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Estimation of the Unknown Inputs and Vertical Forces of the Heavy Vehicle Via Higher Order Sliding Mode Observer

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Abstract—In this work, A new method is developed to estimate the unknown inputs of heavy vehicle. These inputs represent road profile which is very interesting since is used to estimate the vertical forces acting on the wheels. We reconstruct these unknown inputs by using higher order sliding mode observer. First, we estimate speeds and accelerations of heavy vehicle in finite time.

I. INTRODUCTION

Many researches on vehicle-road interaction are performed without taking into account the response of the road profile. Recently, the responses of the road profile to dynamics vehicle forces have been investigated. However,these inputs are not well known and there are some difficulties to have them in real time. Then, it's not easy to estimate the vertical forces on board.

A variety of sensors and instruments can be used for measuring these inputs (LPA, inertial method, profilometers...). However, the sensors are expensive and the measurements are not done on board.

In this work, we propose a new method to evaluate the road profile inputs based on higher order sliding mode observers which are considered as inputs of the developed heavy vehicle model.

First, these inputs are measured by LPA instrument (Longitudinal Profile Analyzer), APL in French, developed at Roads and Bridges Central Laboratory (LCPC in French) and applied to the heavy vehicle model ([10], [11], [7]). Then a third order sliding mode observer is developed to estimate these unknown inputs ([3]). This estimation allow us to reconstruct the vertical forces which are very important to calculate road damage or to evaluate the risk of rollover of the heavy vehicle using the Load Transfer Ratio (see ([8], [9])).

In a validation procedure, we compare the LPA measures with simulation results coming from a developed estimator. We compare also the states and vertical forces estimations with those given by PROSPER simulator developed by Sera-Cd (see ([1])). It's validated by many tests with an instrumented heavy vehicle, as we can see in ([2]).

This paper is organized as follows: section 2 deals with the heavy vehicle model description. The design of observer to estimate the road profile and vertical forces is presented in section 3. Section 4 gives some simulations related to estimation in order to show robustness of a proposed method. Finally, some remarks and perspectives are given in a concluding section.

II. HEAVY VEHICLE MODEL DESCRIPTION

Many studies deal with heavy vehicle modelling which represent a complex mechanical system with nonlinear features. The models used for heavy vehicles are very complicated ([4], [6], [5]). Consequently, it is relatively difficult to define the different parameters of these models. In this paper, we consider the half tractor model with 4 degrees of freedom (dof) composed of two principal bodies: the first, is the unsprung part of the vehicle (unsprung mass, four wheels and front axles), and the second body represents the sprung mass. This model is represented in the figure (1).

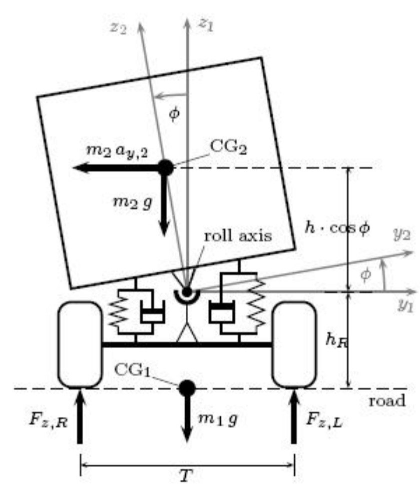

Fig. 1. Heavy vehicle model

This model is derived using Lagrangian's equations.

We can define a dynamic model of the vehicle as :

$$M(q)\ \ddot{q} + C(q,\dot{q})\ \dot{q} + K(q) = F_g \quad (1)$$

where $M \in \Re^{3\times3}$ is the inertia matrix (Mass matrix), $C \in \Re^{3\times3}$ is related to the damping effects, $K \in \Re^4$ is the springs stiffness vector and $F_g \in \Re^3$ is a vector of generalized forces.

$q \in \Re^3$ is the coordinates vector defined by:

$$q = [q_1, q_2, \theta]^T \quad (2)$$

The vertical acceleration of the chassis (tractor's body) is obtained as following:

$$\begin{aligned}\ddot{z} \;=\; & (k_1q_1 + k_2q_2 + (k_1 - k_2)\frac{T_w}{2}sin(\theta) \\ & + B_1\dot{q}_1 + B_2\dot{q}_2 - (B_1 - B_2)\frac{T_w}{2}cos(\theta)\dot{\theta})/M\end{aligned} \quad (3)$$

θ is the tractor roll angle;

q_1, q_2 are respectively the left and right front suspension deflection of the tractor;

z is the vertical displacement of the tractor sprung mass (center of gravity height). $\ddot{z}$ is the vertical acceleration of the sprung mass.

