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A RECEPTION PLATE METHOD OF MEASUREMENT OF THE FREE VELOCITY OF MACHINES IN BUILDINGS

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Abstract
The structure-borne sound power of installed machines requires a measure of both the vibration activity of the machine and the structural dynamics (mobility or impedance) at the contacts with the building structure. The source activity can be obtained as a free velocity, which requires the source to be removed from the installation whilst operating in otherwise normal conditions. There are practical difficulties in duplicating the operating loads on the machine and the free velocity must be measured at each contact point and often for more than one component of excitation. A practical alternative is described where the machine under test is attached to a thin plate, of high mobility, and the spatial average response velocity of the plate is recorded. This allows an estimate of the vibration power received by the plate, which equals the power delivered by the machine. The power is proportional to the sum of the squares of the source free velocity, over the contact points. An experimental determination of reception plate power is described and directly and indirectly measured free velocities are compared.

INTRODUCTION

In characterizing the structure-borne energy transmitted from a source into a structure it is convenient to be able to characterize the source and structure independently of one another, so that installed power transmission can be predicted from separately measured characteristics of the source and structure. One common source characteristic used is the free velocity of the contact points whereby the velocities of the contact points are measured independently of the supporting structure (with the source resiliently mounted).

The method described in this paper uses a reception plate, of a much higher mobility than the source, in place of the normal supporting structure. The steady-state reception plate structure-borne sound power is used in conjunction with the measured effective mobility of the plate to determine the sum of squared-free-velocities of the source.

MEASUREMENT THEORY

The approach is based on the method of Gibbs, Qi and Moorhouse[3]. The complex power transmitted to a receiving structure from a point source is given by[4],

$$W = \frac{1}{2}\left|v_{sf}\right|^2 \frac{Y_R}{\left|Y_S + Y_R\right|^2} \tag{1}$$

where W is the complex power, v_{sf} is the source free velocity and Y_R and Y_S are the receiver mobility and source mobility respectively. Taking the real part of the power and using a highly mobile receiver such that $\left|Y_R\right| >> \left|Y_S\right|$ equation (1) can be reduced to equation (2) where P_{SR} is the power transmitted from the source to the receiver.

$$P_{SR} = \frac{1}{2}\frac{\left|v_{sf}\right|^2}{\left|Y_R\right|^2}\mathrm{Re}\left(Y_R\right) \tag{2}$$

Where there is more than one contact point between the source and receiver, the concept of the effective mobility [6,5] may be used to simplify the description of interaction between the points where effective mobility, $Y^{\Sigma}{}_{Ri}$, is given by,

$$Y^{\Sigma}{}_{Ri} = Y_{Ri,i} + \sum \frac{F_j}{F_i} Y_{Ri,j} \tag{3}$$

In the absence of detailed knowledge of the contact forces, the forces can be assumed to be of equal magnitude and act coherently. If we assume that the contact points are incoherent sources, the effective mobility can be defined as follows [6] [5],

$$\mathrm{Re}\left(Y^{\Sigma}{}_{Ri}\right) \approx \mathrm{Re}\left(Y_{Ri,i}\right) \tag{4}$$

$$\left|Y^{\Sigma}{}_{Ri}\right|^2 \approx \sum_{j=1}^{N}\left|Y_{Ri,j}\right|^2 \tag{5}$$

It is not immediately clear whether a coherent or incoherent assumption would be most appropriate for a typical source and this was investigated further during the experimental measurements. The total power transmitted to the plate is then given by,

$$P_{SR}^{Total} = \frac{1}{2}\sum_{i}^{N}\frac{\mathrm{Re}\left(Y^{\Sigma}{}_{Ri}\right)}{\left|Y^{\Sigma}{}_{Ri}\right|^{2}}\left|v_{sfi}\right|^{2} \tag{6}$$

where N is the total number of contact points.

