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EM-Based Recursive Tracking Algorithm for Near-Field Moving Sources

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Abstract— **In this paper, we address the problem of joint tracking of the direction of arrival (DOA) and range parameters of moving sources in the near-field of an antenna array with the Expectation-Maximization (EM) based recursive algorithm. The main characteristic of the proposed recursive EM approach is to include computation of the gradient of the log-likelihood and some form of the complete-data Fisher information matrix. The proposed recursive algorithm in this work assumes that the parameters of interest are described by a linear polynomial model. Simulation results of the suggested algorithm are also presented in order to illustrate the performance of the algorithms.**

I. INTRODUCTION

The problem of source localization using passive sensor arrays has various applications including radar, sonar, wireless communications, seismology, and electronic surveillance. However, the majority of the localization techniques deals with the case in which the source is assumed to be in the far-field of the array. If the sources are located close to the array (i.e. near-field), the inherent curvature of the waveforms needs to be taken into account. Therefore, the location of each source has to be parameterized in terms of the direction of arrival (DOA) and range parameters. In recent years, there has been lots of research effort on the near-field source localization techniques. Especially, due to many attractive features suitable to the near-field scenario such as consistency, asymptotic unbiasedness, and asymptotic minimum variance, ML approaches have been proposed recently [1]. Moreover, several techniques have been studied to reduce the complexity of the ML estimator including Expectation-Maximization (EM) iterative technique [2], [3]. EM algorithms for estimating constant DOA parameters were discussed in [4], [5], [6], [7]. Furthermore, recursive EM approaches are maintained because of the time consuming and massy calculation characteristic of the EM algorithm. After gathering only a little observation data, the recursive EM algorithm is appropriate for on-line processes i.e. tracking, while the conventional EM algorithm is more suitable for off-line processes. In addition, the recursive version of the EM algorithms was also applied to the time-varying DOA estimation problem [8],[9],[12].

In this paper, we primarily propose a recursive approach to perform maximum likelihood estimation of the time-varying parameters of moving sources in near-field of the antenna array. The proposed approach is based on the recursive form of the EM algorithm which is based on the stochastic approximation procedure applied directly on the parameters of interest. It involves computation of the gradient of the log-likelihood and some form of the complete-data Fisher information matrix [9].

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II. SIGNAL MODEL

We will first describe the time-varying near-field signal model in the sequel. In the near-field scenario under consideration, it is assumed that the source signals are collected by a uniform linear array. M narrow band signals from time-varying directions $\boldsymbol{\theta}(t) = [\theta_1(t), \ldots, \theta_M(t)]^T$ at an array of N sensors. $\boldsymbol{r}(t) = [r_1(t), \ldots, r_M(t)]^T$ represents the unknown range parameters of the mobile sources with non-linear movement. Moreover, $\boldsymbol{\Theta}(t) = [\boldsymbol{\theta}^T(t), \boldsymbol{r}^T(t)]^T$ represents the parameter super-vector to be estimated corresponding to the moving sources. Thus, the signal model for the data observed at the output of the sensors at time instant t is $\boldsymbol{x}(t) \in \mathbb{C}$;

$$\boldsymbol{x}(t) = \boldsymbol{H}(\boldsymbol{\Theta}(t))\boldsymbol{s}(t) + \boldsymbol{u}(t), \qquad t = 1, 2, \ldots \tag{1}$$

where the steering matrix is

$$\boldsymbol{H}(\boldsymbol{\Theta}(t)) = [\boldsymbol{d}(\boldsymbol{\Theta}_1(t), \ldots, \boldsymbol{d}(\boldsymbol{\Theta}_M(t))] \in \mathbb{C}^{N \times M} \quad . \tag{2}$$

