that  $\varepsilon$  approaches the control activation zone  $|\bar{F}_{l_s} x| = 1$ . Eq. (16) bounds the assistance torque to  $TM$  for  $x \in \varepsilon$ .

After having solved the LMI optimisation problem we can compute the central lane strip  $\pm d_{ext}$  (see Fig. 1). It is inside this lane strip that the front wheels of the vehicle will stay for a control activation by  $|\bar{F}_{l_s} x| = 1$  thanks to the invariant set properties of the Lyapunov level curve  $V(x) = x^T P x = V_{ext}$ <sup>5</sup>:

$$d_{ext} = \frac{2d - a}{2} \sqrt{V_{ext} \bar{F}_{l_s} Q \bar{F}_{l_s}^T} + \frac{a}{2}. \quad (17)$$

Moreover the vehicle state  $x$  is confined during the control process in a hypercube defined by  $|x_i| \leq x_i^M$ ,  $x_i^M = \sqrt{Q_{i,i}}$  for  $i = 1, \dots, 6$ , where  $x_i$  is the  $i$ -th element of the state vector  $x$  and  $Q_{i,i}$  is an element of the diagonal of matrix  $Q$ . An upper bound of the motor torque used to bring the vehicle on the right trajectory is given by  $TM_{ext} = \max_{x \in \varepsilon_{ext}} (Kx)$ .

## VI. ROBUSTNESS CONCERNING THE VEHICLE SPEED

The system description given in eq. (1) depends non linearly on the vehicle speed  $v$ . Choosing  $\xi_v \in [-1; 1]$  a parameter that describes the variation of  $v$  between  $v_{min}$  and  $v_{max}$  we can write the following [13]:

$$\frac{1}{v} = \frac{1}{v_0} + \frac{1}{v_1} \xi_v, \quad v \cong v_0 \left(1 - \frac{v_0}{v_1} \xi_v\right), \quad \frac{1}{v^2} \cong \frac{1}{v_0^2} \left(1 + 2 \frac{v_0}{v_1} \xi_v\right). \quad (18)$$

Setting for  $\xi_v = -1$   $v = v_{min}$  and for  $\xi_v = 1$   $v = v_{max}$  it yields for  $v_0$  and  $v_1$ :

$$v_0 = \frac{2v_{min}v_{max}}{v_{max} + v_{min}}, \quad v_1 = -\frac{2(v_{min}v_{max})}{v_{max} - v_{min}}. \quad (19)$$

With the above expressions for  $v$ ,  $1/v$  and  $1/v^2$  we are able to express the matrix  $A$  of system (1) as  $A = A^* + A^{**} \xi_v$ . Hence the matrix  $A(v)$  resides in a matrix polytope.

Let us now have a look at the LMI optimization problem enounced in Section V and to the required modifications due

<sup>4</sup>The LMI optimization problem is done only for the look-ahead measure of the lateral offset. It doesn't change for the look-down case if one sets  $l_s = 0m$ .

<sup>5</sup> $V_{ext}$  is the smallest value such that the ellipsoid  $\varepsilon_{ext} = \{x \in \mathbb{R}^6 : x^T P x \leq V_{ext}\}$  includes the control activation zone  $(L(F^N) \cap |\bar{F}_{l_s} x| = 1)$  (see [9]).

to a vehicle speed  $v \in [v_{min}, v_{max}]$ . One notice that only the matrix  $A$  of the LMI problem depends on the vehicle speed  $v$ . Due to the convex property of the matrix polytope the ineq. (12) holds for any  $v \in [v_{min}, v_{max}]$  if the following hold:

$$Q(A^* \pm A^{**})^T + (A^* \pm A^{**})Q + BY + Y^T B^T < 0. \quad (20)$$

The above considerations mean that if we find the matrix variables  $Q$  and  $Y$  that minimize the LMI problem from Section V for  $A = A^* - A^{**}$  and for  $A = A^* + A^{**}$  then the control vector  $K$  stabilizes the system (1) for any varying speed value  $v \in [v_{min}, v_{max}]$ .

## VII. DRIVING TEST RESULTS

### A. Numerical results

In both cases, look-down ( $l_s = 0m$ ) and look-ahead ( $l_s = 5m$ ) measure of the lateral offset, we have fixed a central lane strip for the “normal driving” of  $d = 1m$  for a vehicle speed  $v \in [12m/s; 16m/s]$ .

