



BENDING WAVENUMBER AND ASSOCIATED DAMPING

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Abstract

This paper outlines and evaluates a non-destructive experimental technique used to obtain in-situ measures of the real part of the bending wavenumber in the two principal directions of a wood joist floor. Measurements confirm the highly orthotropic nature when the floor is bare, without topping. Wavenumbers in the direction parallel to the joists are a function of location between the joists. Lower wavenumbers in this direction can be expected closer to the joists. A floor topping is shown to significantly modify the wavenumbers in the exposed surface; with a topping, the floor is less orthotropic. Changes in measured wavenumbers due to adding a topping are shown to correlate well with the changes in vibration distribution and rates of structure borne attenuation. Finally, measured wavenumbers and vibration levels are combined with theory to obtain an estimate of the loss factor that agrees well with that measured by the power injection method.

INTRODUCTION

The paper begins by providing a brief overview of measuring wavenumbers using the draw-away approach[1]. Measured wavenumbers for the floor, with and without a topping, are compared to assess the effect a topping has on the vibration response of a wood joist floor. Changes in the measured wavenumbers are then compared to vibration mappings of the floor to show the correlation between wavenumbers and vibration gradients. Finally, measured wavenumbers, and vibration mapping results are combined with the necessary theory to obtain estimates of the loss factor using a modified intensity relation.

Using the draw-away approach, an estimate of the real part of the bending wavenumber is obtained from the slope of the best-fit line of measured phase change between a reference accelerometer near a point source and a field accelerometer that

is systematically moved away from the source,

$$\Re(\bar{k}_B) = \frac{d\phi}{dr} \quad \text{Eqn. 1}$$

The phase angle between the field and reference accelerometers (PCB 352C33) is obtained from a cross-spectrum analyzer (B&K 2144). There is no need to correct for phase mismatch between the measurement channels and the near field at the reference point. The phase mismatch is constant for all measurements, and does not affect the slope of the best-fit line (estimate of the wavenumber), only the intercept.

The difficulty of the approach is that the phase has to be “unwrapped” because the phase of the cross-spectrum is always in the principal branch of the arctangent function. However, by selecting a series of closely spaced measurement positions, as shown in Figure 1, it is possible to track the incremental increase in phase between consecutive points. The spacing between measurement points must be less than a half wavelength to unequivocally interpret “wrapping”.

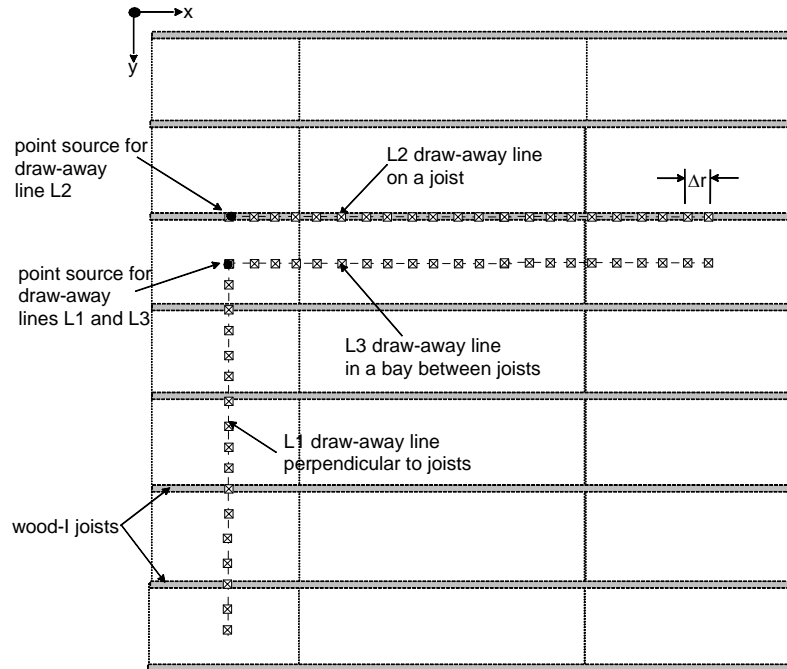


Figure 1: Sketch showing the layout of the three draw-away lines to estimate the change in phase with distance.