Steering matrix consists of M steering vectors $d(\boldsymbol{\Theta}_m(t)) \in \mathbb{C}^{N\times 1}$, $m = 1, \ldots, M$ which is a function of unknown parameter vector $\boldsymbol{\Theta}_m(t) = [\boldsymbol{\theta}_m^T(t), \boldsymbol{r}_m^T(t)]^T$. For the mth source with an array of N sensors, the steering vector can be written as

$$\boldsymbol{d}(\mu_m(t), \zeta_m(t)) = \begin{bmatrix} e^{j(k_{min}\mu_m(t) + k_{min}^2\zeta_m(t))} \\ \vdots \\ 1 \\ e^{j(\mu_m(t)+\zeta_m(t))} \\ e^{j(2\mu_m(t)+4\zeta_m(t))} \\ \vdots \\ e^{j(k_{max}\mu_m(t) + k_{max}^2\zeta_m(t))} \end{bmatrix} . \tag{3}$$

k_{min} and k_{max} denote $-N/2$th and $N/2$th sensors, respectively. The steering vector parameters $\mu_m(t)$ and $\zeta_m(t)$

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are functions of the DOA parameter $\theta_m(t)$ and the range parameter $r_m(t)$ of the mth source as

$$\mu_m(t) = -\frac{2\pi\Delta}{\lambda}\sin\theta_m(t),$$
$$\zeta_m(t) = \frac{\pi\Delta^2}{\lambda r_m(t)}\cos^2\theta_m(t) \tag{4}$$

where λ is the wavelength of wavefronts, Δ is the distance between two successive sensors. We also assume that $M < N$, the and waveforms of the M narrow band signals $\boldsymbol{s}(t) = [s_1(t)\dots s_M(t)]^T \in \mathbb{C}^{M\times 1}$ are unknown and deterministic. Noise process $\boldsymbol{u}(t) \in \mathbb{C}^{N\times 1}$ is independent, identical white complex Gaussian distributed with zero mean and covariance matrix $\nu\boldsymbol{I}$, where ν represents an unknown noise spectral parameter and $\boldsymbol{I}$ is the identity matrix.

Before discussing the development of the proposed REM approach, it is helpful to introduce the assumptions on the signal model (1):

Assumption 1: Let $\boldsymbol{x}(1), \boldsymbol{x}(2), \dots$ be independent observations with $f(\boldsymbol{x};\boldsymbol{\vartheta})$ the probability density function, where $\boldsymbol{\vartheta}$ denotes an unknown parameter vector.

Assumption 2: The augmented data associated with the EM algorithm $\boldsymbol{y}(1), \boldsymbol{y}(2), \dots$ is characterized by the pdf $f(\boldsymbol{y};\boldsymbol{\vartheta})$ [9]. The augmented data $\boldsymbol{y}(t)$, $\mathcal{M}(\boldsymbol{y}(t)) = \boldsymbol{x}(t)$ is a many to one mapping [2]. Let $\boldsymbol{\vartheta^t}$ denote the estimate after t observations.

The problem taken into consideration is the estimation of the direction of arrivals $\boldsymbol{\theta}(t)$ and range parameters $\boldsymbol{r}(t)$ of the time-varying signals recursively from the observation $\boldsymbol{x}(t)$ for a known number of sources. With this problem at hand, we present a recursive ML solution based on the EM algorithm in the sequel.

III. REM Algorithm

We will first introduce the general EM framework and then develop the proposed recursive EM (REM) algorithm.

A. EM Framework

The EM algorithm provides ML estimation of parameters when maximization of the likelihood function may not be feasible directly. It is an iterative procedure which consist of expectation and maximization steps. Although, the EM algorithm is a batch oriented approach, it is desirable to process the received data in a recursive form in order to eliminate the delay, reduce storage requirements and increase the computational efficiency. We therefore consider tracking of near-field parameters via recursive form of the EM algorithm [8].

To be able to easily apply the EM algorithm, the signal model must be formed in terms of the observed data (incomplete data) and a hypothetical data set (complete data). The complete data must be chosen in such a way that: the complete data log likelihood function is easily maximized and the complete data log likelihood function can be easily estimated from the incomplete data [11]. The complete data $\boldsymbol{y}(t)$ and the incomplete data are related by a linear transformation.

