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THE TUNED LIQUID COLUMN DAMPER AS THE COST-EFFECTIVE ALTERNATIVE OF THE MECHANICAL DAMPER WITHIN VIBRATION PRONE CIVIL ENGINEERING STRUCTURES

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Abstract

Modern architecture, limited space in urban area and new developments in building construction techniques have caused an increased need to construct flexible and tall structures. However, many of those structures are vibration prone and even minor dynamic loads like regularly occurring wind gusts may cause occupant discomfort, especially in the upper floors of high-rise buildings. On the other hand, earthquakes and strong winds often cause structural damage or even failure and thus an increased awareness about the vulnerability of modern structures became public. These include large dams and all kinds of light bridges from footbridges to long-span bridges with the need of increased effective structural damping. In the course of the cantilever method of bridge construction, critical states are encountered in windy situations. Consequently, there is a higher demand to protect the structures from all kinds of dynamic loads. Damping in the low frequency range of such vibration prone C.E. structures requires a concentration of energy for its efficient dissipation. The classical tuned mechanical damper (TMD) requires high investments and maintenance fees. In all respects, the tuned liquid column damper (TLCD) is superior, and it is analyzed and, in a first step modally tuned, using a recently established geometrical analogy to the TMD. When sealed, choosing the right gas pressure in chambers above the liquid surface extends the frequency range of application from close-to-zero to about five Hertz. The slightly over-linear gas-spring effect in combination with the averaged turbulent damping of the (relative) fluid flow (verified experimentally), protect the TLCD from overload by detuning. Fine-tuning in state space improves the performance even further. Fuzzy stiffness can be accounted for in the design stage. The result is a robust control in the frequency window around a resonance of the main structure with its effective structural damping dramatically increased.

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INTRODUCTION

The basic idea of most vibration decreasing devices is the absorption of a certain, critical part of the kinetic energy thereby reducing the ductility demand of the main structure and thus prevent it from serious structural damage under severe dynamic loads. In addition, discomfort of inhabitants in tall buildings under light to moderate wind loads must be avoided and the elongation of the lifetime of bridges under traffic and wind loads requires the increase of its effective structural damping. In the last decade, intensive research and development efforts, see Housner et al. [1], Soong & Spencer [2] for reviews, have resulted in the basic concepts of active and passive energy dissipation and in a large number of testing facilities for small, medium or real size experiments, as well as several actual installations all over the world. One major field of practical and successful vibration control is concerned with the application of dynamic vibration absorber. They are broadly categorized as either passive, such as tuned mass dampers (TMD), or active hybrid (ATMD), for reviews see again [1], [2] and Constantinou et al. [3]. Contrary to passive control, active control schemes usually depend on an external energy supply since they accomplish a desired system behaviour by applying active forces to the main structure. In 1989, active mass driver (AMD) have been used to mitigate bending and torsional vibrations in the first full-scale application, an 11-story structure in Tokyo, Japan, see Spencer & Sain [4]. To reduce the need for external power supply, semi-active control devices have been developed which, like passive systems, cannot add mechanical energy into a structural system, but have adjustable damper properties to reduce the system response, e.g. a permanent adaptation of the actual energy dissipation.

Commonly, the structural damping of bridges is extremely low. Since relative motions are small, the direct application of dashpots and/or frictional dampers requires a complicating design. To concentrate the energy consumed from the vibrating bridge, mechanical dampers are properly tuned (TMD) and applied, see Petersen [5], for their general design. The quite expensive reconstruction of the Millennium Bridge in London is described by Dallard et al. [6], where both, dampers and TMD are used to increase the effective structural damping beyond its cut-off value of the synchronization effect observed in the excitation process of pedestrian bridges, see Newland [7]. Nakamura and Fujino [8] solved the problems of the vibration prone Toda Park Bridge by substituting the expensive TMD with tanks and sloshing fluids. Exciting forces by walking pedestrians or by runners are analyzed in detail by Bachmann [9] and applied in the course of this paper.

Sakai et al. [10], and References [11] to [13] developed the tuned liquid column damper (TLCD) with applications to tall buildings. Hochrainer [13] invented a novel active control by pressurizing gas above the liquid column, thus creating an ATLCD as the cheap counterpart to the ATMD, see also Hochrainer and Ziegler [14]. In the passive mode, a sealed piping system with gas pressure in the equilibrium state properly adjusted extends the frequency range of application of TLCD up to about five Hertz.

Xue et al. [15], Shum and Xu [16a] and [16b], considered torsional bridge motions and validated their simulations experimentally. In Ref. [17], damping of bridges is

considered based on Reiterer [18], who developed the detailed model of TLCD interacting with the bridge in coupled oblique bending-torsional motion paralleled by laboratory testing, see also [19]. His investigations, see [17] too, include the Millennium Bridge and the Toda Park Bridge beside other bridges, and thus the illustration of the effective damping action of TLCD. A novel pipe-in-pipe design of TLCD is described, that reduces vibrations with dominating vertical amplitude.
Tuning of TLCD is done in two steps. By means of an analogy between TMD and TLCD, worked out in detail in [13] and [18], modal tuning is performed by a transformation of the classical Den Hartog [20] formula. In a second step, fine-tuning in state space with the Den-Hartog-parameter as starting values is recommended.

Development of tuned liquid column damper (TLCD)

Beside the popular TMD, a novel innovative concept of structural protection by TLCD has shown to be effective in reducing structural vibrations. Basically, a TLCD consists of a rigid, U-shaped piping system that is smoothly integrated into a building and partially filled with liquid, preferably water. Similar to TMD, the vibration decreasing capability is based on an energy transfer from the supporting host structure to the TLCD, thereby inducing a relative motion of the water column. Finally, the energy is dissipated by viscous and turbulent fluid damping, which can be regulated and thus optimized by the insertion of hydraulic resistances, e.g. orifice plates. Several small-scale experiments have proven that it is adequate for dynamic investigations to model the TLCD as an SDOF-oscillator. In many respects TLCD exceed by far the capabilities of other vibration reducing devices. Their main advantages comprise of low installation costs, no moving mechanical parts, easy application to new buildings or in retrofitting existing structures, a simple tuning mechanism which allows for in situ adaptation to modified (degraded) building dynamics and virtually no maintenance requirements. A water reservoir for water supply or fire fighting e.g. might be modified such that it can act as TLCD without causing significant additional cost or weight. Due to their salient features, TLCD have caused an increased research interest in the last decade, resulting in both, analytical and experimental analyses, see e.g. Hochrainer [13], Hochrainer et al. [21], Reiterer [18], Reiterer & Hochrainer [22], Adam et al. [23], Chang & Hsu [24], Shum & Xu [16] or Gao et al. [25]. Hitchcock et al. [26], has extended the traditional uni-directional design to the bi-directional liquid column damper, and hybrid systems have been investigated, e.g. for active orifice control see Haroun et al. [27], Yalla et al. [28] and Yalla & Kareem [29]. A semi-active MR-TLCD control system using a magneto-rheological fluid was recently proposed to counteract the vibrations of wind-excited tall buildings, Ni et al. [30]. Kagawa et al. [31] report on real size applications of a 9-story steel structure, equipped with semi active TLCD. Teramura & Yoshida [32] implemented a bi-directional vibration control system based on TLCD in a 26-story, 106m high hotel in Japan. Hochrainer [13] invented an ATLCD. A short outline is given by Hochrainer & Ziegler [14]. Recently, Reiterer [19] and Reiterer & Hochrainer [33], proved experimentally and by computer simulation, the effectiveness of TLCD when applied to vibration prone long-span bridges, where they appear superior to alternative countermeasures, especially for pedestrian or wind induced vibrations.