The suspension is modeled as the combination of spring and damper elements as shown in the figure (2).

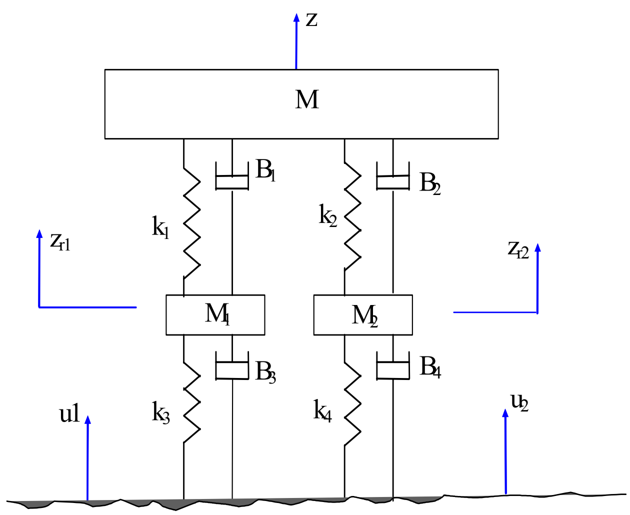

Fig. 2. Suspension model

The tractor chassis (sprung mass) with the mass M is suspended on its axles through two suspension systems. We can also show that the tire is modeled by springs (represented by k_i in the figure 2) and damper elements (represented by B_i in the figure 2). The wheel masses are represented by M_1 and M_2. At the tire contact, we have the road profile represented by the inputs u_1 and u_2, which are considered as the heavy vehicle inputs.

z_{r1} and z_{r2} represent respectively the vertical displacement of the left and right wheel of the tractor front axle. These can be calculated using the following equations:

$$\begin{cases} z_{r1} = z - q_1 - \frac{T_w}{2}\sin(\theta) - r\cos(\theta) \\ z_{r2} = z + q_2 - \frac{T_w}{2}\sin(\theta) - r\cos(\theta) \end{cases} \quad (4)$$

where T_w is the tractor width and r is the wheel radius. The vertical acceleration of the wheels are given by:

$$\begin{cases} \ddot{z}_{r1} = (B_1\dot{q}_1 - k_1\frac{T_w}{2}\sin(\theta) - B_1\frac{T_w}{2}\cos(\theta)\dot{\theta} \\ \qquad +k_1q_1 - k_3z_{r1} + k_3u_1)/m_1 \\ \\ \ddot{z}_{r2} = (B_2\dot{q}_2 + k_2\frac{T_w}{2}\sin(\theta) + B_2\frac{T_w}{2}\cos(\theta)\dot{\theta} \\ \qquad +k_2q_2 - k_4z_{r2} + k_4u_2)/m_2 \end{cases} \quad (5)$$

Using these equations, the unknown inputs can the be written as following:

$$\begin{cases} u_1 = (m_1\ddot{z}_{r1} - B_1\dot{q}_1 + k_1\frac{T_w}{2}\sin(\theta) \\ +B_1\frac{T_w}{2}\cos(\theta)\dot{\theta} - k_1q_1 + k_3z_{r1})/k_3 \\ \\ u_2 = (m_2\ddot{z}_{r2} - B_2\dot{q}_2 - k_2\frac{T_w}{2}\sin(\theta) \\ -B_2\frac{T_w}{2}\cos(\theta)\dot{\theta} - k_2q_2 + k_4z_{r2})/k_4 \end{cases} \quad (6)$$

The vertical forces F_{ni} , $i = 1..2$ acting on the wheels are calculated using the following expression:

$$F_{ni} = F_{ci} + k_i(u_i - z_{ri}), \quad i = 1..2 \quad (7)$$

where F_{ci} is the static load and k_i is the spring stiffness. We assume that the force generated by damping effect is neglected compared to the spring force.

The next section deals with the development of the high order sliding mode observer to estimate all states of the system and to identify the road profile and the vertical forces of the heavy vehicle.