Moreover, even for the application of the REM algorithm, the augmented data would be chosen with the following relation between the augmented data $\boldsymbol{y}_m(t)$ and the incomplete data $\boldsymbol{x}(t)$

$$\boldsymbol{x}(t) = \sum_{m=1}^{M}\boldsymbol{y}_m(t)\ . \tag{5}$$

The augmented data is obtained via separating the array output $\boldsymbol{y}(t)$ into its components as given below,

$$\boldsymbol{y}(t) = [\boldsymbol{y}_1^T(t)\dots\boldsymbol{y}_m^T(t)\dots\boldsymbol{y}_M^T(t)]^T \tag{6}$$

The incomplete data consists of M independent Gaussian vectors having mean $d(\boldsymbol{\Theta_m})s_m(t)$ and each with identical covariance $\nu_m\boldsymbol{I}/M$, thus the augmented data is given by

$$\boldsymbol{y}_m(t) = d(\boldsymbol{\Theta_m})s_m(t) + u_m(t), \qquad 1 \le m \le M\ \ . \tag{7}$$

Motivation behind this choice is that if one could somehow observe each of the incident waves separately, the estimation of its near-field parameters would be straightforward by performing M parallel maximization.

The logarithmic likelihood function of the augmented data is given as below [9]

$$\begin{aligned}\log f(\boldsymbol{y}(\boldsymbol{\theta});\boldsymbol{\vartheta}) \;=\; & -\sum_{m=1}^{M}\Bigg[N\log\pi + Nlog\left(\frac{\nu}{M}\right)\\ & + \left\{\frac{M}{\nu}\Big(\boldsymbol{y}_m(t) - d(\boldsymbol{\theta}_m)s_m(t)\Big)^H\right.\\ & \times \left.\Big(\boldsymbol{y}_m(t) - d(\boldsymbol{\theta}_m)s_m(t)\Big)\right\}\Bigg]\ .\end{aligned} \tag{8}$$

$(.)^H$ denotes the Hermitian transpose of a vector. The batch EM algorithm makes use of the log-likelihood function of the augmented data (8) to obtain ML estimates of the source parameters.

B. Time-varying Parametric Model

In the development of the REM approach, it is assumed that the parameters of interest are described by a linear polynomial model as,

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + t\boldsymbol{\theta}_1 \tag{9}$$
$$\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{r}_1 \tag{10}$$

where $\boldsymbol{\theta}_0 = [\theta_{01},\dots,\theta_{0M}]^T$, $\boldsymbol{\theta}_1 = [\theta_{11},\dots,\theta_{1M}]^T$, $\boldsymbol{r}_0 = [\boldsymbol{r}_{01},\dots,\boldsymbol{r}_{0M}]^T$, $\boldsymbol{r}_1 = [\boldsymbol{r}_{11},\dots,\boldsymbol{r}_{1M}]^T$. The direction of arrivals and the ranges are shown together in $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1^T,\dots,\boldsymbol{\Theta}_m^T\dots,\boldsymbol{\Theta}_M^T]^T$ and here $\boldsymbol{\Theta}_m = [\theta_{0m},\theta_{1m},r_{0m},r_{1m}]^T$. Since the recursive expectation maximization algorithm is only used for estimation of angle and range parameters, we therefore consider only the unknown parameter vector $\boldsymbol{\Theta}$ in the development of REM procedures rather than the complete unknown set $\boldsymbol{\vartheta} = [\boldsymbol{\theta}(t)^T\ \boldsymbol{r}(t)^T\ \boldsymbol{s}(t)^T\ \boldsymbol{\nu}]$. The problem we address in the sequel is the recursive estimation of the time-varying near-field parameters.

C. REM Approach

The REM approach we propose here uses the stochastic approximation approach which can be thought as a stochastic generalization of an optimization procedure namely the steepest descent method. In this approach,the true Hessian matrix inverse provides an adaptive step in a recursion to lead to an asymptotically optimal search direction.

This REM algorithm maximizes the augmented log likelihood using a stochastic approximation recursion at iteration t, given by

$$\boldsymbol{\vartheta}^{t+1} = \boldsymbol{\vartheta}^{t} + \varepsilon_t \boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^t)^{-1}\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t) \tag{11}$$

where ε_t is a decreasing step size and

$$\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^t) = E[-\nabla_{\vartheta}\nabla_{\vartheta}^T logf(\boldsymbol{y};\boldsymbol{\vartheta})|\boldsymbol{x}(t), \boldsymbol{\vartheta}]|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t}, \tag{12}$$

$$\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t) = \nabla_{\vartheta} logf(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} \tag{13}$$

represent the augmented information matrix and gradient vector, respectively, both evaluated at point ϑ^t. Moreover, $\nabla_{\boldsymbol{\vartheta}}$ is a column gradient operator with respect to $\boldsymbol{\vartheta}$.