MEASURED WAVENUMBERS

This section presents wavenumber data for the two principal directions of the wood joist floor shown in Figure 1 when the floor is bare and when a topping is applied. The bare floor consisted of 1.21 x 2.42 x 0.018 m sheets of oriented strand board (OSB) applied with the long axis at right angles to 305 mm deep wood I-beam joists spaced 406 mm on center. The gypsum board ceiling was attached to the bottom of the joists via resilient channels spaced 610 mm on center. To this floor three toppings were systematically added. First was 25mm gypsum concrete on a

resilient interlayer, second was 25mm gypsum concrete applied directly to the OSB, third was a gypsum fiber board raft applied over the OSB but without mechanical connection. Details of these constructions can be found elsewhere[2].

The bare floor is assumed orthotropic. Parallel to the joists, one would expect the effective stiffness to increase as the line defined by the field points and source is moved toward a joist, with the maximum occurring immediately above a joist and the minimum midway between two joists. To test this hypothesis two draw-away lines were measured parallel to the joists – one immediately on top of a joist and the second midway between the joists, as shown in Figure 1. A third draw-away line was oriented perpendicular to the joists.

Figure 2 compares the measured in-situ wavenumbers along the three draw-away lines. Perhaps it is most interesting to compare the wavenumbers for the direction perpendicular to the joists (draw-away line L_1) with those parallel to the joists and midway between the joists (draw-away line L_3). Of particular interest is the fact that the two wavenumber curves cross, indicating that in the frequency range (315 to 500 Hz) around the crossing point the floor might be effectively isotropic because of the similarity in the OSB wavenumbers in the x and y-principal directions.

Above about 500 Hz in the direction parallel to the joists, the OSB wavenumber between the joists is higher than in the direction perpendicular to the joists indicating that the bending stiffness of the OSB is lower in the direction parallel to the joists than perpendicular. This is fully consistent with OSB panels having a Young's modulus (and stiffness) that is greater in the long axis of the panel which is oriented perpendicular to the joists when installed on a framed floor. Below about 315 Hz (frequency of first OSB sub-panel cross-mode), the wavenumber is greater in the direction perpendicular to the joists than in the direction parallel to the joists. This indicates that below about 315 Hz the stiffness of the OSB is now greater in the direction parallel to the joists than perpendicular. Also, the wavenumber measured in the bay tends to converge to the wavenumber measured on the joist as the frequency decreases indicating that below the first cross mode of the OSB sheathing the stiffness parallel to the joists is significantly affected by the joists.

Figure 3 shows the wavenumbers measured parallel and perpendicular to the joists but when the floor has a topping. It is evident that a topping modifies the vibration response (and hence wavenumbers) of the exposed surface. There are three important things to notice. First, with an applied topping the wavenumbers of the exposed surface are similar in the two orthogonal directions (parallel and perpendicular to the joists), for frequencies above about 200 Hz. The floor appears considerably less orthotropic than the bare floor. Second, below about 200 Hz the measured wavenumbers indicate the floor is still orthotropic. Here the wavenumbers parallel to the joists are controlled by the joists as they were for the bare floor. Third, the gypsum fiberboard raft (formed from two 12.7mm plies secured together by a viscoelastic material and staples spaced 200mm on centre) exhibited the largest wavenumbers of all the toppings (because it is the least stiff). While the bonded 25mm gypsum concrete exhibited the lowest (because it was the most stiff).