III. ESTIMATION OF THE ROAD PROFILE AND VERTICAL FORCES

A. Strong observability test

In this section, we develop the third order sliding mode observer to estimate the heavy vehicle dynamic states and the road profile which allow us to reconstruct the vertical forces ([3]). Before developing such observer, we need to verify the strong observability of the system ([13]). Let us rewrite the dynamic model (1) as:

$$\begin{cases} \dot{x} = Ax + D \\ y = Cx \end{cases} \quad (8)$$

where $x \in \Re^3$ is the state vector, the matrix A is of dimension $\Re^{3\times3}$, C is a matrix in $\Re^{3\times3}$, D is a vector in $\Re^3$ composed of non linearities of the system.

Definition 1: $s_0 \in C$ is called an invariant zero of the triple $\{A; C; D\}$ if rank $R(s_0) < n + rank(D)$, where R is the Rosenbrock matrix of system (8):

$$R = \begin{bmatrix} SI - A & -D \\ C & 0 \end{bmatrix}$$

Definition 2: System (8) is called (strongly) observable if for any initial state $x(0)$, $y(t) \equiv 0$ ($\forall\, t \geq 0$) implies $x(t) \equiv 0$ ($\forall\, t \geq 0$).

The following statements are equivalent ([3]).

(i) The system (8) is strongly observable.

(ii) The triple $\{A; C; D\}$ has no invariant zeros.

From these definitions, it's has been shown that the system (8) is strongly observable.

B. Third order sliding mode observer

In order to develop the observer, we rewrite the dynamic model defined in (1) in the state form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{q} = M^{-1}(F_g - C(x_1, x_2)x_2 - K(x_1)) \\ y = x_1 \end{cases} \tag{9}$$

where the state vector $x = (x_1, x_2)^T = (q, \dot{q})^T$, and $y = q$ is the measured outputs vector of the system.

Thus, we obtain:

Before developing the sliding mode observer, let us consider the following assumptions(see [3]):

1. The state is bounded ($\|x(t)\| < \infty$, $\forall\, t \geq 0$).

2. The system is inputs bounded ($\exists$ a constant $\mu \in \Re$ such as: $|u_i| < \mu$, $i = 1..2$).

3. The generalized forces F_g are bounded ($\exists$ a constant $\zeta \in \Re$ such as: $F_{gi} < \zeta$, $i = 1..3$).

4. In the vector $C(x_1, x_2)x_2$, we find quadratics elements in x_2. We can then bound this vector as:

$$\| C(x_1, x_2)x_2 \| \leq c_1 \| x_2 \|^2 \tag{10}$$

This last assumption is coming from mechanical and physical properties of our system and because of boundness of the real signals (position, speed, acceleration).

We propose in this work the following third order sliding mode observer ([3]):

$$\left\{ \begin{array}{l} \dot{v}_0 = -\lambda_0 \left|z_0 - y\right|^{2/3} sign(z_0 - y) + v_1 \\ \dot{v}_1 = -\lambda_1 \left|z_1 - z_0\right|^{1/2} sign(z_1 - z_0) + v_2 \\ \dot{v}_2 = -\lambda_2 sign(z_2 - z_1) \end{array} \right| \tag{11}$$

where v_0, v_1 and v_2 are respectively the estimate of x_1, x_2 and $\dot{x}_2$.

This observer permit to estimate both positions, speeds and accelerations of the system. λ_0, λ_1 and $\lambda_2 \in \Re^3$ are the gains of the observer.

The Jerk of the system is bounded, satisfying the inequality:

$$f^+ \geq\ 2 \mid \dddot{y} \mid \tag{12}$$

Chosen the ith components of λ_0, λ_1 and λ_2 as: $\lambda_0^i = 3\sqrt[3]{f^+}$, $\lambda_1^i = 1.5\sqrt{f^+}$, $\lambda_2^i = 1.1 f^+$, $i = 1..3$, we obtain the convergence in finite time t_0 ([3]), both the positions, speeds and accelerations v_0, v_1 and v_2.

Let consider the convergence of the acceleration of the body for $t > t_0$. According to the equation 3, we estimate this acceleration as follows:

$$\hat{\ddot{z}} = (k_1\hat{q}_1 + k_2\hat{q}_2 + (k_1 - k_2)\frac{T_w}{2} sin(\hat{\theta}) + B_1\hat{\dot{q}}_1 \tag{13}$$

$$+B_2\hat{\dot{q}}_2 - (B_1 - B_2)\frac{T_w}{2} cos(\hat{\theta})\ \hat{\dot{\theta}})/M \tag{14}$$

where (^) represents the estimated symbol.