Since the augmented data associated with the EM algorithm is characterized by the hypotethical data (complete data) assumed to arrive to the sensors separately instead of observed data (incomplete data), the augmented data $\boldsymbol{y}_m$ therefore have a more simple form than the observed data $\boldsymbol{x}$. Therefore, the calculation of the augmented data information matrix $\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^t)$ can be performed in a more simple way [9].

Choosing a proper step size is a critical issue for the algorithms tracking ability. However, the step size ε_t is chosen as a small positive constant in this work assuming the sources are moving slowly.

Moreover, the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t);\boldsymbol{\nu}^t)$ corresponding to the mth source DOAs $\boldsymbol{\theta}_m$ is given as below;

$$\begin{aligned}
&\frac{\partial}{\partial\theta_{0m}} logf(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} = \\
&\frac{2}{\boldsymbol{\nu}^t} Re\left[(\boldsymbol{x}(t)\boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H (d'(\boldsymbol{\Theta}_m^t)s_m^t)\right] \\
&\frac{\partial}{\partial\theta_{1m}} logf(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} = \\
&\frac{2}{\boldsymbol{\nu}^t} Re\left[(\boldsymbol{x}(t)\boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H (d'(\boldsymbol{\Theta}_m^t)s_m^t)\right].
\end{aligned}$$

Here the indices $0m$ and $1m$ refer to $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$ in equation 9. Similarly, the components of the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t);\boldsymbol{\nu}^t)$ corresponding to the mth source range parameters $\boldsymbol{r}_m$th are

$$\begin{aligned}
&\frac{\partial}{\partial r_{0m}} logf(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} = \\
&\frac{2}{\boldsymbol{\nu}^t} Re\left[(\boldsymbol{x}(t)\boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H (d'(\boldsymbol{\Theta}_m^t)s_m^t)\right] \\
&\frac{\partial}{\partial r_{1m}} logf(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} = \\
&\frac{2}{\boldsymbol{\nu}^t} Re\left[(\boldsymbol{x}(t)\boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H (d'(\boldsymbol{\Theta}_m^t)s_m^t)\right].
\end{aligned}$$

Here the indices $0m$ and $1m$ refer to $\boldsymbol{r}_0$ and $\boldsymbol{r}_1$ in equation 10. Furthermore, the derivatives of the steering matrix with respect to DOA parameter and the range parameter are given as follows, respectively

$$\begin{aligned}
\boldsymbol{d}'(\boldsymbol{\Theta}_m^t) &= \partial\boldsymbol{d}(\boldsymbol{\Theta}_m^t)/\partial\theta_m|_{\theta_m=\theta_{0m}^t+t\theta_{1m}^t} \qquad (14)\\
&= \left.\frac{\partial e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}}{\partial\theta_m}\right|_{\theta_m=\theta_{0m}^t+t\theta_{1m}^t} \\
&= j(-k\frac{2\pi\Delta}{\lambda}cos\theta_m - 2k^2\frac{\pi\Delta^2}{\lambda r_m}cos\theta_m \cdot sin\theta_m) \\
&\times \quad e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)} \\
\boldsymbol{d}'(\boldsymbol{\Theta}_m^t) &= \partial\boldsymbol{d}(\boldsymbol{\Theta}_m^t)/\partial r_m|_{r_m=r_{0m}^t+tr_{1m}^t} \qquad (15)\\
&= \left.\frac{\partial e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}}{\partial r_m}\right|_{r_m=r_{0m}^t+tr_{1m}^t} \\
&= j(-2k^2\frac{\pi\Delta^2}{\lambda r_m^2}cos^2\theta_m) \times \\
&\quad e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}
\end{aligned}$$