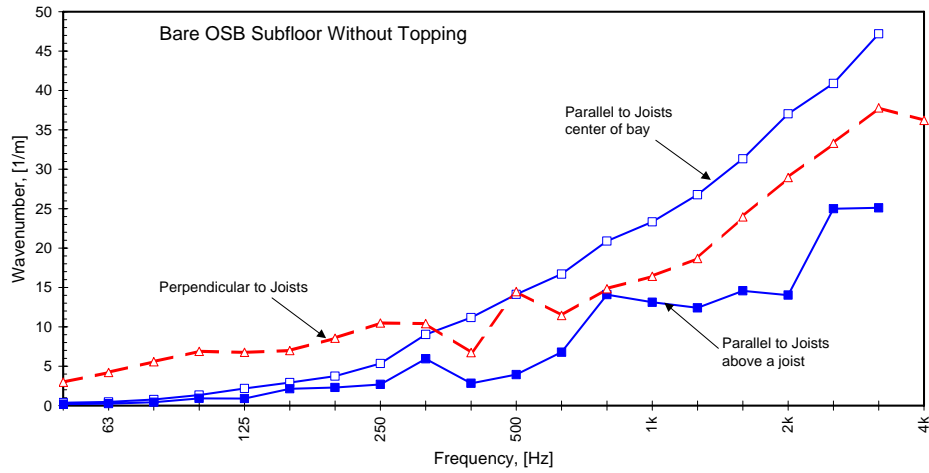


Figure 2: Estimates of the in-situ wavenumber for the direction perpendicular to the joists, k_y , using draw-away line L_1 and parallel to the joists, k_x , using draw-away lines L_2 and L_3 .

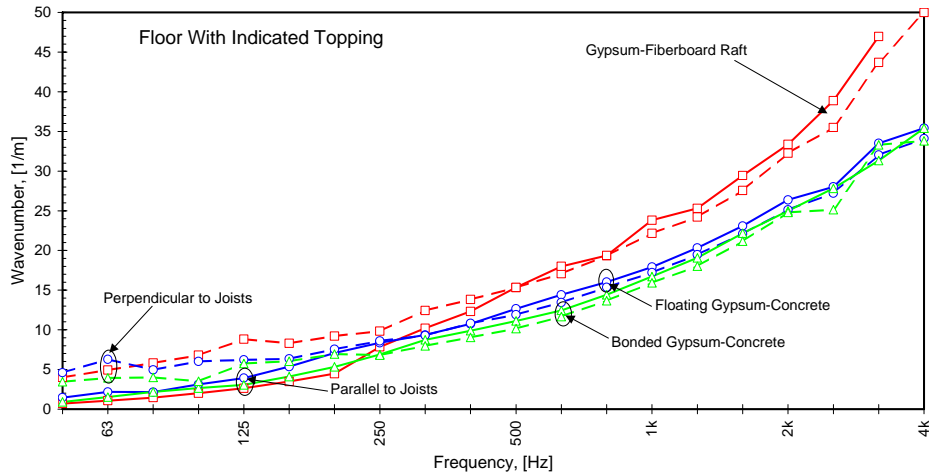


Figure 3: Estimates of the in-situ wavenumber for floor with the indicated topping in the direction perpendicular to the joists, k_y , using draw-away line L_1 and parallel to the joists, k_x , using draw-away line L_3 .

VIBRATION MAPS

In this section, vibration maps of the floor with and without a topping are compared to assess the effect a topping has on the distribution of vibration energy. These changes are qualitatively related to the change in wavenumber.

Figure 4 (top) shows the vibration response of the bare floor (i.e., without a topping) at 1kHz when excited by the point source used for wavenumber measurements. The mapping clearly shows the orthotropic nature of the floor – contours of equal acceleration are not circular. Instead, they are more elliptical with the long axis parallel to the joists implying there is considerably less propagation attenuation parallel to the joists than perpendicular to the joists.