Tuning of TLCD

Tuning of the TLCD in the design stage is performed in several steps. At first, the linearized computer model is tuned with respect to a selected mode of the structure using the analogy to TMD-tuning and applies, properly transformed, Den Hartog's optimal parameter, [20]. Improvements of the performance in MDOF-structures are achieved by considering the neighbouring modes as well, in a state space optimisation. This fine-tuning renders the parameters slightly modified. Analogously, two or more TLCD counteracting a single selected mode in parallel action, turn out with differently adjusted tuning parameters. Further, such a state space parameter optimization may include structural uncertainty of the building, e.g. its fuzzy stiffness, by generalizing the performance index. Final adjustments are easily made in the course of in-situ testing. The TLCD in its passive mode considerably reduces steady state vibrations similarly to an increase in the effective structural damping. Reduction of transient peaks in the early period of the strong motion phase of earthquakes, requires active control of the gas-spring, rendering the damper for short time an ATLCD. A sufficient condition based on cut-off damping must be checked, to save any consideration of the vertical ground (floor) acceleration, see again [18] and [22].

THE TMD–TLCD ANALOGY

Main SDOF-structure with TMD

Considering the representative model sketched in Fig. 1 under both, base and force excitation, renders Eq. 1. To refer to the (even actively controlled) tuned mass damper (TMD), all parameter are denoted by a star,

$$\left(1+\mu^*\right)\ddot{w}+\mu^*\ddot{u}^*+2\zeta_s^*\Omega_s^*\dot{w}+\Omega_s^{*2}w=-\left(1+\mu^*\right)\ddot{w}_g+F(t)\big/M^* \quad , \ \mu^*=m^*\big/M^*<6\% \ ,$$

$$\ddot{u}^*+\ddot{w}+2\zeta_A^*\omega_A^*\dot{u}^*+\omega_A^{*2}u^*=-\ddot{w}_g \tag{1}$$

$$\Omega_S^*=\sqrt{K^*/M^*} \ , \ \omega_A^*=\sqrt{k^*/m^*} \ , \ \zeta_S^*=c^*/2M^*\Omega_S^* \ .$$

Tuning of the passive TMD, i.e. finding the optimal values of the absorber parameter, frequency ratio and linear damping coefficient, is classically done by applying Den Hartog's optimization criterion (with light damping of the main system even neglected) [20],

$$\delta_{opt}^*=\omega_A^*\big/\Omega_S^*=1\big/\left(1+\mu^*\right) \ , \ \zeta_{A,\,opt}^*=\sqrt{3\mu^*\big/8\left(1+\mu^*\right)} \tag{2}$$

Equation (2) is derived under conditions of time harmonic forcing and minimizing the dynamic displacement magnification factor of the main system. The same parameter apply in case of time harmonic base acceleration if the magnification factor of the total acceleration is minimized, Warburton [34] and Soong & Dargush [35]. To reduce peaks in the transient response, an active control mechanism can be added, but is not discussed further in this context. Hochrainer [13], p. 44, studied the influence of small (modal) structural damping and confirmed the results of Warburton [36] that the optimal values in Eq. (2) should not be changed for broadband excitation. TMD have been installed in tall buildings, bridges and towers for response control of primarily wind induced external loads, despite of their complicating design and maintenance requirement, see Holmes [37] and EERC [38] for listings of worldwide installations.

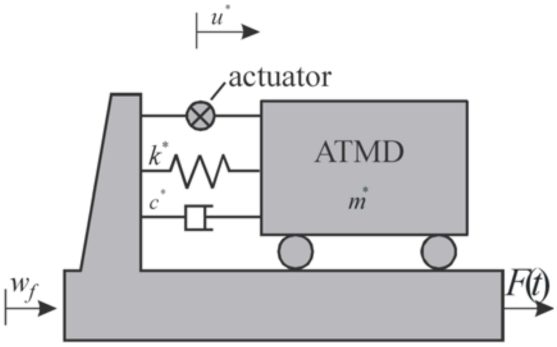

Figure 1. (Active) Tuned mechanical damper attached to a main SDOF-system (supporting springs and its horizontal base acceleration not shown). Floor displacement $w_f=w_g+w$. ATMD: Actuator force changes acceleration of the absorber mass

Main SDOF-structure with TLCD

The TMD in Fig. 1 is substituted by a sealed TLCD, illustrated in Fig. 2. Considering the floor displacement w_f, the equation of the ideal fluid flow in the rigid piping system is derived by projecting Euler's vector equation of motion on its relative velocity and subsequently integrating over the length of the liquid column, i. e. over the arc of the relative streamline in its instant configuration, keeping time constant. The absolute acceleration of the fluid particles, in general is delineated into the guiding acceleration, the Coriolis component and the relative acceleration. Since the Coriolis acceleration is orthogonal to the relative velocity of the fluid motion, it does not at all contribute to the equation, which may be called a generalized Bernoulli equation, cf. Ziegler [39], p.497,

$$\int_1^2 \frac{\partial' v_{rel}}{\partial t} ds' + \frac{\dot{u}_2^2}{2} - \frac{\dot{u}_1^2}{2} = -g\left(z_2 - z_1\right) - \frac{p_2 - p_1}{\rho} - \int_1^2 \left(\vec{a}_g . \vec{e}_t'\right) ds' \ , \ \dot{u}_1 = \dot{u}_2 = \dot{u} \ , \tag{3}$$

where, in the course of integration, the rule of partial differentiation is used

$$\vec{a}_{rel} . \vec{e}_t{}' = \frac{\partial' v_{rel}}{\partial t} + \frac{\partial}{\partial s'}\left(\frac{v_{rel}^2}{2}\right)$$

The guiding acceleration in tall buildings simply reduces to the horizontal total floor acceleration, (the effect of any vertical acceleration must be checked separately),

$$a_g = a_A = \ddot{w}_f = \ddot{w}_g + \ddot{w} \tag{4}$$

A is the origin of the moving frame. For a moving bridge cross-section see Fig. 10. In sealed passive TLCD, the gas-spring effect follows approximately the quasi-static polytropic law, [39], p. 88. Linearization with respect to the equilibrium pressure, the reference gas pressure applies to both, the symmetric left and right volumes in Fig. 2, renders the pressure difference in Eq. (3) sufficiently well approximated within a limited range of the liquid stroke,

$$p_2 - p_1 = 2np_0 \frac{u}{H_a} + o\left(u^3\right), 1 \le n \le 1.4, \max|u| / H_a < 0.3, V_0 = A_H H_a \tag{5}$$