As the receiver is a uniform plate and the size of the plate is large compared to the spacing between the contact points we may further assume that the ratio of the real part of the effective mobility to the square of the magnitude is similar at each contact point such that the mean value is given by,

$$\frac{\mathrm{Re}\left(Y^{\Sigma}{}_{R}\right)}{\left|Y^{\Sigma}{}_{R}\right|^{2}} = \frac{\sum_{i=1}^{N}\left[\frac{\mathrm{Re}\left(Y^{\Sigma}{}_{Ri}\right)}{\left|Y^{\Sigma}{}_{Ri}\right|^{2}}\right]}{N}. \tag{7}$$

This allows equation (6) to be further simplified to express the total power transmitted as the product of the effective mobility of the contact points and the sum of squared free velocities,

$$P_{SR}^{Total} = \frac{1}{2}\frac{\mathrm{Re}\left(Y^{\Sigma}{}_{R}\right)}{\left|Y^{\Sigma}{}_{R}\right|^{2}}\sum_{i}^{N}\left|v_{sfi}\right|^{2}. \tag{8}$$

It can also be shown [2] that the total structure borne power transmitted to a reception plate is given by,

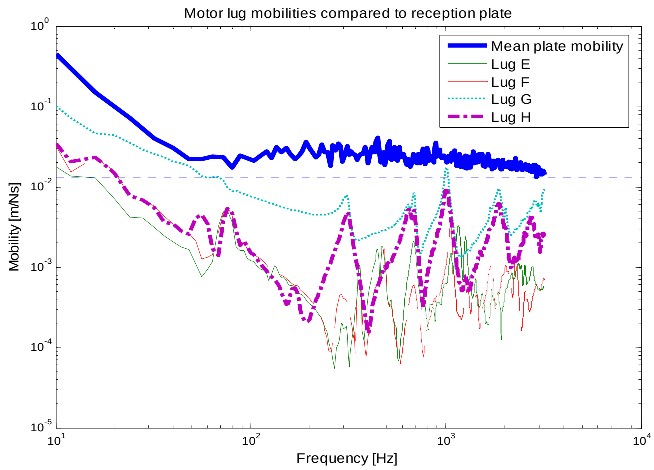
$$P_{SR}^{Total} = \omega\eta_{R}\ddot{m}_{R}S_{R}\left\langle v_{R}^{2}\right\rangle \tag{9}$$

where η_R is the total loss factor of the receiving plate of mass per square meter $\ddot{m}$ and surface area S_R and a spatial average of mean square plate velocity $\left\langle v_{R}^{2}\right\rangle$.

All of the terms on the right hand side of equation (9) are measurable, as is the effective mobility of the plate. Hence, by substituting the total power and effective mobility into equation (8) the sum of the squared free velocities of the source contact points may be obtained.

EXPERIMENTAL MEASUREMENTS

Experimental measurements were made on a 300W electric induction motor, from a domestic appliance, as a source connected at four points to a 1m x 2m high mobility perforated steel reception plate. The point mobility of the plate was measured at a number of randomly selected positions. The average plate point mobility was relatively flat above 50 Hz with a value of approximately 2x10^{-2} m/Ns. The average plate mobility was greater than the motor attachment points at all frequencies and was typically more than 10dB above the motor mobility allowing the $|Y_R| >> |Y_S|$ condition



to be applied.

Figure 1 –Average plate mobility, motor lug mobilities and plate characteristic mobility

Figure 1 shows the average plate mobility and the motor attachment point mobilities. Also shown is the calculated characteristic mobility [2] obtained from,

$$\mathrm{Re}\{\mathbf{Y}\} = \frac{1}{8\sqrt{B'\ddot{m}}}, \mathrm{Im}\{\mathbf{Y}\} = 0, \qquad (10)$$

$\ddot{m}$ is the mass per unit area and B' is the bending stiffness of the plate. The measured average mobility is on average 3 dB more than the calculated characteristic mobility. This is probably due to the use of a perforated plate. The equivalent solid plate specific mass and bending stiffness were calculated using the method of Soler and Hill [8] but there is some doubt whether these static estimates are valid for dynamic behavior[1].