To prevent the singularity and simplify the iterations instead of the whole block diagonal matrix, $\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)$ with only diagonal components of $\boldsymbol{\ell}_{EM}(\boldsymbol{\vartheta}^t)$ is used in Eq. (11). The components of diagonal matrix $\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)$ corresponding to the mth source, $m = 1, \ldots, M$, DOA parameters and range parameters are given in (16) and (17) respectively:

$$\begin{aligned}
&diag[\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)]|_{\theta_0} = \\
&\frac{2}{\boldsymbol{\vartheta}^t} Re\left[(-d''(\boldsymbol{\Theta}_m^t)s_m^t)^H(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}_m^t)^H\right. \\
&\left. + M\|d'(\boldsymbol{\Theta}_m^t)s_m^t)\|^2\right] \\
&diag[\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)]|_{\theta_1} = \\
&\frac{2t^2}{\boldsymbol{\vartheta}^t} Re\left[(-d''(\boldsymbol{\Theta}_m^t)s_m^t)^H(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}_m^t)^H\right. \\
&\left. + M\|d'(\boldsymbol{\Theta}_m^t)s_m^t)\|^2\right]
\end{aligned} \tag{16}$$

$$\begin{aligned}
&diag[\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)]|_{r_0} = \\
&\frac{2}{\boldsymbol{\vartheta}^t} Re\left[(-d''(\boldsymbol{\Theta}_m^t)s_m^t)^H(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}_m^t)^H\right. \\
&\left. + M\|d'(\boldsymbol{\Theta}_m^t)s_m^t)\|^2\right] \\
&diag[\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)]|_{r_1} = \\
&\frac{2t^2}{\boldsymbol{\vartheta}^t} Re\left[(-d''(\boldsymbol{\Theta}_m^t)s_m^t)^H(\boldsymbol{x}(t)\boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}_m^t)^H\right. \\
&\left. + M\|d'(\boldsymbol{\Theta}_m^t)s_m^t)\|^2\right]
\end{aligned} \tag{17}$$

where the second derivatives of the DOA parameters and the range parameters are given as follows, respectively

$$\begin{aligned}
\boldsymbol{d}''(\boldsymbol{\Theta}_m^t) &= \partial^2\boldsymbol{d}(\boldsymbol{\Theta}_m^t)/\partial\theta_m^2|_{\theta_m=\theta_{0m}^t+t\theta_{1m}^t} \\
&= \left.\frac{\partial^2 e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}}{\partial\theta_m^2}\right|_{\theta_m=\theta_{0m}^t+t\theta_{1m}^t} \\
&= j\left(k\frac{2\pi\Delta}{\lambda}sin\theta_m - 2k^2\frac{\pi\Delta^2}{\lambda r_m}(cos^2\theta_m - sin^2\theta_m)\right) \\
&\times e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m + k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}
\end{aligned}$$

$$+\left(k\frac{2\pi\Delta}{\lambda}cos\theta_m+2k^2\frac{\pi\Delta^2}{\lambda r_m}(cos\theta_m\cdot sin\theta_m)\right)^2 \times e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)} \tag{18}$$

$$\begin{aligned}\boldsymbol{d}''(\boldsymbol{\Theta}_m^t) &= \partial^2\boldsymbol{d}(\boldsymbol{\Theta}_m^t)/\partial r_m^2|_{r_m=r_{0m}^t+tr_{1m}^t}\\ &= \left.\frac{\partial^2 e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}}{\partial r_m^2}\right|_{r_m=r_{0m}^t+tr_{1m}^t}\\ &=\left(j(2k^2\frac{\pi\Delta^2}{\lambda r_m^3}cos^2\theta_m)-(k^2\frac{\pi\Delta^2}{\lambda r_m^2}cos^2\theta_m)\right)\\ &\quad\times e^{j(-k\frac{2\pi\Delta}{\lambda}sin\theta_m+k^2\frac{\pi\Delta^2}{\lambda r_m}cos^2\theta_m)}.\end{aligned} \tag{19}$$

