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# EVALUATION OF ELASTIC MODULUS BY RESONSCE FREQUENCY AND EULER-BEROUILLI EQUATION

Kisoo Kang[*1], Koungsuk Kim[2], Manyong Choi[1], Youngjune Kang[3], Seongmo Yang[3] and Dongpyo Hong[3]

[1] Korea Research Institute of Standard and Science,
P.O. Box 102, Yuseong, Daejeon, 305-600 Republic of Korea
[2] Department of Mechanical Design Engineering, Chosun University,
375 Seosuk-dong, Gwangju, 501-759 Republic of Korea
[3] Division of Mechanical & Aerospace System Engineering, Chonbuk National University,
664-14, Duckjin-dong, Jeonju, Jeonbuk, 561-756 Republic of Korea
[*] kskang@kriss.re.kr

## Abstract

Various testing technique to determine the properties of film material have been investigated such as Nano indentation test, Bulge test, Micro tensile test and so on. They each have strength and weakness in the preparation of test specimen and the analysis of experimental result. Recently, optical techniques have been developed for measuring mechanical properties. Among these methods, an effective technique is the use of a cantilever beam load by various means. The study proposes the elastic modulus evaluation technique of a cantilever beam by vibration analysis based on time-average electronic speckle pattern interferometry (TA-ESPI) and Euler-Bernoulli equation of cantilever beam. Elastic material properties critically affect the vibration behaviour of structures. The observation can be inverted, leading to the idea that the vibration behaviour of a particular material can be used to determine the material's elastic properties. The principle is the foundation of all vibration-based identification methods which used the Euler-Bernoulli beam theory to link the elastic modulus with the specimen's natural frequency.

## INTRODUCTION

Material properties, which play a major role in the mechanical behavior of structures and the properties have been standardized with previous testing methods such as the tensile test and bending test. However, the material properties of thin film may not be the same as those of bulk films. Moreover, the deposition process of a thin film affects the material properties of the film. Thus, it is important to determine the mechanical properties of thin material to predict the performances of micro structure devices. Because thin films have thickness of the order of microns, the measurement methods used for bulk materials become inappropriate[1]. Various testing techniques to determine the properties of thin film have been investigated: Nano indentation test[2], Bulge test[3], Micro tensile test[4] and so on. They each have strengths and weaknesses with respect to test specimen preparation and experimental result analysis, and recently, optical techniques have been developed for measuring mechanical properties. Among these methods, the cantilever beam loading method, in which the load is applied by various means, has become an effective technique. Comella and Scanlon [5] determined the stiffness and elastic modulus of an array of aluminum cantilever beams that were deflected by atomic force microscopy (AFM). Tsai and Fang [1], and Kang et al [6] used the optical method to measure the resonance frequency of a cantilever beam to determine its elastic modulus.

This paper proposes an elastic modulus evaluation technique of a cantilever beam by vibration analysis based on time-average electronic speckle pattern interferometry (TA-ESPI) and Euler-Bernoulli equation. Elastic material properties critically affect the vibration behavior of structures. We applied the reverse of this idea, in which the vibration behavior of a particular material can give the material's elastic properties. The principle is the foundation of all vibration-based identification methods which use the Euler-Bernoulli beam theory to link the elastic modulus with the specimen's natural frequency. TA-ESPI is a type of laser speckle interferometry with the advantage of non-contact, non-destructive, high-resolution and whole-field measurement. The technique has been developed as a common measurement method for vibration mode visualization and surface displacement. A harmonically vibrating object has the maximum surface displacement at its resonance frequency. The amplitude of the vibration is directly proportional to TA-ESPI fringe order. The number of the TA-ESPI fringe order is a clue for finding the resonance frequency at each vibration mode shape. Thus, the elastic modulus of a test material can be readily estimated from the measured resonance frequency and Euler-Bernoulli equation. The TA-ESPI vibration analysis technique is able to give the elastic modulus of particular materials, requiring simple preparation process and analysis.