The modal density of the reception plate was also calculated to ensure a relatively constant modal density over the frequency range considered and is plotted in Figure 2.

The plate loss factors were estimated using the reverberation time, T, of the plate vibration which was measured in 1/3 octave bands from 50 Hz to 5kHz using the reverse-integrated impulse-response method[7]. The loss factors were estimated using[9],

$$\eta = \frac{2.2}{f \cdot T} \qquad (11)$$

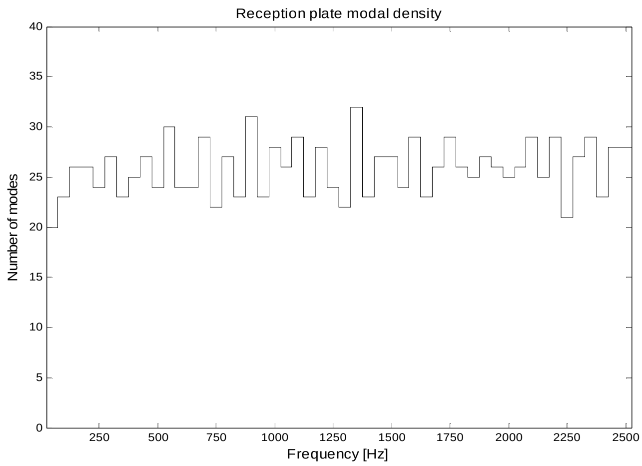


Figure 2 – Modal density of reception plate

The measured point and transfer mobilities for four plate contact points are shown in Figure 3. The four point mobilities are all similar to the average plate mobility (Figure 1), whilst the transfer mobilities decay as the frequency increases (i.e. as the distance between the points relative to wavelength increases).

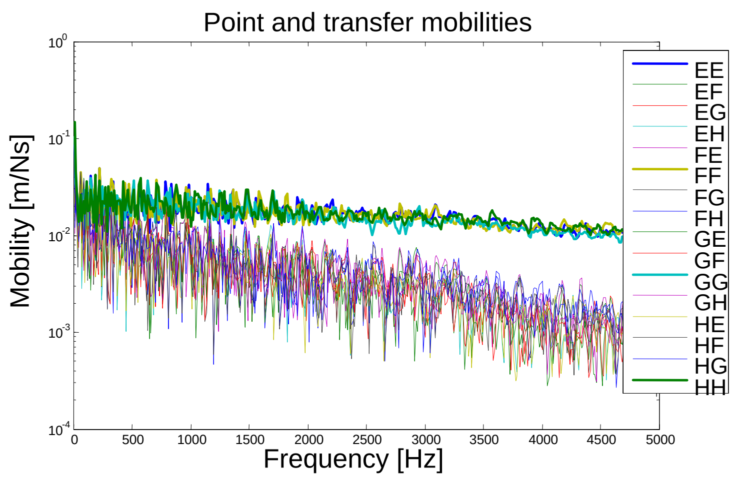


Figure 3 – Point and transfer mobilities for reception plate contact points (points E, F, G, H)

The complex effective mobility, Y^{Σ}_{Ri}, at each contact point was calculated using equation (3) and the mean effective mobility ratio, $\frac{\mathrm{Re}\left(Y^{\Sigma}_{R}\right)}{\left|Y^{\Sigma}_{R}\right|^{2}}$, was calculated for both coherent and incoherent force assumptions using equations (4), (5) and (7).

The magnitudes of the complex effective mobilities are plotted in Figure 4. Note the low frequency dip in mobility calculated using the coherent assumption due to destructive interference between the point mobility and the three transfer mobilities where the spacing between contacts is of the order of one-half bending wavelength.