When the $\boldsymbol{\Theta}^{t+1}$ parameter is estimated, the signal and noise parameters are calculated by means of maximum likelihood estimation with respect to $\boldsymbol{\Theta}^{t+1}$ and $\boldsymbol{x}(t)$ given as follows;

$$\begin{aligned} s^{t+1} &= \boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}\boldsymbol{x}(t)\\ \nu^{t+1} &= \frac{1}{N}tr\left[\boldsymbol{P}(\boldsymbol{\theta}^{t+1})^{\perp}\widehat{\boldsymbol{C}}_x(t)\right]\end{aligned} \tag{20}$$

where $\boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}$ is the pseudo inverse of $\boldsymbol{H}(\boldsymbol{\theta}^{t+1})$ and $\boldsymbol{P}(\boldsymbol{\theta}^{t+1})^{\perp} = \boldsymbol{I} - \boldsymbol{P}(\boldsymbol{\theta}^{t+1})$ denotes the orthogonal complement of the following projection matrix: $\boldsymbol{P}(\boldsymbol{\theta}^{t+1}) = \boldsymbol{H}(\boldsymbol{\theta}^{t+1})\boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}$ and $\widehat{\boldsymbol{C}}_x(t) = \boldsymbol{x}(t)\boldsymbol{x}(t)^H$.

The update equation then has the following form,

$$\boldsymbol{\Theta}^{t+1} = \boldsymbol{\Theta}^t + \varepsilon_t\boldsymbol{\ell}_{EM}(\boldsymbol{\Theta}^t)^{-1}\boldsymbol{\gamma}(\boldsymbol{x}(t),\boldsymbol{\Theta}^t) \tag{21}$$

The steps of the proposed algorithm are summarized as follows;

Step1: Take initial values of DOA, $\boldsymbol{\theta}^0$ and range $\boldsymbol{r}^0$ parameters.

Step2: Calculate the gradient vector for DOAs and range parameters by (9) and (10).

Step3: Calculate the $\tilde{\boldsymbol{\ell}}_{EM}(\boldsymbol{\vartheta}^t)$ augmented data information matrix for DOAs and range parameters by (16) and 17).

Step4: Update the parameters by using (21).

Step5: Update the signal and noise parameters $\boldsymbol{s}^t$, $\boldsymbol{\nu}^t$ by (20) instead $\boldsymbol{\theta}^t$ using $\boldsymbol{\Theta}^t$.

The proposed REM approach is convenient for slowly moving sources, however it suffers from the value of the step-size. The step-size must be chosen properly in order to get more accurate parameter estimates. Moreover, the signal and noise variance are also estimated together with azimuth and range parameters due to structure of the algorithm.

IV. Simulations and Results

The near-field scenario is taken into consideration for deterministic signals received from unitary line array consisting of 5 sensors, and 2 sources in the simulation. The moving sources emit signals at different locations, i.e., have different directions of arrival and range parameter values. In this scenario, the targets (sources) are followed by 10000 time steps and the experiments are repeated 100 times. The azimuth angle is defined in degree and, the range is defined in Δ/λ.

Received signals and process noise are updated by using the observed data at every time instant, and then updated values are used while updating the parameter vector to be estimated. The augmented data matrix and the gradient vector are calculated at each step of the algorithms. For the step size an appropriate value is chosen to provide the stable operation.

The proposed algorithm is tested for a range of SNR values which changes from 0 to 40 dB. The results obtained from the simulations are presented in related figures. In all cases, the following MSEs are used for the $\boldsymbol{\theta}$ and $\boldsymbol{r}$

$$\begin{aligned} MSE_{\theta_m} &= \frac{1}{N}\sum_{n=1}^{N}(\theta_m-\hat{\theta}_{m_n})^2, \qquad m=1,2,\ldots,M\\ MSE_{r_m} &= \frac{1}{N}\sum_{n=1}^{N}(r_m-\hat{r}_{m_n})^2, \qquad m=1,2,\ldots,M\end{aligned} \tag{22}$$

Considering the proposed approach, true movement trajectories and the estimated trajectories of two non-linear moving sources are shown in the figure 1. The tracking of both DOA and range parameter values of the sources are illustrated in this figure. The tracking trajectory is calculated by virtue of the real azimuth angles and range parameter values of the both sources and the estimated values for the time instants. The mean square error (MSE) of the estimated direction of the arrival values and range parameter values of the non-linear moving sources for SNR value change from 0 to 40 dB in the near-field are given in figure 2 and figure 3, respectively.