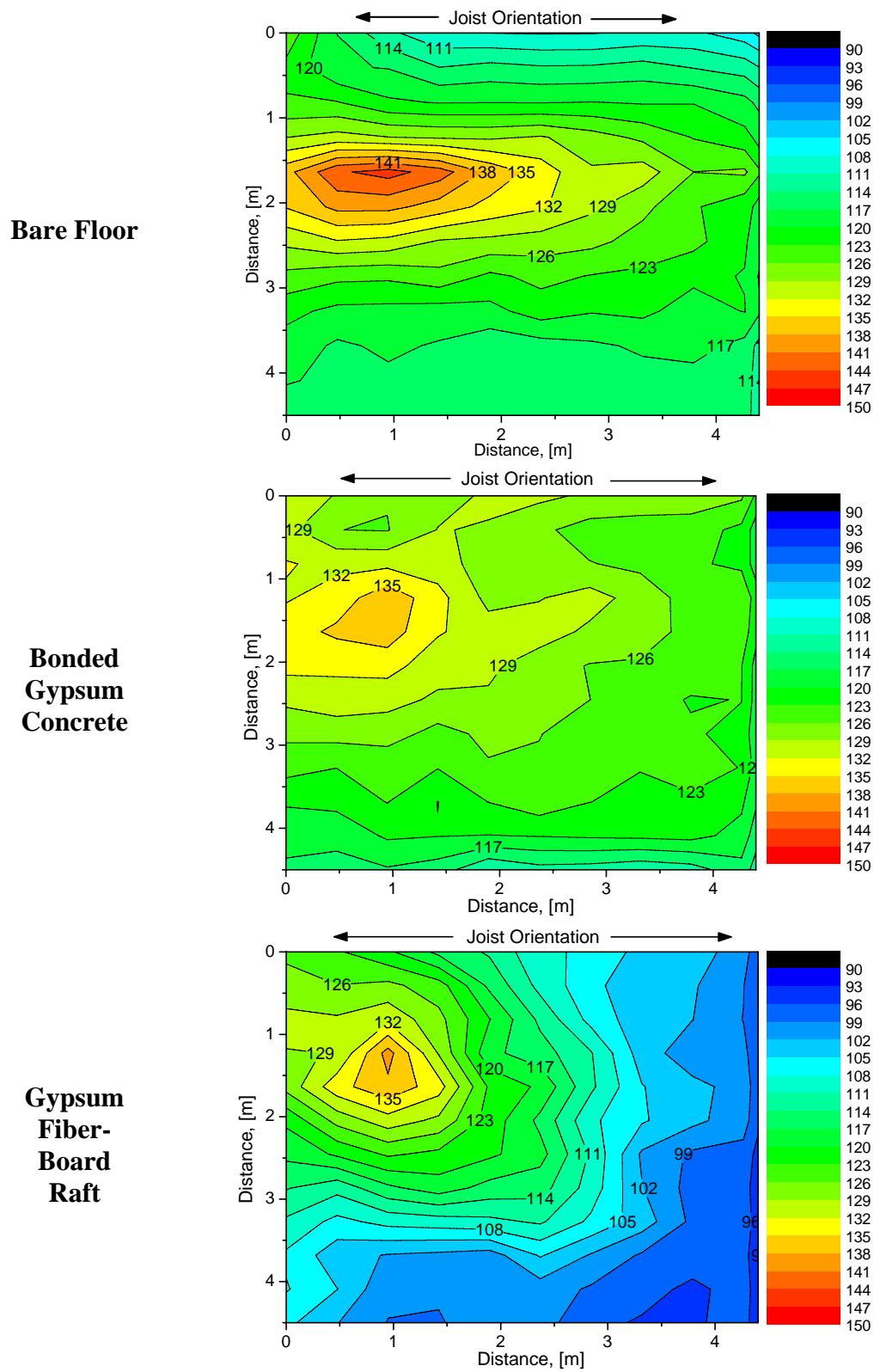


Figure 4: Contour map of measured acceleration level (dB arb.) at 1kHz for the wood joist floor with the indicated topping, if present. A point source is located at (0.85,1.44) m.