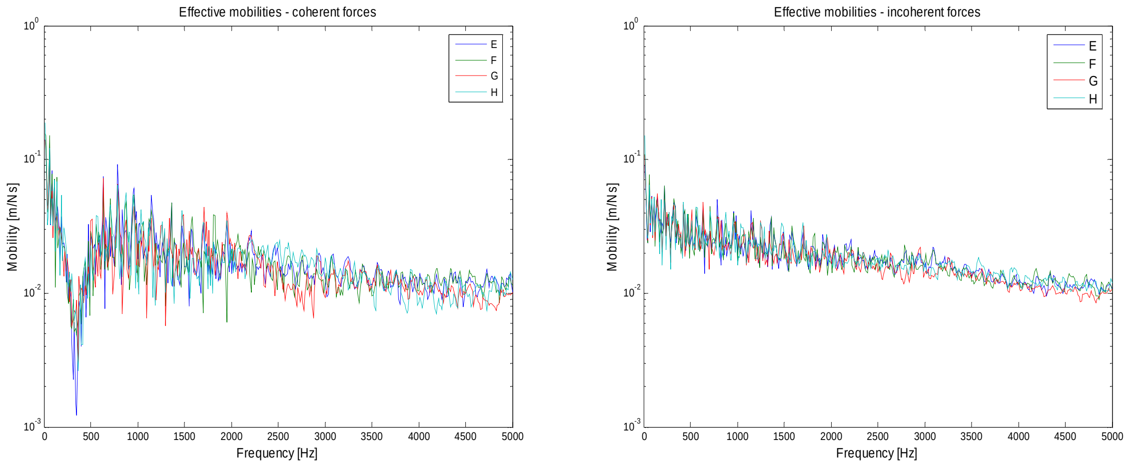


Figure 4 - Effective mobilites using coherent and incoherent force assumptions

The source motor was mounted on the plate and the plate velocities were recorded using five miniature accelerometers attached to the plate at randomly selected positions, avoiding the main axes of symmetry of the plate, to reduce the likelihood of being placed on a nodal point for a large number of plate modes. Narrowband values were obtained using FFT analysis and also 1/3 octave bands. The total power transmitted to the plate was calculated using equation (9) for a plate specific mass $\ddot{m}_R$, and surface area S_R, of 6.24 kg.m^{-2} and 2 m^2 respectively. The total power derived from the plate velocity was substituted into equation (6) with the mean effective mobility ratio to calculate the sum of the squared contact point free velocities for both motor speeds and with both coherent and incoherent force source assumptions.

DIRECT MEASUREMENT COMPARISON

In order to validate the calculated estimate of the sum of free velocities squared, the free velocities at the contacts were measured directly. The motor was suspended resiliently and the velocities of the contact points were measured. Measurements were repeated to obtain results for two motor speeds. Initial measurements were made using FFT analysis and a comparison of the directly measured sums of squared free velocities, and those determined from the plate vibration, are shown in Figure 5 as narrowband values. Comparisons of the free velocities determined using the coherent and incoherent force assumptions showed little difference except at the low frequencies where the half-wavelength dip dominated the coherent force effective mobilities. For the simple case reported in this paper the coherent force assumption gave a closer prediction of the measured free velocity at low frequencies although there was little difference in the results between the two assumptions. Further comparisons between the directly measured and calculated sum of squared free velocities were made using plate velocities measured using a 1/3 octave band analyzer. Results of these measurements are shown in Figure 6.