## PRINCIPLE

### Time Average Electronic Speckle Pattern Interferometry

Holographic interferometry has been a powerful technique in the measurement of vibration mode shape and surface displacement. However, the analysis of the measurements is very complicated. Following this technique, several speckle interferometry techniques were developed. Their recording and reconstruction processes were fairly simple. TA-ESPI is based on Electronic Speckle Pattern Interferometry (ESPI), which also is a common measurement method for vibration mode shape and surface displacement. The ESPI fringe pattern represents both in-plane and out-of-plane displacement.

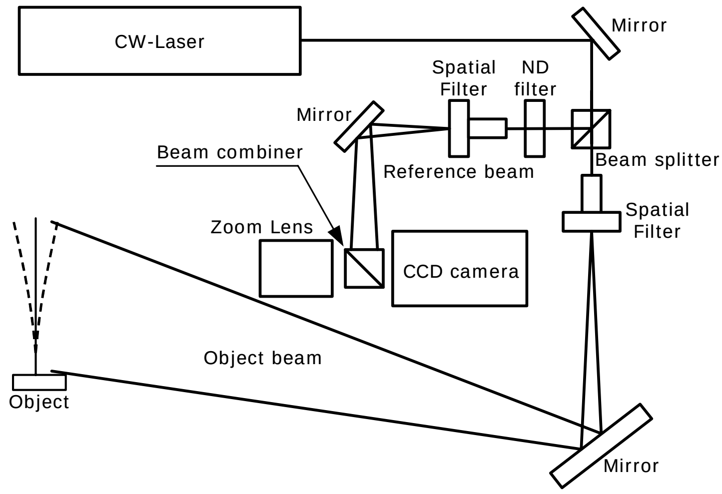


*Figure 1 – Out of plane displacement sensitive ESPI interferometer*

The interferometer sensitive to out-of-plane displacement is used as shown in Fig. 1. The detector plane of the camera is located in the image plane of the speckle interferometer. Under these conditions, the output signal from the CCD camera, as obtained from the object in its initial state, is recorded in the computer memory. The object is then displaced, and the live camera signal is subtracted or added electronically from the stored signal. The areas of the two images where the speckle pattern remains constant will give a resultant signal of zero, whereas the areas where the pattern changed will give nonzero signals[7]. In order to visualize a vibration mode shape, the interferometer is operated in the time-average mode. In this mode, the images are recorded and added together while the object is vibrating. A fringe pattern of the moving object is formed with an exposure time lasting over several periods of the vibrational motion. To understand the formation of fringe pattern (vibration mode shape), consider the intensity of before displacement $U_{steady}(x,y)$ as given by equation

$$U_{steady}(x,y) = A(x,y)e^{i\phi(x,y)} \tag{1}$$

where, $A(x,y)$ is amplitude and $\phi$ is the phase difference between the reference beam and object beam before the displacement. The displacement in a harmonically vibrating object is a periodic function of time and at a given instant t, the irradiance in the image plane is given by $U_t(x,y)$ .

$$U_t(x,y) = A(x,y)e^{i\left[\phi(x,y)+\frac{4\pi}{\lambda}a(t)\right]} \tag{2}$$

The function $a(t)$ represents the position of a given point on the object at time $t$ . The intensity is averaged over time $\tau$ to obtain

$$U_{TA}(x,y) = A(x,y)\frac{1}{T}\int_0^T e^{i\left[\phi(x,y)+\frac{4\pi}{\lambda}a(t)\right]}dt \tag{3}$$

The average is evaluated for a sinusoidal vibration, $a(t) = a_0 \sin \omega t$, and it can be assumed that $2\pi/\omega << \tau$, which means that the average is over several oscillations. The average reduces to

$$U_{TA}(x,y) = A(x,y)e^{i\phi(x,y)}\, J_0\left[\frac{4\pi}{\lambda}a_0\right] \tag{4}$$

where, $J_0$ is the zero-order Bessel function and $a_0$ is the amplitude of the vibrating object. The value of $U_{TA}(x,y)$ averaged over many speckle patterns is constant over the whole image, but the contrast of the speckle is seen to vary as the value of the $J_0^2$ function varies. The correlation fringe patterns thus observed map out the variation in $a_0$. A harmonically-excited object has the maximum surface displacement at its resonance frequency. The amplitude of the object is directly proportional to the number of the TA-ESPI fringe order, which is a clue for finding the resonance frequency at each vibration mode shape.