Figure 4 (middle) shows that with a bonded 25mm thick gypsum concrete topping, vibration levels become considerably more uniform and the propagation attenuation (change in vibration level with distance from the point source) is considerably reduced for both directions parallel and perpendicular to the joists.

Figure 4 (bottom) shows the vibration response for 25mm thick gypsum fiberboard raft. Here the contours most closely resemble circles centred about the source. Compared to the gypsum concrete topping, the rate of propagation attenuation is considerably greater in the gypsum fiber-board raft, and is attributed to two factors: increased internal loss factor due to the viscoelastic material between the two plies, and large wavenumber. As shown in the next section, propagation attenuation is a function of the product of the wavenumber and loss factor.

The figures have shown that the vibration response of the exposed surface of a floor can be significantly modified by adding a topping. Not only will the space average levels change (typically they will decrease) but the rates of propagation attenuation parallel and perpendicular to the joists will change, too. The general trend is that with a topping the rate of attenuation parallel and perpendicular to the joists will be more similar than for the bare floor.

ESTIMATING THE LOSS FACTOR

In this section, the measured wavenumbers and the vibration mappings are used to obtain estimates of the loss factor for the floor.

Consider a point source exciting an orthotropic plate. Suppose that we know the displacement at all points on this plate and that the wavenumbers in the two principal directions are known, too. We can then apply the intensity relation to estimate the loss factor of the plate. Define two contours that completely enclose the point source such that all points on that contour have the same phase. In other words define contours r_1 and r_2 such that

$$k_B(\theta)r_1(\theta) = \phi_1 \text{ and } k_B(\theta)r_2(\theta) = \phi_2 \quad \text{Eqn. 2}$$

where $\phi_1 < \phi_2$, and are constants for angles, θ . This immediately implies that the contours r_1 and r_2 must be inversely proportional to the slowness contour which is defined as the $k_B(\theta)/\omega$. If we select ϕ_1 and ϕ_2 to be greater than one, the effect of the near field can be avoided. The power, Π , flowing across the contours is given by

$$\Pi_1 = \oint (\vec{I}_1 \cdot \hat{n}_1)_1(\theta) d\theta \text{ and } \Pi_2 = \oint (\vec{I}_2 \cdot \hat{n}_2)_2(\theta) d\theta \quad \text{Eqn. 3}$$

where \hat{n} is the unit normal to the contour of integration. In the limit that there are no losses there must be conservation of energy so the power injected is equal to the power through the two contours; $\Pi_{\text{injected}} = \Pi_1 = \Pi_2$, but we are interested in the case where there are internal losses so this will not hold. Instead, we need to write[3],

$$\Pi_{\text{injected}} = \Pi_1 e^{-\frac{\phi_1 \eta}{2}} = \Pi_2 e^{-\frac{\phi_2 \eta}{2}} \quad \text{Eqn. 4}$$

where η is the loss factor. Equating the expressions and solving for η gives,

$$\eta = \frac{1}{\left(\frac{\phi_2 - \phi_1}{2}\right) \log e} \log \left(\frac{\Pi_1}{\Pi_2} \right) \quad \text{Eqn. 5}$$

In the limit that the plate is isotropic, the wavenumber and contour of integration are no longer angular dependent because they are constants, and the direction of power flow is radial i.e., normal to a circle centred at the source. This allows for simplification, and after some manipulation gives,

$$\eta = \frac{2}{k_B(r_2 - r_1) \log e} \log \left(\frac{\langle v_1^2 \rangle r_1}{\langle v_2^2 \rangle r_2} \right) \quad \text{Eqn. 6}$$

Letting r_1 and r_2 be large, such that the wave front approximates that of a plane wave, gives the result often in textbooks[4],

$$\eta = \frac{10\lambda}{13.6(r_2 - r_1)} \log \left(\frac{\langle v_1^2 \rangle}{\langle v_2^2 \rangle} \right) \quad \text{Eqn. 7}$$

To implement the approach one must know the angular dependent wavenumber as this defines the slowness (and integration) contour. Estimates can be obtained from measures in the two principal directions (parallel and perpendicular to the joists). Vibration levels along the contours of integration are generally found through interpolation of vibration levels measured on a regular grid (like those used to generate the maps above). With these input data it is possible to evaluate Eqn. 5 to obtain an estimate of the loss factor.