The estimation error is obtained from (3) and (13) as follows:

$$\tilde{\ddot{z}} = (k_1\tilde{q}_1 + k_2\tilde{q}_2 + (k_1 - k_2)\frac{T_w}{2} sin(\tilde{\theta}) + B_1\tilde{\dot{q}}_1 \tag{15}$$

$$+B_2\tilde{\dot{q}}_2 - (B_1 - B_2)\frac{T_w}{2}(cos(\theta)\dot{\theta} - cos(\hat{\theta})\hat{\dot{\theta}}))/M$$

where (~) represents the estimated error symbol.

We can easily remark, related to the equation (15) and after convergence of v_0, v_1 and v_2, that the estimation error $\tilde{\ddot{z}}$ converges toward 0 in finite time $t > t_0$.

Assuming that the initial conditions of speeds and positions are known, we are able to do a double integration to the acceleration $\hat{\ddot{z}}$ in order to obtain the vertical displacement $\hat{z}$ of the chassis. On another hand, and from the equations 4, we estimate the vertical displacements of the wheels using the following equations:

$$\begin{cases} \hat{z}_{r1} = \hat{z} - \hat{q}_1 - \frac{T_w}{2}\sin(\hat{\theta}) - r\cos(\hat{\theta}) \\ \hat{z}_{r2} = \hat{z} + \hat{q}_2 - \frac{T_w}{2}\sin(\hat{\theta}) - r\cos(\hat{\theta}) \end{cases} \tag{16}$$

At time $t > t_0$, all positions including the vertical displacement $\hat{z}$ of the chassis are estimated. In such case and related to the equations (16), we reconstruct in finite time these displacements.

To estimate the road profile first, it's necessary to estimate in finite time the vertical acceleration of the wheels. We can notice that it's possible with a double derivation of the equation 16 since all others states are observed for $t > t_0$.

Let consider now, the equation 6. We can remark that since all states and vertical accelerations of the wheels are now known, we can show the convergence of the road profile $\hat{u}_1$ and $\hat{u}_2$ in finite time.

The finite time estimation of the road profile and the vertical displacement of the wheels, allow finally to reconstruct the vertical forces in finite time using the following equation:

$$\hat{F}_{ni} = F_{ci} + k_i(\hat{u}_i - \hat{z}_{ri}), \quad i = 1..2 \tag{17}$$

In the next section, we give some simulation results to test and validate the method.

IV. ESTIMATION RESULTS

We give in this section some results related to the estimations of the states, the road profile and the vertical forces, in order to verify the robustness of the proposed approach.

Several road profile with different amplitudes and frequencies are then measured by the LPA to excite the system.

The LPA instrument is represented in the figure (3).

Fig. 3. Longitudinal Profile Analyser (APL in french)

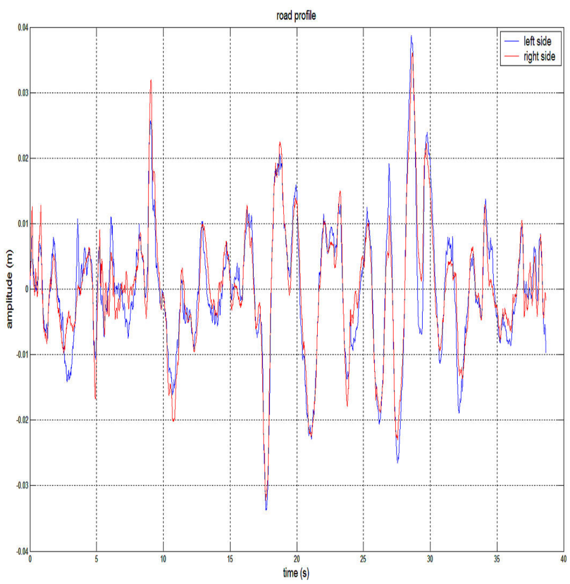

Fig. 4. Measured road profile

The figure (4) shows an example of the left and right road profile measured by this instrument.

First, let see if the states are well estimated. In the top of the figure (5), we show the estimation of the suspension deflection and the roll angle.

We remark that these positions are well and fast observed compared to those given by PROSPER's simulator.

The estimations of its speeds are shown in the bottom of the same figure (5).

We notice that the speeds are also correctly estimated in finite time with small errors.

Normally and as we announced previously, the well estimation of the states, allow to reconstruct the vertical acceleration of the body.

In the figure (6), we show the result of this reconstruction. We can notice that this acceleration converges in finite time to the true one given by PROSPER's Simulator.