### Determination of Elastic Modulus

The natural vibration frequency can be calculated from the governing equation of a cantilever, an Euler-Bernoulli beam model. Assuming that beam thickness is uniform along its length, the *n*th natural frequency is given by[8]

$$\omega_n = \frac{c_n^2}{l^2}\sqrt{\frac{E\,I}{m}} \tag{5}$$

where, $c_n$ is the *n*th eigenvalue of the governing equation for a cantilever, $l$ is the length of the beam, $E$ is the elastic modulus, $I$ is the moment of inertia, and $m$ is the line density of the beam material. Rewriting equation (5):

$$E = \frac{m\,\omega_n^2\, l^4}{c^4\, I} \tag{6}$$

Hence, elastic modulus can be readily determined from equation (6)

## EXPERIMENTAL SETUP

Figure 2 shows the schematic of the experimental apparatus that is used to determine the resonance frequency of a cantilever beam. The specimens used for the experiment are stainless steel 304 (STS304), manufactured by Nilaco Co., Japan. Its geometry is 90 *mm* length, 5 *m*m width and 0.1 *mm* thickness. The specimens are excited by a loud speaker from its back and the exciting frequency is controlled by a function generator. The dynamic response of the cantilevers is measured by the TA-ESPI. A continuous Nd:YAG laser, as the illuminating source ($\lambda$ = 532 nm), is delivered by an optical fiber into a sensor, and the out of plane displacement sensitive interferometer is set up inside the sensor. Successive time-averaged images are added to the reference image when the beam is vibrating, and are displayed on computer screen in real time. The history of the vibration mode shape, according to the change of exciting frequency, is observed, and the resonance frequency at the maximum fringe order and the maximum displacement of the object are distinguished readily. The nodal line of the first mode shape appeared nearby the clamped side, and the line was difficult to be distinguished due to the limitation of the viewing direction. Resonance frequencies given at the 2nd, 3rd, 4th, 5th and 6th vibration mode shape are used to estimate the elastic modulus.

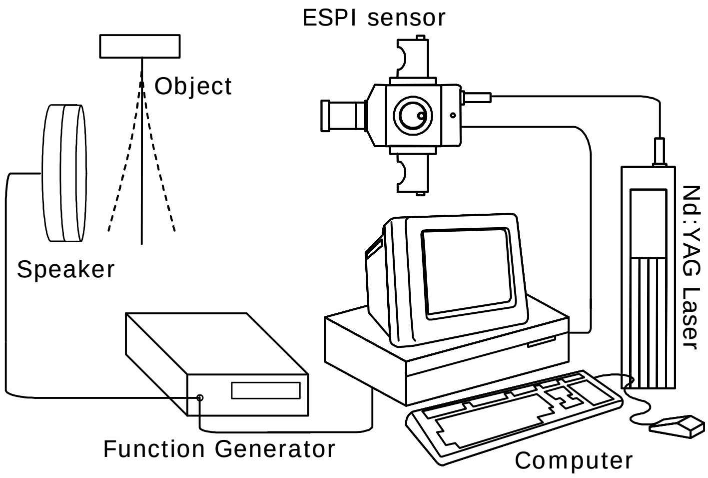


*Figure 2 - The schematic of experimental arrangement*

## REULSTS AND DISCUSSION

### Determination of Elastic Modulus

The specimens of STS304 were slowly cut off by wire cutting. The elastic modulus of the specimen at Matweb was 197 GPa and its density, measured with calibrated system, is 8042.67 kg/$m^3$. Table 1 shows the comparison of resonace frequencies between TA-ESPI and FEM. Elastic modulus of STS 304, estimated with equation (6) and each resonance frequencies from TA-ESPI, are averaged and evaluated as 194.53 GPa. There is just 1.25% error between the elastic modulus of Matweb and that of the proposed technique. In Table 1, resonance frequencies of FEM obtained with elastic modulus(197 GPa) of Matweb are compared with those of TA-ESPI, wich provides the maximum 1.65 % error. On the contrary, results of FEM excuted with the estimated(194 GPa) from equation (6) show the maximum 0.76% error, compared with