V. Conclusion

In this study, the REM algorithm is proposed to estimate the directions of arrival and the range parameters of the near-field sources. The computer simulations expose that the calculation load changes with the variable step size and the calculation process takes less time by choosing an appropriate step size for the proposed approach. The number of iteration steps affects the computation time for the proposed algorithm. For time-varying parameters, the tracking ability

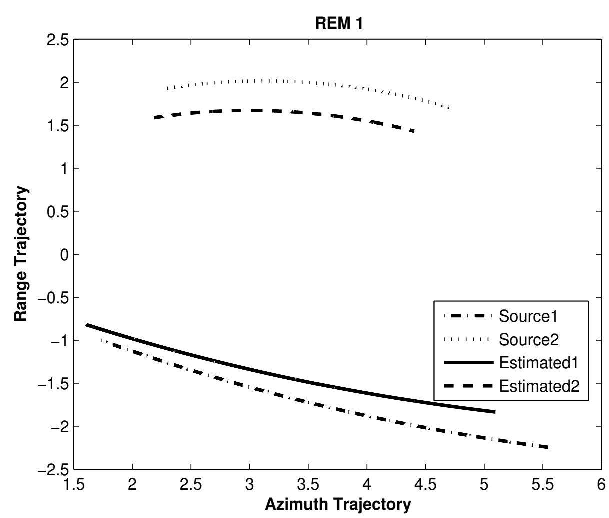

Fig. 1. True movement trajectory and estimated trajectory of the non-linear moving sources

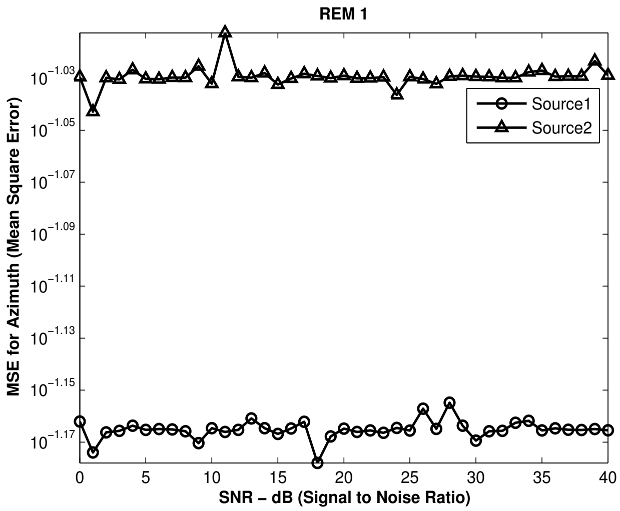

Fig. 2. MSE for DOA parameters of the non-linear moving sources

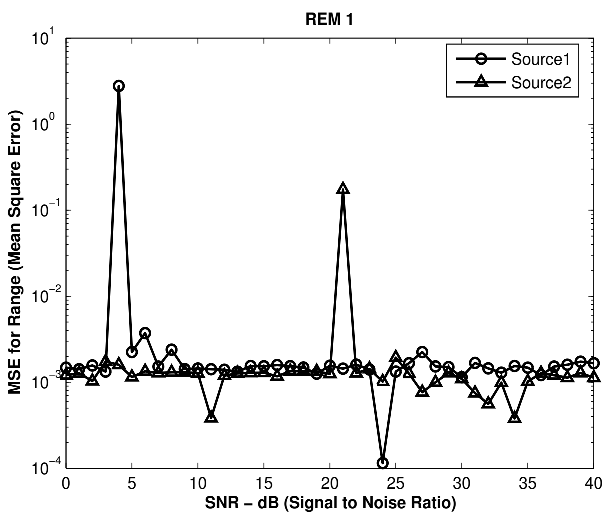

Fig. 3. MSE for range parameters of the non-linear moving sources

of a stochastic approximation procedure depends mainly on the dynamics of the true parameters, the gain matrix, and the step size [10]. Therefore, choosing suitable initial values plays an important role for performance of the algorithm. The DOAs and the range parameters both change at the same time so the movement of the objects is non-linear. the estimated trajectories follow the true movement trajectories at close range. It can be inferred that the MSEs for the DOAs and also the range parameters of the sources do not decrease too much by the increasing SNR values. Besides, the MSE values of the range parameters of both sources is very close to each other with changing SNR values.

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