Performing the integration of the non-stationary term in Eq. (3), and just considering the horizontal floor acceleration, Eq. (4), assigned and further, adding the experimentally verified pressure loss through averaged turbulent damping to the right hand side of Eq. (3), yield the equation of relative fluid motion in the symmetrically designed TLCD in passive action when attached to a horizontally moving frame, see again Fig. 2, and Hochrainer [40] for detailed derivations,

$$\ddot{u} + \delta_L \dot{u}|\dot{u}| + \omega_A^2 u = -\kappa \ddot{w}_f \ , \ \omega_A = 2\pi f_A = \sqrt{\frac{2g}{L_{eff}}\left(\sin\beta + \frac{h_0}{H_a}\right)} \ , \ h_0 = np_0/\rho g \tag{6}$$

$$L_{eff} = 2H + B A_H / A_B \ , \ \kappa = (B + 2H\cos\beta)/L_{eff} \ , \ \rho_{water} = 1000\, kg/m^3 \ .$$

Similarly to the pendulum-type of a TMD, -the length of the mathematical pendulum is equivalent to half of the length of the fluid column for same linear frequency, -any vertical floor acceleration adds parametric forcing to Eq. (6). However, with sufficient damping understood, parametric resonance does not occur. Reiterer & Ziegler [41] provide detailed experimental and numerical verifications. If the turbulent damping is equivalently linearized, a sufficient condition for damping is derived by requiring one and the same dissipated energy per unit cycle in steady state vibrations. It turns out to be proportional to both, the amplitude of the time harmonic vibration of the fluid and the mean turbulent loss factor. The maximum stroke of the fluid motion when substituted enters the inequality with the cut-off value of the most critical parametric resonance present,

$$\zeta_A = \frac{4\left(\max|u|\right)}{3\pi}\delta_L > \zeta_{A,0} = \frac{\max\left|\ddot{v}_g\right|/g}{4\left(1 + h_0/H_a \sin\beta\right)} \tag{7}$$

The gas-spring effect in sealed TLCD lowers the required cut-off damping even further. If the inequality (7) holds true, any effects of the vertical excitation become negligible. Conservation of momentum of the fluid mass renders the control force acting on the main system in horizontal motion, Fig. 2,

$$F_x = m_f\left(\ddot{w}_f + \bar{\kappa}\ddot{u}\right) \ , \ \bar{\kappa} = (B + 2H\cos\beta)/L_1 \ , \ m_f = \rho A_H L_1 \ , \ L_1 = 2H + B A_B/A_H \tag{8}$$

The horizontal displacement of the fluid center of mass is determined by $\bar{\kappa} u$. Substructure synthesis of TLCD and main SDOF-structure according to Fig. 2, renders the equivalent to Eq. (1), Eqs. (6) and (8) are both considered,

$$(1+\mu)\ddot{w} + \bar{\kappa}\mu\ddot{u} + 2\zeta_S \Omega_S \dot{w} + \Omega_S^2 w = -(1+\mu)\ddot{w}_g + F(t)/M \ , \ \mu = m_f/M < 6\% ,$$

$$\ddot{u}+\kappa\ddot{w}+\delta_L|\dot{u}|\dot{u}+\omega_A^2\,u=-\kappa\ddot{w}_g\ ,\ \Omega_S=\sqrt{K/M}\ ,\ \zeta_S=c/2M\Omega_S<<1 \tag{9}$$

In view of setting-up the equation of motion for a regular main MDOF-structure with multiple TLCD attached, the equivalently linearized Eq. (9) with respect to the actual turbulent damping, is cast in its matrix form,

$$\tilde{M}_S\begin{bmatrix}\ddot{w}\\ \ddot{u}\end{bmatrix}+\tilde{C}_S\begin{bmatrix}\dot{w}\\ \dot{u}\end{bmatrix}+\tilde{K}_S\begin{bmatrix}w\\ u\end{bmatrix}=-\begin{bmatrix}M+m_f\\ \kappa\end{bmatrix}\ddot{w}_g\ ,\ \tilde{M}_S=\begin{bmatrix}M+m_f & \bar{\kappa}m_f\\ \kappa & 1\end{bmatrix}, \tag{10}$$

$$\tilde{C}_S=diag\left[2\zeta_S\Omega_S M\quad 2\zeta_A\omega_A\right]\ ,\ \tilde{K}_S=diag\left[M\Omega_S^2\quad \omega_A^2\right].$$

Hochrainer [13], inspected the coupled system of equations of the main SDOF-system with either one, a TMD, Eq. (1), or a linearized TLCD, Eq. (10), attached, and deduced their (geometric) analogy, which is solely determined by the geometry factors κ and $\bar{\kappa}$, Eqs. (6) and (8), and the active absorber mass,

$$\mu^*=m^*/M^*=\mu\frac{\kappa\bar{\kappa}}{1+\mu(1-\kappa\bar{\kappa})}<\mu=m_f/M\ ,\ \Omega_S^*=\Omega_S\frac{1}{\sqrt{1+\mu(1-\kappa\bar{\kappa})}}\ ,\ m^*=\kappa\bar{\kappa}m_f\ ,$$

$$\delta_{opt}=\frac{\delta_{opt}^*}{\sqrt{1+\mu(1-\kappa\bar{\kappa})}}=f_{A,opt}/f_S=\frac{\sqrt{1+\mu(1-\kappa\bar{\kappa})}}{1+\mu}\ ,\ \zeta_{A,opt}=\zeta_{A,opt}^*=\sqrt{\frac{3\kappa\bar{\kappa}\mu}{8(1+\mu)}} \tag{11}$$

The remaining impulsive fluid-mass must be regarded as dead weight load of the main structure lowering somewhat its natural frequency. Numerical simulations of the dynamic magnification factor (DMF) convincingly approve the analogy, Fig. 3.

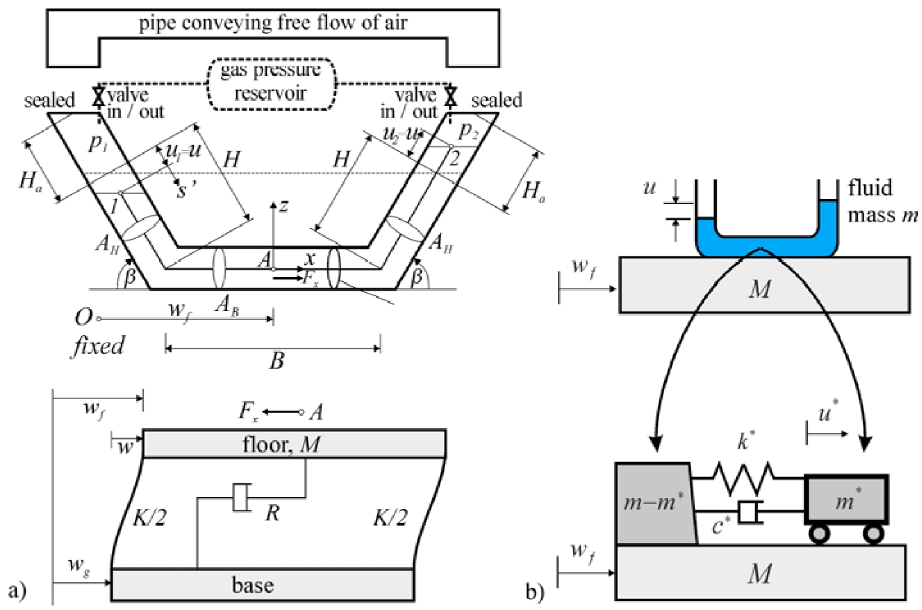

Figure 2. - a) Symmetrically designed TLCD rigidly attached to the displaced floor of the SDOF-main system. Instant relative streamline from point 1 to 2. Gas pressure reservoir for active control schematically added; and - b) the (passive) TMD-(linearized) TLCD-analogy.