TA-ESPI. These results obviousely depicts the propsed technique provides more suitabel elastic modulus and also guarantees the repeatability of results because the technique need not destruct a specimen.

*Table 1 Comparison of resonance frequency between FEM and TA-ESPI (Hz)*

| | 2nd | 3rd | 4th | 5th | 6th |
|---|---|---|---|---|---|
| TA-ESPI | 61.0 | 171.7 | 336.0 | 556.7 | 832.0 |
| FEM at 197 GPa | 61.9 | 173.3 | 340.1 | 563.6 | 843.9 |
| Error (%) | 1.43 | 0.97 | 1.19 | 1.22 | 1.65 |
| FEM at 194 GPa | 61.5 | 172.1 | 337.3 | 557.6 | 833.2 |
| Error (%) | 0.76 | 0.26 | 0.39 | 0.17 | 0.14 |

**Geometric Influence of Specimen**

The geometric condition of Euler-Bernoulli beam ($l/t \geq 10$ and $l/w \geq 10$) limits the preparation of specimen and the condition has influence on the determination of elastic modulus. The geometric condition needs to be investigated in experiment. Elastic modulus of STS 304 is estimated with each ratio of length to width ($L/W$) which has 8, 10, 12, 14, 16 and 18. Figure 3 shows the estimated elastic modulus of each resonance frequencies by TA-ESPI. Although standard deviation at $L/W$ = 8 is 3.16 GPa, others have maximum 1.94 GPa, which means the repeatability of experiments is reliable. There is a boundary of elastic modulus according to the change of $L/W$ ratio.

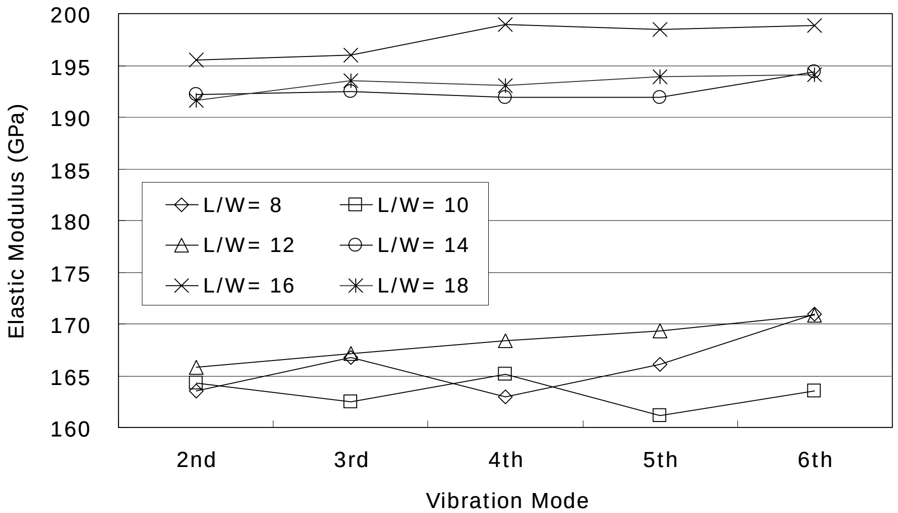


*Figure 3 - Comparison of elastic modulus at resonance frequencies according to the change of ratio of length to width.*