Figure 5 shows the loss factor for the wood joist floor with a bonded 25mm gypsum-concrete topping measured using the power injection method[5] and using the wavenumber method described above. In general, the agreement is good.

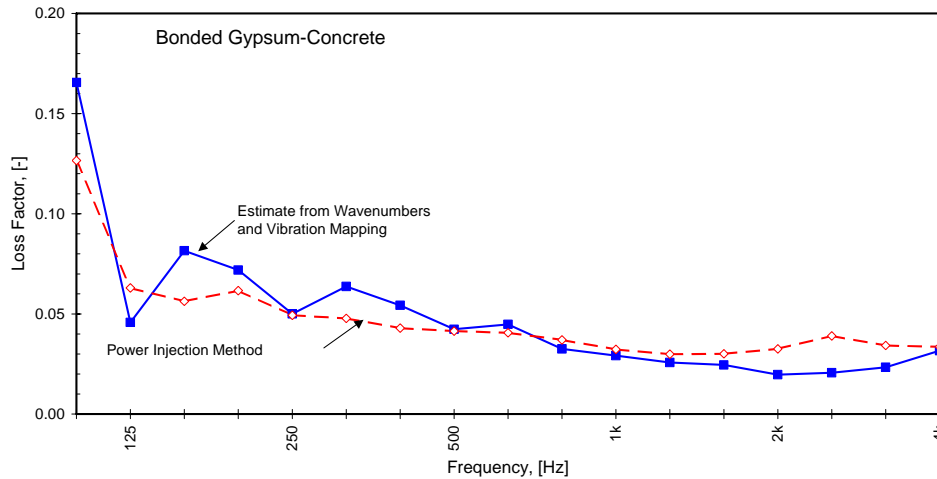


Figure 5: Loss factor of the floor with the bonded gypsum concrete topping measured using the wavenumber method (described above) and power injection method.

DISCUSSION AND CONCLUSIONS

Wavenumber measurements along draw-away lines parallel and perpendicular to the joists confirmed the highly orthotropic nature of a wood joist floor. Stiffness in the direction parallel to the joists is partially determined by the complex interaction of the OSB floor sheathing and the joists. Below the cut-on frequency of the first cross mode, the joists significantly affect the stiffness of the OSB floor sheathing parallel to the joists. Above the frequency of the first cross mode, the effect of the joists is significantly reduced and as the bending wavelength becomes much smaller than the joist spacing, the wavenumber (and bending stiffness) approaches that of the OSB floor sheathing in isolation[1]. Wavenumbers in the direction parallel to the joists also approached those measured on OSB in isolation[1].

Measurements on the exposed surface of the floor having a topping suggest that both bonded and floating toppings can significantly modify the vibration response. The toppings examined make the floor less orthotropic and for frequencies above about 200 Hz the floor appears essentially isotropic. Below this frequency, the floor appears orthotropic – significantly stiffer parallel to the joists.

In addition to making the floor more isotropic, the bonded gypsum concrete topping also significantly reduced the wavenumber relative to that measured on the bare OSB perpendicular to the joists and parallel to the joists but in the sub-panel bay. Whereas, a raft formed from gypsum fiberboard increased the wavenumber.

The rate of structure borne attenuation across the floor surface is proportional to the product of the wavenumber and internal loss factor. This allows the loss factor of an orthotropic floor to be obtained using a modified intensity relation developed here. Comparison of the change in wavenumbers and vibration distribution due to adding a topping suggests that the most effective toppings for controlling flanking transmission will have a high wavenumber and high internal loss factor so that propagation attenuation is maximized.

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