The limits of the “normal driving” set  $L(F^N)$  are given in Table I. To achieve acceptable maximum bounds for the state variables during the control activation, the limits for the side slip angle  $\beta$ , yaw rate  $r$  and lateral offset  $y_L$  have been set for  $l_s = 0m$  lower than for  $l_s = 5m$ . On the contrary, we have needed higher normal bounds for the steering angle  $\delta_f$  and for the steering angle rate  $\dot{\delta}_f$  in the look-down case than in the look-ahead case. This shows that with a look-down measure of the lateral offset, for an initial state nearer to the origin, we can only ensure maximum bounds comparable with the maximum bounds for the look-ahead case (see Table II). In order to achieve this result we have even to use a larger range of values for the steering angle and its derivative for  $l_s = 0m$  than for  $l_s = 5m$ . Hence, we had to employ for  $l_s = 0m$  a faster control law than for  $l_s = 5m$  in order to provide the same performances.

The system trajectories are guaranteed to stay during the control activation below the limits given in Table II. We have obtained as maximum bounds for the assistance steering torque  $23.73Nm$  ( $l_s = 0m$ ) and  $23Nm$  ( $l_s = 5m$ ). According to the numerical results, the trajectories of the front wheels of the vehicle won't exceed during the assistance control  $d_{ext} = 1.38m$  ( $l_s = 0m$ ) and  $d_{ext} = 1.46m$  ( $l_s = 5m$ ), respectively. We have considered that the driver is inattentive for a steering torque  $\sigma_1$  below  $1Nm$  and we have set the security deactivation limit  $\sigma_2$  to  $3Nm$ .

TABLE I

LIMITS OF “DRIVING NORMAL” ZONE.

$l_s(m)$	$\beta^N(rad)$	$r^N(rad/s)$	$\psi_L^N(rad)$	$y_L^N(m)$	$\delta_f^N(rad)$	$\dot{\delta}_f^N(rad/s)$
0	0.0043	0.0872	0.0174	0.3	0.0157	0.0436
5	0.0087	0.1047	0.0174	0.5	0.0087	0.0349

TABLE II

LIMITS OF MAXIMAL ZONE.

$l_s(m)$	$\beta^M(rad)$	$r^M(rad/s)$	$\psi_L^M(rad)$	$y_L^M(m)$	$\delta_f^M(rad)$	$\dot{\delta}_f^M(rad/s)$
0	0.0181	0.1875	0.0639	0.67	0.0318	0.1796
5	0.0240	0.2172	0.0478	0.68	0.0221	0.0965

### B. Test environment

The test track we have used is located in Satory, 20Km west of Paris. The site is 3.5Km long with various road profiles including a straight lane, a tight bend and a squabble. The experimental vehicle has been equipped with a CORE-VIT that measures the side slip angle  $\beta$ , an Inertial Navigation System to measure the yaw rate  $r$  and an odometer for the vehicle speed  $v$ . The front wheels steering angle  $\delta_f$  has been obtained from an optical encoder. The driver torque is measured by a load cells sensor integrated to the steering wheel. For the look-down measure of the lateral offset we have used a differential GPS, for which lanes markers are digitalized each 5cm. The look-ahead lateral offset is measured with a video camera that can detect lane markers [14]. The assistance torque is obtained from a DC motor mounted on the steering column.

### C. Practical implementation results

We discuss in this section the first practical results of the implementation of the steering assistance on the vehicle. The theoretical results have been verified for the two computed control feedback laws corresponding to  $l_S = 0m$  and to  $l_S = 5m$ . As expected, for a driver torque bellow  $\sigma_1$  the steering assistance has switched on as soon as the front wheels have crossed the lane strip  $d$ . The assistance torque has brought back the vehicle to the center of the lane without exceeding the computed lane strip  $d_{ext}$ , and then keeping the vehicle on the lane all the time during regulation (see Fig. 3 (a) and Fig. 7 (a)).

The trajectories for the look-down lateral offset came back into the central lane strip very fast, hence the deactivations took place for a driver torques of about  $1Nm$  (see Fig. 3 (b)). For the look-ahead measured lateral offset the third deactivation ( $t = 92.5s$ ) occurred when the front wheels of the vehicle were still outside the lane strip  $d$ , thus the driver had to provide a  $3Nm$  torque (see Fig. 7 (b)). He felt in that case a small resistance on the steering wheel, which indicates that the vehicle was about to go off the lane and it was corrected by the steering assistance.

The torque values of the assistance control remained bellow  $10Nm$  (see Fig. 3 (b) and Fig. 7 (b)). We notice an offset of the average assistance torque in the negative side, which corresponds to a compensation of the road bank angle.

During the control activation all the state variables have stayed below the maximum computed values  $x^M$ , and for the most of the time even inside the “normal driving” zone  $L(F^N)$  (see Figs. 4, 5, 6 and Figs. 8, 9, 10).