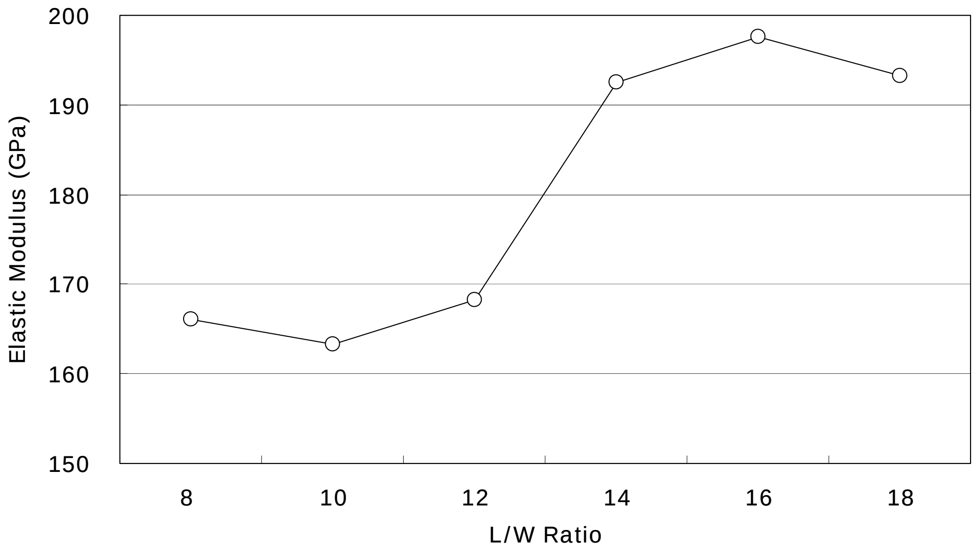


*Figure 4 - Comparison of elastic modulus at each L/W ratio*

Figure 4 shows the elastic modulus of each $L/W$ ratio averaged with resonance frequencies. The average elastic modulus of $L/W$ = 8, 10 and 12 is evaluated as 165.78 GPa and that of $L/W$ = 14, 16 and 18 is 194.53 GPa. There is the difference of elastic modulus, 30 GPa between two data set. Because Euler-Bernoulli beam theory recommends the length to be 10 times more than the thickness and width, it is assumed that elastic modulus in $L/W$ =8, 10 and 12 is not reliable. In comparison of its value with Matweb, its assumption is also reasonable. The results of Fig. 4 mean that the length should be 14 times more than the width for accurate estimation of the constant in the proposed technique.

## CONCLUSION

Elastic modulus is an important mechanical property of a material for predicting stress and structure dynamics, although it has been difficult to determine accurately and conveniently, thus far. In this study, a new technique for evaluation of the elastic modulus of a vibrating cantilever beam was presented. The method utilized time average electronic speckle pattern interferometry(TA-ESPI), which can extract directly the resonance frequencies, and its elastic modulus can be readily determined from classical beam theory. A comparison between the result from the proposed technique and the data showed excellent agreement. The advantages of the technique are that elastic modulus can be determined in a single measurement by using a prepared cantilever and that this technique is more economical than other techniques. Because the approach is very simple, it can be applied as a supplement to the other elastic modulus measurement technique.

## REFERENCES

[1] B.T. Comella, M.R. Poon, "The determination o the elastic modulus of a microcantilever beam using atomic force microscopy", J. Mater. Sci., 35, 567-572 (2000)

[2] D.J. Inman, *Engineering Vibration*. (Prentice-Hall Inc., NewYork, 1994)

[3] G.L. Cloud, *Optical Methods of Engineering Analysis*. (Cambridge University Press, London, 1990)

[4] Hsin-Chang Tsai, Weileun Fang, "Determining the Poisson's ratio of thin film materials using resonant method", Sensor and Actuators A, 103, 377-383 (2003)

[5] K. HØgmoen, O.J. LØkberg, "Detection and measurement of small vibration using electronic speckle pattern interferometry," Applied Optics, 16, 1869-1875 (1977)

[6] www.matweb.com

[7] Xin Kang, C.J. Tay, C. Quan, "Evaluation of Youngs modulus of a vibrating beam by optical method", Opt. Eng., 42, 10, 3053-3058 (2003)