Since the vertical acceleration is well estimated, the double

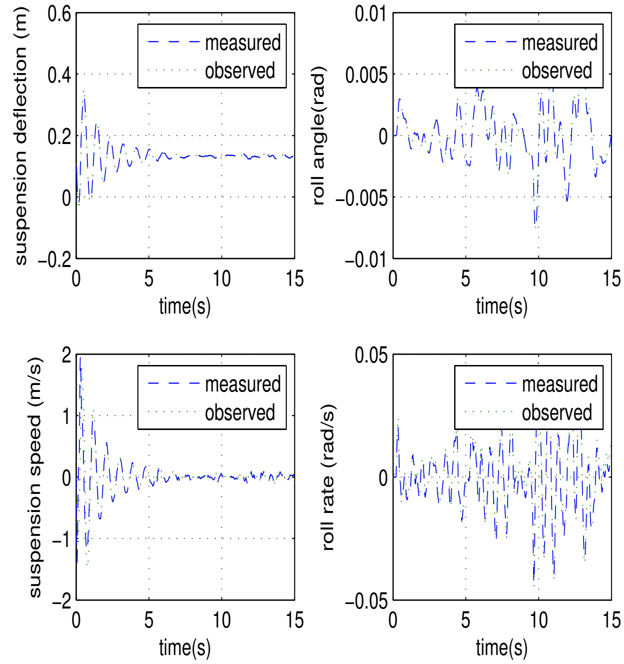

Fig. 5. Estimation of the states

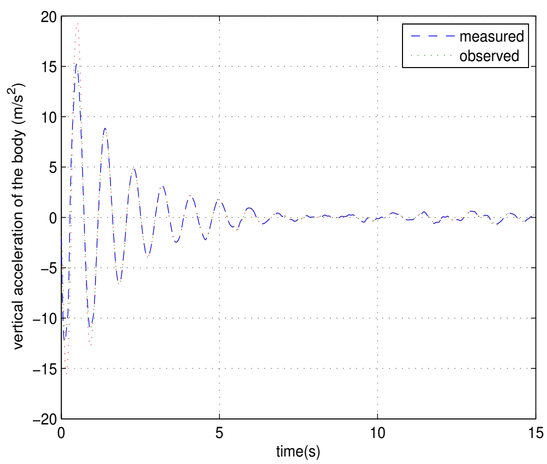

Fig. 6. Estimation of the vertical acceleration of the body

integration as shown in the figure (7), gives an accurate estimation of the height center of gravity of the body compared to the true one.

As we said in the previous section, using the equation 4, and if we can estimate the positions vector q and the center height of gravity of the body z, we can calculate the vertical displacements of the wheels z_{r1} and z_{r2}. This estimation is then shown is the figure (8).

We can notice that these displacements are well and in finite time reconstructed. The result about the estimation of the road profile is shown in the figure (9).

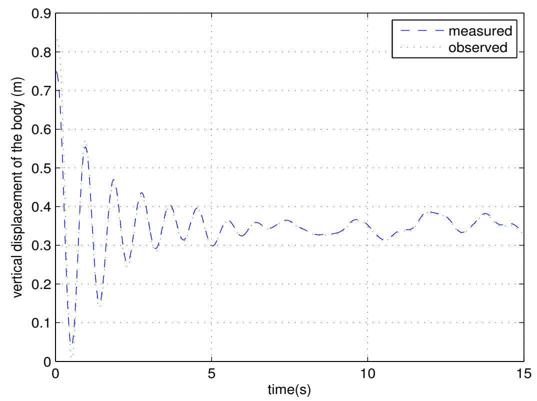

Fig. 7. Estimation of the center height of gravity of the body

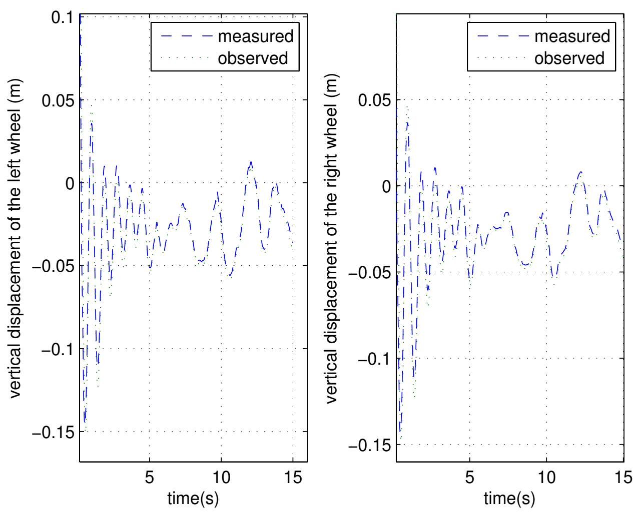

Fig. 8. Estimation of the vertical displacements of the wheels

We notice that these signals are estimated with small errors compared to those measured by the LPA instrument.