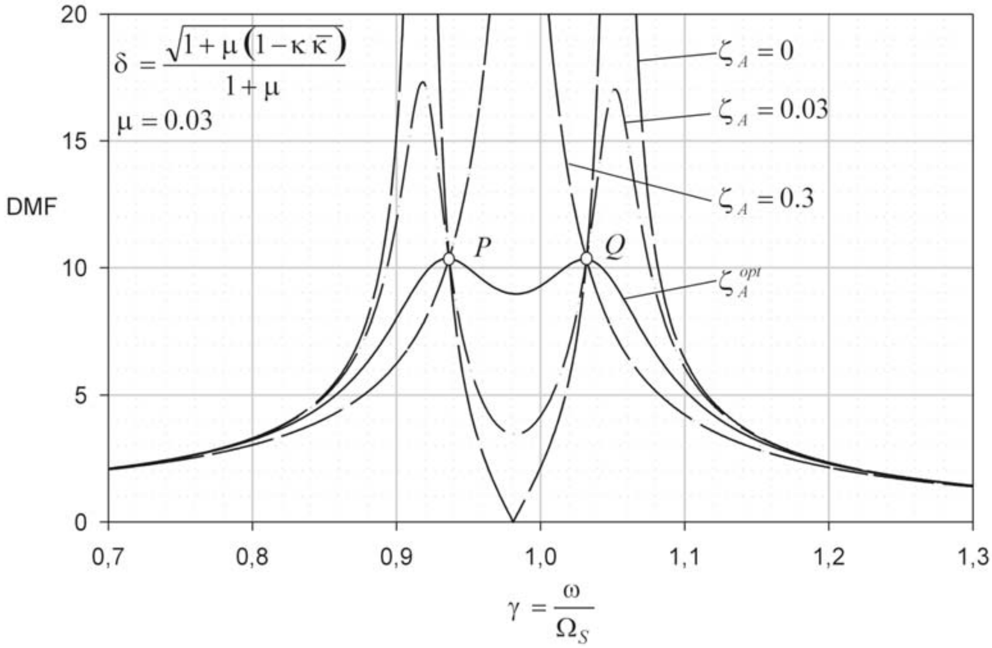

Figure 3. - Frequency response curves of force excited SDOF - (linearized) TLCD system. Active mass = 63% of fluid mass. Conjugate TMD and the analogy considered. Undamped SDOF-main system

Control of main MDOF-structure by a single TLCD

For illustrative purpose, a simple *N*-DOF shear frame building is considered with a single TLCD attached to the *i*-th floor, forced by wind-gusts and base acceleration. The modal matrix of the main system is assumed known and the absolute floor displacements are represented by their modal series. However, the right hand side of the resulting system of modal equations decouples approximately only under the severe assumption of well-separated natural frequencies. Under these conditions, Hochrainer and Adam [42] derived the following approximating and linearized system of decoupled equations, cf. Eq. (10) under the assumption that in the vicinity of the j-th natural frequency the floor displacements become separable,

$$\begin{pmatrix} 1+\mu & \mu\bar{\kappa}/\phi_{ji} \\ \kappa\phi_{ji} & 1 \end{pmatrix}\begin{pmatrix} \ddot{q}_j \\ \ddot{u} \end{pmatrix} + \begin{pmatrix} 2\zeta_S\Omega_S & 0 \\ 0 & 2\zeta_A\omega_A \end{pmatrix}\begin{pmatrix} \dot{q}_j \\ \dot{u} \end{pmatrix} + \begin{pmatrix} \Omega_S^2 & 0 \\ 0 & \omega_A^2 \end{pmatrix}\begin{pmatrix} q_j \\ u \end{pmatrix} = -\begin{pmatrix} L_j \\ \kappa \end{pmatrix}\ddot{w}_g + \begin{pmatrix} F_j \\ 0 \end{pmatrix}$$

$$\vec{w} = q_j(t)\vec{\phi}_j \ , \ m_j = \vec{\phi}_j^T\,\tilde{M}\,\vec{\phi}_j \, , \ L_j = \frac{\vec{\phi}_j^T\,\tilde{M}\,\vec{r}_s + \phi_{ji}\,m_f}{m_j}, \ \mu = \frac{\phi_{ji}^2\,m_f}{m_j} \, , \ F_j = \frac{\vec{\phi}_j^T\,\vec{F}(t)}{m_j} \qquad (12)$$

Note the modified participation factor, and most importantly, the modal mass ratio μ are identified, where $\tilde{M}$ = the mass matrix and $\vec{r}_s$ = the static influence vector that describes the rigid body motion for single point base excitation. Since Eq. (12) can be transformed to account for the conjugate TMD attached to the *i*-th story, by substituting for the motion of the absorber mass of the conjugate TMD, and by multiplying with the diagonal matrix, respectively,

$u^* = u/\kappa\phi_{ji}$, $diag\left[1/\left(1+\mu\left(1-\kappa\bar{\kappa}\right)\right) \quad 1/\kappa\phi_{ji}\right]$,

all new modal quantities turn out adjusted. Under wind-type load, the effectiveness of the equivalent TMD is exactly the same as in the SDOF system, thus the optimal solution is directly applicable. Soong & Dargush [35] considered seismic excitation of the MDOF-system and found the transition to the modal counterpart non-optimal. Nevertheless, the transformation of the TLCD to an equivalent TMD is always possible by considering its active mass-star, Eq. (11), and stiffness rendering the absorber frequency. Thus any optimization procedure developed for TMD systems is applicable to the TLCD, independent of the separation of the natural frequencies. Even nonlinear main structures can be investigated along these lines.

STATE SPACE OPTIMIZATION OF MULTIPLE TLCD

The *N*-DOF main system with a number of $n << N$ TLCD installed at proper locations, is described by the set of matrix equations, Eq. (12) extended to its hyper matrix form, single point excitation of the base understood,

$$\tilde{M}_S \begin{bmatrix} \ddot{\vec{w}} \\ \ddot{\vec{u}} \end{bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{0} \\ \tilde{0} & \tilde{C}_f \end{bmatrix} \begin{bmatrix} \dot{\vec{w}} \\ \dot{\vec{u}} \end{bmatrix} + \begin{bmatrix} \tilde{K} & \tilde{0} \\ \tilde{0} & \tilde{K}_f \end{bmatrix} \begin{bmatrix} \vec{w} \\ \vec{u} \end{bmatrix} = -\begin{bmatrix} \tilde{M}\,\vec{r}_S + \tilde{L}\,\tilde{M}_f\,\vec{i} \\ \tilde{\kappa}\,\vec{i} \end{bmatrix} \ddot{w}_g + \begin{pmatrix} \vec{F}(t) \\ 0 \end{pmatrix} \tag{13}$$

The sparse position matrix with dimension $N \times n$ and the static influence vector are

$$\tilde{L} = \begin{bmatrix} 1 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \textit{DOF to be influenced} \\ \\ \\ \end{matrix} \;, \quad \vec{r}_S = \vec{i} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \end{bmatrix}^T , \tag{14}$$