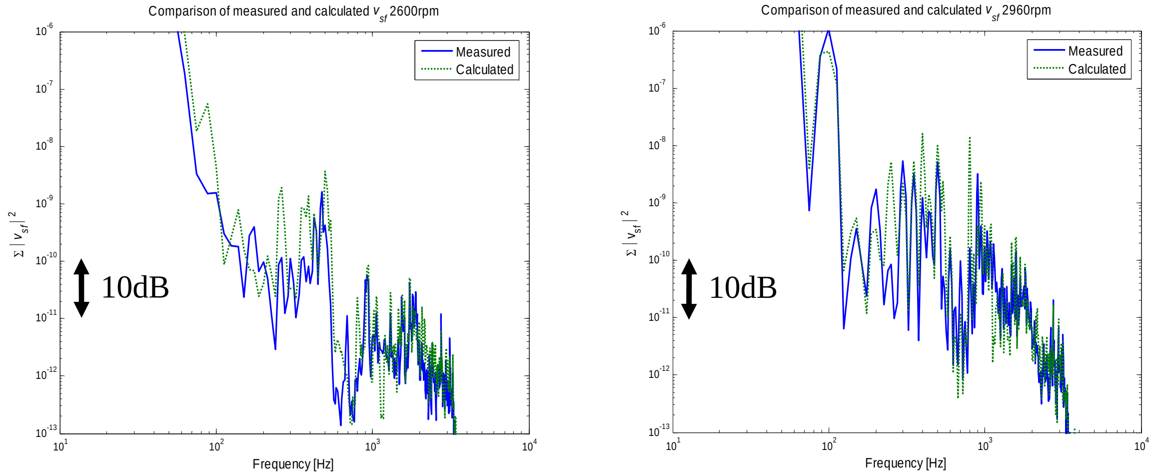


Figure 5 – Sum of squared free velocities for 2600rpm and 2960rpm motor speed

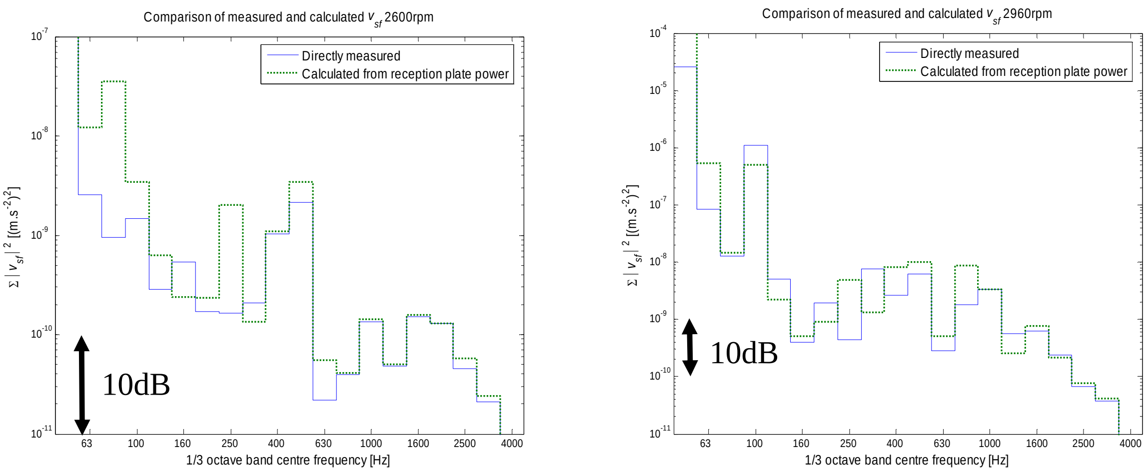


Figure 6 – 1/3 octave band sum of squared free velocities for 2600rpm and 2960rpm motor speed

At the higher motor speed the agreement is within 5dB at most frequencies with discrepancies of 10dB at some frequencies. At the lower motor speed, there are greater discrepancies, particularly at low frequencies, which is likely the result of signal to noise problems.

CONCLUSIONS

The technique described in this paper offers a simple practical method for the characterization of the free velocity of a vibration source. In particular it is shown that an estimate of source free velocity can be obtained from broadband plate velocity measurements, providing the characteristics of the plate are known. This makes the technique attractive as a method for routine measurement of source strength and could prove useful when predicting vibrational energy transmitted from installed machines into buildings and structures or the transmission of vibrational energy from source components into machine frames in the modeling of virtual acoustic prototypes.

The method has here been applied to a case of multiple point contacts. However, the method could be applied to cases of line or area contacts by discretizing the contact line / area and assigning free velocities to the mesh.

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