Generally speaking, we haven't noticed great differences between the two implemented control laws. However, in the look-down case ( $l_S = 0m$ ) the activation is more restricted due to the fact that the bound of the lateral offset is smaller. Moreover the differential GPS is an expensive solution to measure the lateral offset, since it requires a very precise map. The GPS signal is in addition not very reliable because of the trees and other obstacles that can come between the vehicle and the antennas. Therefore we prefer the look-ahead

measure of the lateral offset which makes use of camera and of vision algorithms.

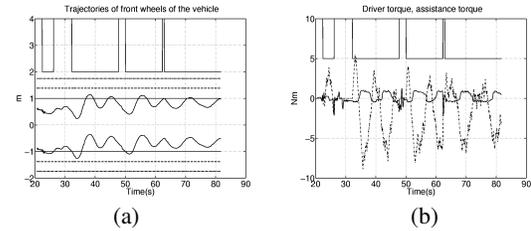


Fig. 3. (a) Trajectories of the front wheels (continuous draw), predefined lane strip  $\pm d$  (continuous draw), computed driving lane strip  $\pm d_{ext}$  (dash draw), lane border (dash-dot draw), assistance activation on 2 ( $l_S = 0m$ ). (b) Driver torque (continuous draw), assistance torque (dash-dot draw), assistance activated on 5 ( $l_S = 0m$ ).

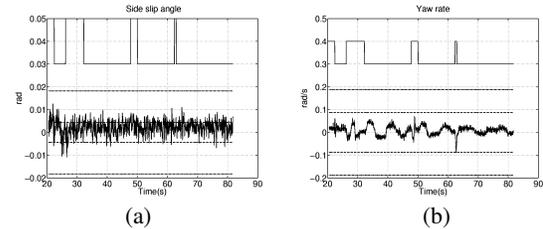


Fig. 4. (a) Side slip angle  $\beta$  (continuous draw), “normal driving” value  $\beta^N$  (dash draw), maximal computed bound  $\beta^M$  (dash-dot draw), assistance activation on 0.03 ( $l_S = 0m$ ). (b) Yaw rate  $r$  (continuous draw), “normal driving” value  $r^N$  (dash draw), maximal computed bound  $r^M$  (dash-dot draw), assistance activated on 0.3 ( $l_S = 0m$ ).

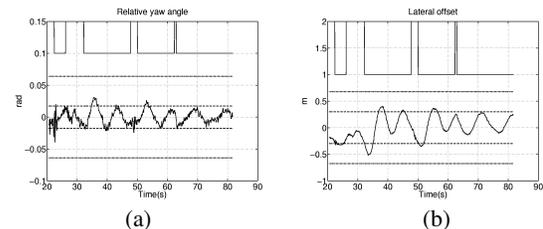


Fig. 5. (a) Relative yaw angle  $\psi_L$  (continuous draw), “normal driving” value  $\psi_L^N$  (dash draw), maximal computed bound  $\psi_L^M$  (dash-dot draw), assistance activation on 0.1 ( $l_S = 0m$ ). (b) Lateral offset  $y_L$  (continuous draw), “normal driving” value  $y_L^N$  (dash draw), maximal computed bound  $y_L^M$  (dash-dot draw), assistance activated on 1 ( $l_S = 0m$ ).

## VIII. CONCLUSIONS

In the present paper we have described the implementation of an automated steering assistance. This assistance activates for a driver lack of attention as soon as the front wheels cross a predefined lane strip on the center of the lane. It brings the vehicle back to the center of the lane keeping the front wheels inside a security zone during the regulation.

We have developed and implemented the above steering assistance for a look-down and for a look-ahead measure of the lateral offset. For the look-down case we had to implement a faster control law to achieve the same results as for the look-ahead case. This is probably due to the fact that “look-ahead” is linked to anticipation. However, for the practical implementation there are no clear differences between the two controllers.

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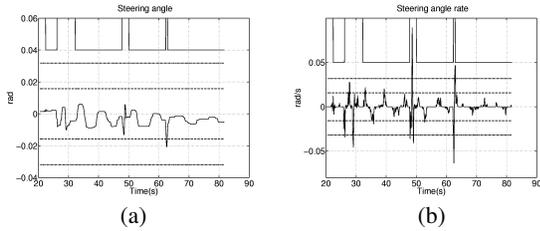


Fig. 6. (a) Steering angle  $\delta_f$  (continuous draw), “normal driving” value  $\delta_f^N$  (dash draw), maximal computed bound  $\delta_f^M$  (dash-dot draw), assistance activation on 0.04 ( $l_S = 0m$ ). (b) Steering angle rate  $\dot{\delta}_f$  (continuous draw), “normal driving” value  $\dot{\delta}_f^N$  (dash draw), maximal computed bound  $\dot{\delta}_f^M$  (dash-dot draw), assistance activated on 0.05 ( $l_S = 0m$ ).