$\uparrow$ *number of TLCD*

apparent in Eq. (13), enter the generalized mass matrix as well,

$$\tilde{M}_S = \begin{bmatrix} \tilde{M} + \tilde{L}\,\tilde{M}_f\,\tilde{L}^T & \tilde{L}\,\tilde{M}_f\,\tilde{\bar{\kappa}} \\ \tilde{\kappa}\,\tilde{L} & \tilde{I} \end{bmatrix} \tag{15}$$

M, *C* and *K* are mass-, light damping- (even non classical) and stiffness matrix of the main system and the following diagonal matrices of the linearized TLCD model are self explanatory,

$$\tilde{M}_f = diag\left[m_{f1}, \ldots, m_{fn}\right], \qquad \tilde{K}_f = diag\left[\omega_{A1}^2, \ldots, \omega_{An}^2\right], \qquad \tilde{\boldsymbol{\kappa}} = diag\left[\kappa_1, \ldots, \kappa_n\right],$$

$$\tilde{\bar{\boldsymbol{\kappa}}} = diag\left[\bar{\kappa}_1, \ldots, \bar{\kappa}_n\right], \; \tilde{C}_f = diag\left[2\zeta_{A1}\omega_{A1}, \ldots, 2\zeta_{An}\omega_{An}\right] \tag{16}$$

To make the tools of control theory applicable, Eq. (13) is converted to the state space by means of the (transposed) state vector given below, and its time derivative,

$$\dot{\vec{z}} = \left(\tilde{A} + \tilde{B}\,\tilde{R}\right)\vec{z} - \vec{e}_g\,\ddot{w}_g + \tilde{E}_f\,\vec{F}(t), \; \vec{e}_g^{\,T} = \begin{bmatrix} \vec{0} & \vec{0} & \tilde{M}_S^{-1} \begin{pmatrix} \tilde{M} + \tilde{L}\,\tilde{M}_f \\ \tilde{\kappa} \end{pmatrix} \vec{i} \end{bmatrix} \tag{17}$$

$$\tilde{z}^T = \left[\vec{w}\ \vec{u}\ \dot{\vec{w}}\ \dot{\vec{u}} \right],\ \ \tilde{E}_f = \left(\tilde{0} \quad \tilde{0} \quad \tilde{M}_S^{-1} \begin{pmatrix} \tilde{I} \\ \tilde{0} \end{pmatrix} \right)^T$$

The resulting system matrix ($A + B\,R$), apparent in Eq. (17), is kept separated since, at this stage, the hypermatrix, $R = diag[0\ \kappa_f\ \ 0\ C_f\ ]$ contains the yet unknown linear TLCD design parameter.

Frequency response optimization for MDOF structures with several TLCD installed

Tuning in frequency space is preferable since the performance index J can simply be defined as weighed squared area of the frequency response function. Low cost optimization results if the frequency range is infinitely extended and a positive semi-definite weighing matrix $\tilde{Q}$ is selected, Müller and Schiehlen [43], p.249, state vector of main system apparent,

$$J = \int_{-\infty}^{\infty} \bar{z}_S^T(\omega)\,\tilde{Q}\,\bar{z}_S(\omega)\,d\omega = 2\pi\,\vec{e}_0^{\,T}\,\tilde{P}\,\vec{e}_0 \to \min\,,\ \bar{z}_S(\omega) = \left[i\,\omega\,\tilde{I} - \left(\tilde{A} + \tilde{B}\,\tilde{R} \right) \right]^{-1} \vec{e}_0\,,$$

$$\vec{e}_0 = \vec{e}_g \ \text{ or } \ \vec{e}_0 = \tilde{E}_f\,\vec{F}_0\,,\ \bar{z}_S^T = \left[\vec{w}\ \dot{\vec{w}} \right],\ \left(\tilde{A} + \tilde{B}\,\tilde{R} \right)^T \tilde{P} + \tilde{P}\left(\tilde{A} + \tilde{B}\,\tilde{R} \right) = -\tilde{Q} \qquad (18)$$

The result of solving Eq. (18) turns out to render the decay rate of free vibrations in time domain optimal, with the initial conditions (in vector form) $z_0\ /\ \sqrt{2\pi} = e_0$ assigned. That means, optimization becomes independent of any specific and common time function of the wind gusts. The matrix solution $\tilde{P}$ of the algebraic Lyapunov equation in Eq. (18) is numerically evaluated by means of the software MATLAB [44]. The minimum search of the performance index in Eq. (18) is best performed by the MATLAB optimization toolbox, "*fminsearch*", when substituting Den Hartog´s modal tuning parameters, as discussed in previous sections, as start values. For instance, in practical applications, it becomes necessary to split a modally tuned single TLCD into several TLCD in parallel connection with subsequently following fine-tuning in the state space, Eq. (18).

Fuzzy main system parameter

The above reported parameter optimization by Eq. (18), may include structural uncertainty, e.g. in stiffness $\Delta\tilde{K}$, by generalizing the performance index. For instance, the main systems with the extreme variations of the parameters are considered in addition to the ideal system by adding the associated performance indices

$$J = J_K + J_{K+\Delta K} + J_{K-\Delta K} \to \min \qquad (19)$$

J_K refers to the performance index of the wanted ideal main system. Minimum of Eq. (19) is searched analogous to Eq. (18).

APPLICATIONS OF SEALED PASSIVE TLCD TO BUILDINGS

Passive control by TLCD is illustrated for a real, wind loaded office building and, as well, for a base isolated tall building under seismic load. In the latter case the remaining, extremely low frequency vibration should be efficiently damped.

A 76-story office building

The vibration reduction by TLCD of an important benchmark building under severe wind excitation, Yang et al. [45]], is presented. The steel building under consideration is located in Sidney, Australia. It is a 76-story high slender office tower with a quadratic cross-section and a height to width ratio of 7.3, ($42\times 42\times 306$ m). It has a total mass / volume = $153\times 10^6\,\mathrm{kg}\;/510\times 10^3\,\mathrm{m}^3$ ratio. The first five natural frequencies are 0.16, 0.77, 1.99, 3.79 and 6.40 Hz, each mode having a light modal damping coefficient of ζ_S = 1% assigned. The wind force data acting on the benchmark model were determined experimentally in a boundary layer wind tunnel facility. Based on a finite element analysis, a plane building model with 76 degrees of freedom was developed and made available in digital form. Modal analysis of the building response has revealed, that for wind excitation the building dynamics is dominated by the first vibration mode. Consequently, only a single tuned liquid column damper with a liquid mass of 500,000 kg is placed on the top floor to counteract the basic vibration mode. Thus, the water mass is chosen about 45% of the top floor mass or approximately 0.33% of the total mass of the building. The optimal frequency and damping ratios of the passive TLCD are either given by Eq. (11) or determined by minimising the performance index, Eq. (18). The optimal frequency and the equivalent proportional damping result, $f_{A,opt}=0.16\,\mathrm{Hz}$ and $\zeta_{A,opt}=5.5\,\%$. Viscous damping turns out somewhat lower than the Den Hartog value. The effective liquid column length is set to L_{eff} = L_1 = 25 m ($B=20$ m and $\beta=45°$) in the piping system with uniform cross sectional area, Fig. 2a, and the gas volume is 114 m^3. Thus, the fictitious height H_a = 8 m results from Eq. (6). With the safety factor of two, the partial gas volume (2 $max|u|\,A_H$) is referred to the pipe's cross sectional area, the remaining gas volume can be designed according to the conditions at the constructional site. However, since the cross-sectional area of A_B = A_H = 20 m^2 is much too large for a single TLCD design, up to ten TLCD in parallel connection should actually be attached to the top of the building. Fine-tuning in state space by again minimising the performance index, Eq. (18), renders the optimal parameters modified. For three pairs of TLCD in a symmetrical package arrangement, i.e. six TLCD with A_B = A_H = 3.33 m^2 each, the optimal parameter result, $f_{A(1)}=0.15\,\mathrm{Hz}$, $f_{A(2)}=0.16\,\mathrm{Hz}$, $f_{A(3)}=0.168\,\mathrm{Hz}$, $\zeta_{A(1,2)}=2.5\,\%$, $\zeta_{A(3)}=2.7\,\%$, i.e. fine tuning changes frequencies slightly but the equivalent viscous damping coefficients of the fluid flow turn out dramatically lowered. The performance index shown in Fig. 4 illustrates the expected increase of robustness of the passive control. The prize to be paid is the magnification of the strokes of the fluid columns.