It's important to notice that the estimation's error can be corrected with a best choose of the parameter f^+.

From the estimation of the vertical displacement of the wheels and the road profile, we can now reconstruct the vertical forces.

The result is shown in the figure (10).

We remark that the vertical forces of the vehicle converge in finite time to the true one coming from PROSPER's simulator.

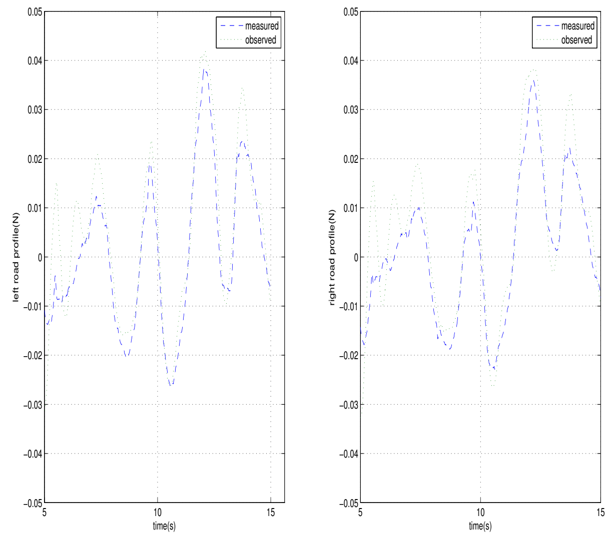

Fig. 9. Estimation of the road profile

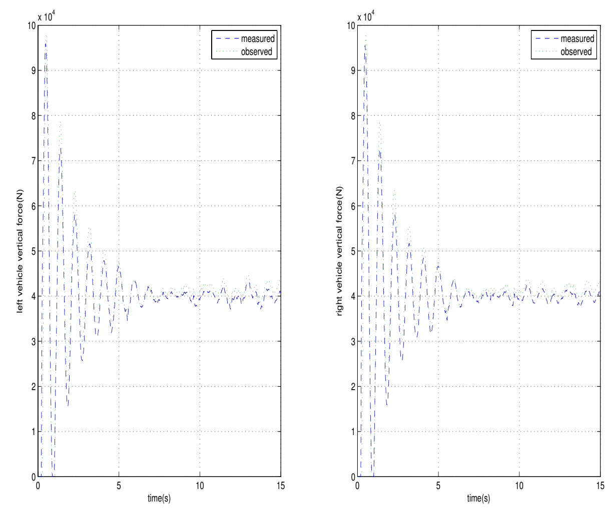

Fig. 10. Estimation of the vertical forces

V. CONCLUSIONS AND FUTURE WORKS

This paper proposes a method to estimate the vertical forces applied on the wheels of the heavy vehicle. These forces depend on the road profile under each wheel. The first step of the work is then to estimate these unknown inputs. To reach this aim, we develop the third order sliding mode observer. The particularity of this observer is that, it permits to estimate in finite time all states of the system including accelerations. The setting of an estimator needs a model. In this work, we develop a half tractor's model with 4 degrees of freedom because it is simple but sufficiently specified for the concerned application. It's validated using PROSPER's simulator. In the second step of the work, and in order to be able to use third order observer, we need to show the strong observability of the system. Since this is verified, we develop in the following step the observer. Different simulations are done and showed that the suspension deflection, roll angle and its speeds and accelerations converge quickly and in finite time. This well estimation allow to reconstruct the acceleration of the height center of gravity of the body and the acceleration of the wheels. This allow also to estimate well and fast the vertical displacements of the wheels. The simulations results show an accurate reconstruction of the road profile. Finally and from previous estimation, the figure showed that the convergence of the vertical forces is of quality compared to PROSPER's simulator result.

Future works:

The future work will concern the application of this method to the instrumented heavy vehicle (Tractor+semi-trailer) to estimate the road profile and vertical forces under each wheel on board.

VI. Acknowledgements

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