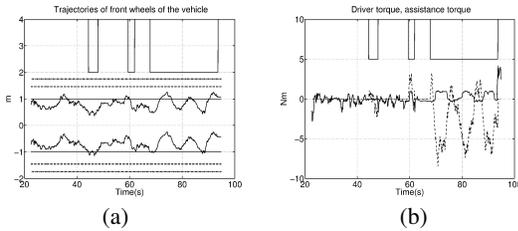


Fig. 7. (a) Trajectories of the front wheels (continuous draw), predefined lane strip  $\pm d$  (continuous draw), computed driving lane strip  $\pm d_{ext}$  (dash draw), lane border (dash-dot draw), assistance activation on 2 ( $l_S = 5m$ ). (b) Driver torque (continuous draw), assistance torque (dash-dot draw), assistance activated on 5 ( $l_S = 5m$ ).

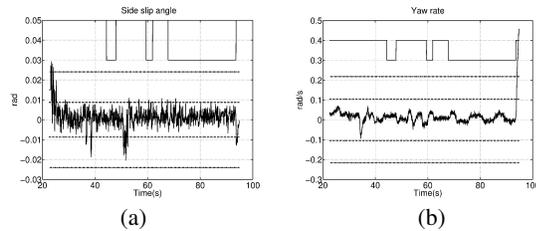


Fig. 8. (a) Side slip angle  $\beta$  (continuous draw), “normal driving” value  $\beta^N$  (dash draw), maximal computed bound  $\beta^M$  (dash-dot draw), assistance activation on 0.03 ( $l_S = 5m$ ). (b) Yaw rate  $r$  (continuous draw), “normal driving” value  $r^N$  (dash draw), maximal computed bound  $r^M$  (dash-dot draw), assistance activated on 0.3 ( $l_S = 5m$ ).

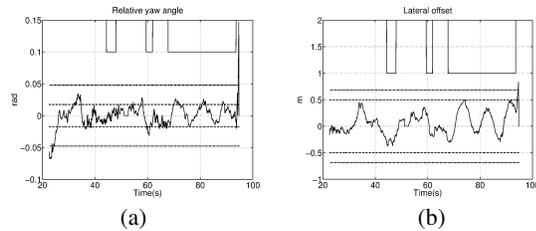


Fig. 9. (a) Relative yaw angle  $\psi_L$  (continuous draw), “normal driving” value  $\psi_L^N$  (dash draw), maximal computed bound  $\psi_L^M$  (dash-dot draw), assistance activation on 0.1 ( $l_S = 5m$ ). (b) Lateral offset  $y_L$  (continuous draw), “normal driving” value  $y_L^N$  (dash draw), maximal computed bound  $y_L^M$  (dash-dot draw), assistance activated on 1 ( $l_S = 5m$ ).

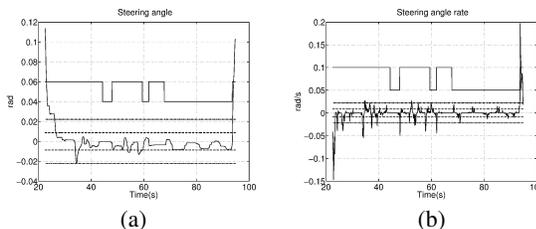


Fig. 10. (a) Steering angle  $\delta_f$  (continuous draw), “normal driving” value  $\delta_f^N$  (dash draw), maximal computed bound  $\delta_f^M$  (dash-dot draw), assistance activation on 0.04 ( $l_S = 5m$ ). (b) Steering angle rate  $\dot{\delta}_f$  (continuous draw), “normal driving” value  $\dot{\delta}_f^N$  (dash draw), maximal computed bound  $\dot{\delta}_f^M$  (dash-dot draw), assistance activated on 0.05 ( $l_S = 5m$ ).

TABLE III  
VEHICLE PARAMETERS AND THEIR VALUES

Parameter		Value
$B_S$	steering system damping coefficient	15
$c_{f0}$	front cornering stiffness	40000 N/rad
$c_{r0}$	rear cornering stiffness	35000 N/rad
$I_S$	inertial moment of steering system	$0.05\text{kg}\cdot\text{m}^2$
$J$	vehicle yaw moment of inertia	$2454\text{kg}\cdot\text{m}^2$
$K_P$	manual steering	1
$l_f$	distance from CG to front axle	1.05m
$l_r$	distance from CG to rear axle	1.56 m
$l_S$	look-ahead distance	5m
$a$	vehicle width	1.5m
$m$	total mass	1600 kg
$R_S$	steering gear ratio	14
$v$	longitudinal velocity	[12;16]m/s
$\eta_t$	tire length contact	0.13m
$\nu$	adhesion	1
$L$	lane width	3.5m

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