76-story office building with fuzzy stiffness

The optimization procedure for the split, six TLCD, may include structural uncertainty, say in stiffness, $\Delta K=\pm 15\%$, by means of the performance index, Eqs.

(18) – (19). Tuning under these uncertain stiffness conditions of the main system yields the TLCD parameter, $f_{A(1)} = 0.186$, $f_{A(2)} = 0.157$, $f_{A(3)} = 0.131\,\mathrm{Hz}$, $\zeta_{A(1)} = 3$, $\zeta_{A(2)} = 3.23$, $\zeta_{A(3)} = 3.41\,\%$. The advantage of the robust optimization becomes apparent in Fig. 5 where an additional gain of $4\,\mathrm{dB}$ at the critical resonance frequency is achieved over the action of the single TLCD.

76-story office building under wind gust load

The system response, with turbulent damping of the optimized TLCD taken into account, is simulated for the recorded Yang et al. [45], 15 min wind load (wind speed set to 47.25 m/s), by means of MATLAB's *lsim* function with one result exemplarily displayed in Fig. 6. From visual inspection of Fig. 6 it is apparent that the level of vibration has been considerably (by about 50%) reduced. With equivalent linear viscous damping of the TLCD understood, the maximum fluid displacement amplitude is estimated to be $\max|u| = 0.95\ \mathrm{m}$ which is well within the acceptable limits. Since sufficient structural response reduction is achieved by the passive action of the TLCD, it becomes unnecessary to add active control.

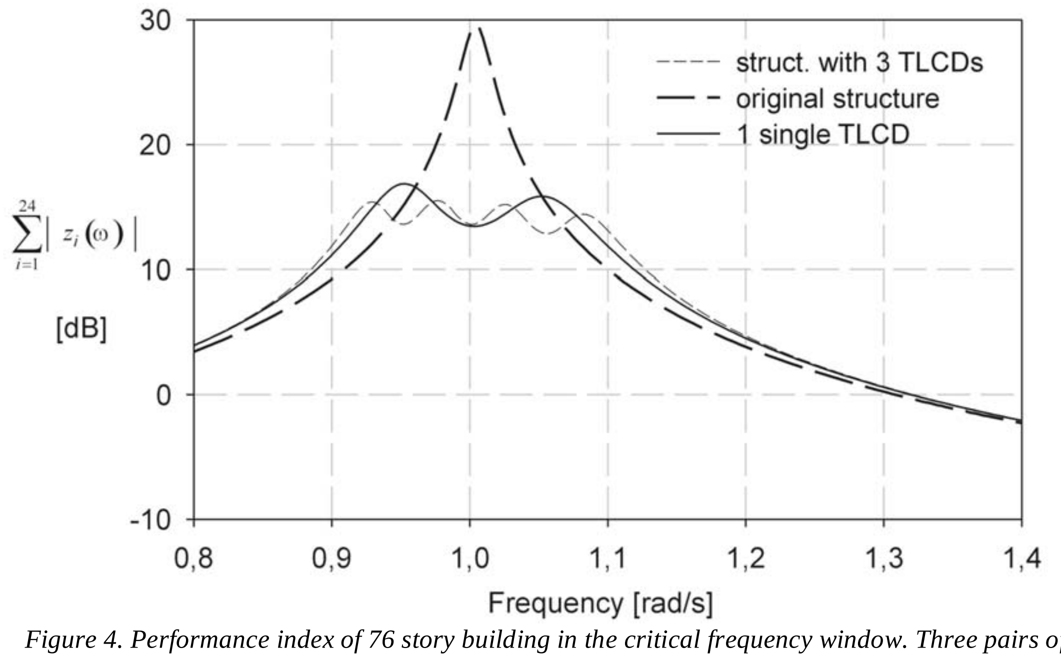

Figure 4. Performance index of 76 story building in the critical frequency window. Three pairs of TLCD increase robustness of the passive control

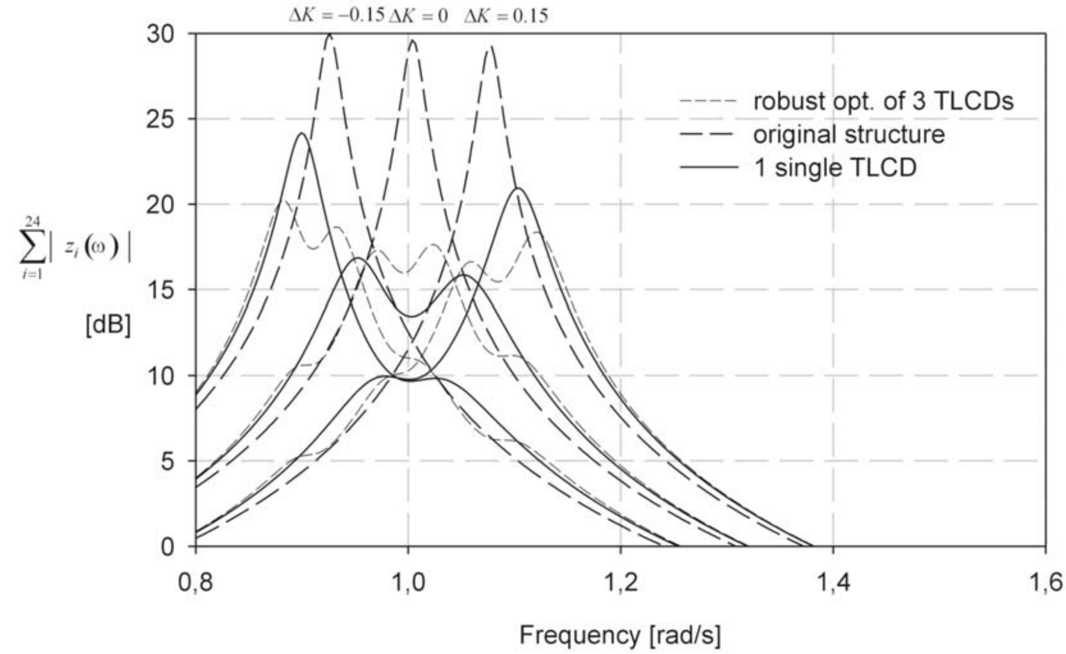

Figure 5. - Performance index of original and uncertain structures ($\Delta K = \pm 15\%$), equipped with a single or with three pairs of TLCD in parallel action. Critical frequency window, [13].

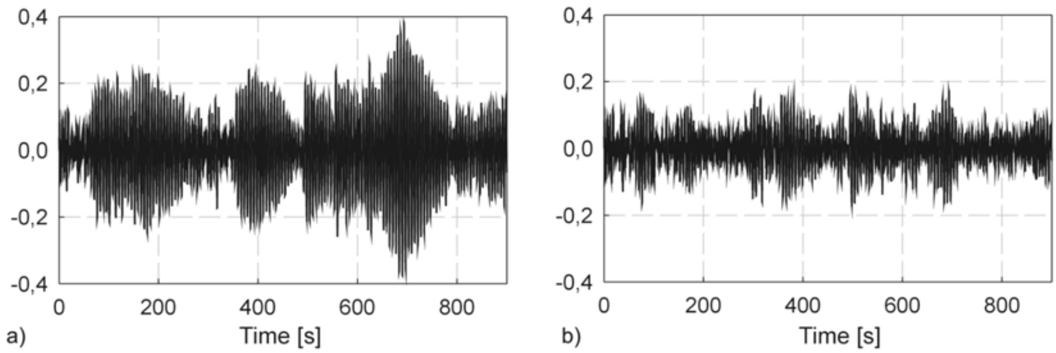

Figure 6. - Top floor acceleration in g/10 for the simulated 15 min wind load, a) original building, b) building with single optimized TLCD on top (with turbulent damping). [13].

Base isolated structure effectively damped by a TLCD

Chopra [46], p. 744, considers a five-story benchmark frame with base isolation, sketched in Fig. 7. A sufficient amount of damping is required to control the new basic mode, namely the rigid body motion of the base isolated system with altogether six degrees of freedom. Base slab and floors have one and the same mass of m = 45,000 kg and total mass $M = (5 + 1)\, m$ = 270,000 kg, that is slightly less to the one considered by Chopra. Homogeneous field stiffness and stiffness of the isolation system are chosen such that the periods listed in Fig. 8 result. The amount of water in the TLCD, located in the base slab, is given by selecting the rather large mass ratio $\mu = m_f / M = 3\%$. The TLCD with constant cross-section, parameter

$\kappa = \bar{\kappa} = 0.95$, in a first step is Den Hartog optimized, Eq. (11), with respect to the new basic mode. The latter is approximately a rigid body motion of the whole building with the assigned period T_b = 2.0 s. Thus the parameters of the TLCD result
$\delta_{A,opt} = 0.97$, $\zeta_{A,opt} = 10$ % .

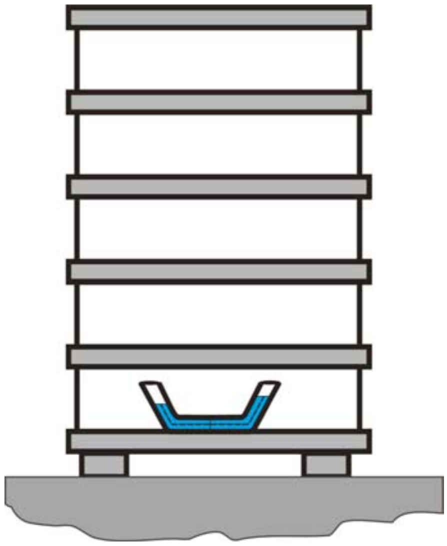

Figure 7. Five-story-benchmark frame, base isolated, TLCD-damping of rigid body mode.

Selecting the alternative and simpler performance index to be minimized

$$J = \sum_{i=1}^{6} \int_0^\infty w_i^2(\omega) d\omega \rightarrow \min \tag{20}$$

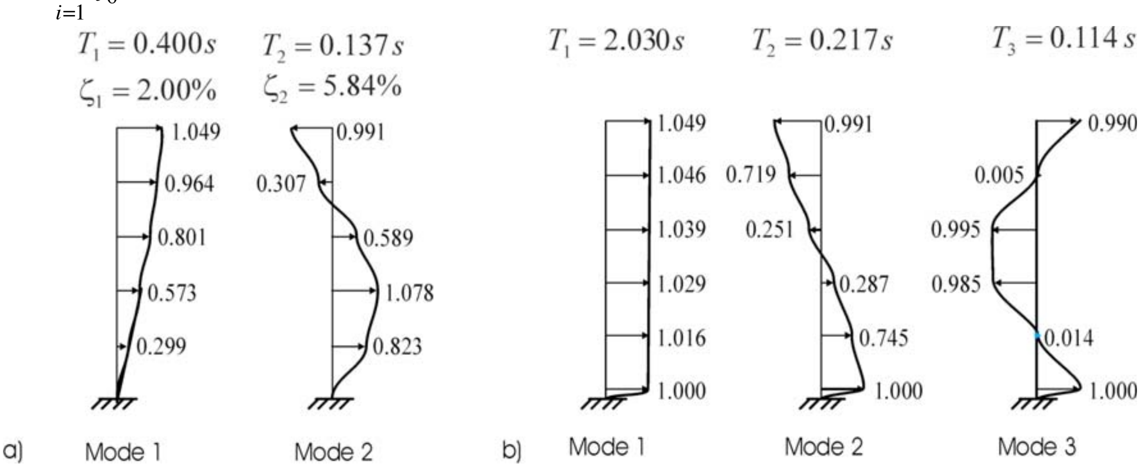

Figure 8. Changing natural mode shapes by base isolation, Chopra [46].

and performing fine tuning analogous to Eq. (18) keeps the Den Hartog frequency ratio unchanged but lowers the equivalent linear damping coefficient to $\zeta_{A,\,opt}$ = 7.9%. Table 1 records the increase of the structural damping by base isolation achieved by Chopra [46] with base isolation damping (with extremely low non-classical damping), and alternatively, by the application of the TLCD in the basement. One or two low frequency modes are created and indicated in Table 1. The huge gain in the performance index of the base isolated structure without isolation damping but with the optimized TLCD attached is illustrated in Fig. 9. Note the positive effect of fine-tuning visible in Fig. 9. Further increase in robustness is achieved by splitting the much too large TLCD in smaller ones in parallel connection, improvements discussed above and not detailed for this illustrative application.

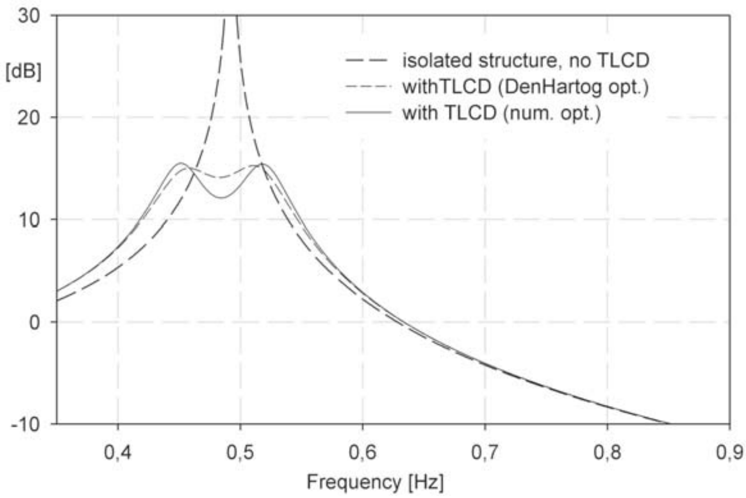

Figure 9. - Performance index in the critical frequency window for the base isolated building without isolation damping. Single optimized TLCD in passive mode applied in the base slab.

Fixed base, Chopra [46]			Base isolated, Chopra [46], with isolation damping			‘no’ isolation damping		Base isolated, ‘no’ isolation damping, but with TLCD in base slab		
mode	$T[s]$	$\zeta[\%]$	mode	$T[s]$	$\zeta[\%]$	$T[s]$	$\zeta[\%]$	mode	$T[s]$	$\zeta[\%]$
								1	2,21	4,19
			1	2,029	9.58	2,029	0,01	2	1,93	3,80
1	0,400	2,00	2	0,217	5,64	0,217	3,60	3	0,218	3,63
2	0,137	5,84	3	0,114	7,87	0,114	7,01	4	0,114	7,01
3	0,087	9,20	4	0,080	10,3	0,080	9,93	5	0,081	9,92
4	0,068	11,8	5	0,066	12,3	0,066	12,1	6	0,066	12,1
5	0,059	13,5	6	0,059	13,6	0,059	13,6	7	0,059	13,6

Table 1. Effective modal linear structural damping coefficients.

NOVEL DESIGN OF AN ACTIVELY CONTROLLED TLCD

Hochrainer [13] proposed a sealed TLCD with controlled gas supply from a standby high pressure gas vessel using reduction valves and a simple bang-bang control, Fig. 2. Consequently, the pressure difference in Eqs. (3) and (5) is changed by adding the newly activated pressure difference due to gas injection and removal, and substitution renders Eq. (6) generalized through the action of the active pressure supply,

$$\Delta p_a \big/ p_{init} = \left(1 + \Delta m_{gas} / m_{gas}\right), \; p_2 - p_1 = \Delta\, p_a + 2\, n p_0\, u \,/ H_a \qquad (21)$$

$$\ddot{u}+\delta_L\,\dot{u}|\dot{u}|+\omega_A^2\,u=-\kappa\,\ddot{w}_f-\Delta\,\overline{p}_a\,,\quad \Delta\,\overline{p}_a=\Delta\,p_a/\rho L_{eff} \tag{22}$$

A standard feedback control problem results when the state equation, with active pressure added to Eq. (17), is combined with the output equation,

$$\dot{\vec{z}}=\left(\tilde{A}+\tilde{B}\,\tilde{R}\right)\vec{z}-\vec{e}_g\,\ddot{w}_g+\tilde{E}_f\,\vec{F}(t)+\tilde{E}_a\,\Delta\vec{\overline{p}}_a\,,\quad \tilde{E}_a^T=\left[\tilde{0}\quad\tilde{0}\quad-\tilde{M}_S^{-1}\begin{pmatrix}\tilde{0}\\ \tilde{I}\end{pmatrix}\right] \tag{23}$$

$$\vec{y}=\tilde{C}_r\,\vec{z}+\tilde{D}_{eff}\,\vec{F}_{eff}+\tilde{D}_a\,\Delta\vec{\overline{p}}_a\,,\quad \Delta\vec{\overline{p}}_a^{\,T}=\left[\Delta\overline{p}_{a,1},\ldots,\Delta\overline{p}_{a,n}\right].$$

The output matrices D_{eff} and D_a depend on the actual output quantity of interest and both matrices vanish if floor displacements or velocities are calculated. Since the optimal passive TLCD parameter are known (thus the matrix R remains unchanged), just the optimal control law remains to be constructed. Since the classical linear quadratic regulator (LQR) design, outlined, e.g. in Föllinger [47], Lewis and Syrmos [48], is a straightforward approach to optimal control and, for sufficiently long observation times, becomes a linear optimal feedback control, with constant $\tilde{P}$ defined by the standard Riccati matrix equation, we choose,

$$\Delta\vec{\overline{p}}_a=-\tilde{S}^{-1}\,\tilde{E}_a^T\,\tilde{P}\,\vec{z}_S\,,\quad \left(\tilde{A}+\tilde{B}\,\tilde{R}\right)^T\tilde{P}+\tilde{P}\left(\tilde{A}+\tilde{B}\,\tilde{R}\right)-\tilde{P}\,\tilde{E}_a\,\tilde{S}^{-1}\,\tilde{E}_a^T\,\tilde{P}+\tilde{Q}=\tilde{0} \tag{24}$$

The well-established Luenberger estimators, or Kalman filters, see e.g. Levine [49] may be used as state-estimating filters, to reconstruct the full state vector from the practicably available scarce measured input data. Each ATLCD is dedicated to a pre-selected mode, similar to the passive TLCD design to save on gas consumption. The pressure input to the multiple ATLCD should be modeled as a first order low pass process with properly assigned cut-off frequencies, to avoid that the first TLCD, that is designed to mitigate the first two neighboring vibration modes, starts to operate at higher frequencies, and so on. Since a simple bang-bang control strategy of the active pressure is most practicable, a robust nonlinear control law should be applied, (Wu et al. [50]),

$$\Delta\vec{\overline{p}}_a=\begin{Bmatrix}+\max\Delta\,\vec{\overline{p}}_a\ldots\ldots\tilde{S}^{-1}\,\tilde{E}_a^T\,\tilde{P}\,\vec{z}_S<0\\ -\max\Delta\,\vec{\overline{p}}_a\ldots\ldots\tilde{S}^{-1}\,\tilde{E}_a^T\,\tilde{P}\,\vec{z}_S>0\end{Bmatrix} \tag{25}$$

MECHANICAL MODEL OF A CONTINUOUS BRIDGE WITH SEVERAL TLCD ATTACHED

Bridges with low structural damping are forced to, more or less coupled, oblique bending and torsional vibrations. Intensity of excitation increases with the action of traffic flow, trains moving sinusoidally on their tracks and/or at critical speed, gusty wind, dense population of walking pedestrians and runners. If lateral horizontal and torsional motions dominate, U-shaped liquid column dampers tuned with respect to frequency and energy absorption (TLCD) are ideally suited to increase the effective structural damping of the bridge. The mechanical model is developed in steps, starting with the in-plane rigid body motion of the cross-section with a TLCD attached. Such a three degree-of-freedom rigid frame carrying the single D.O.F. absorber is analyzed computationally and experimentally under severe forcing by walking pedestrians. Modal tuning with respect to the dominating horizontal displacement and, alternatively, with respect to a dominating rotation, is performed analogous to the classical Den Hartog tuning of a mechanical damper (TMD). Extension to modal analysis of the multiple-degree-of-freedom (MDOF) main structure with several TLCD attached is subsequently done using the Ritz-Galerkin approximation. Fine-tuning in the state space renders the effective damping characteristic of the bridge more robust and optimal.

Substructure synthesis of a single TLC

The rigid, symmetrically designed U-shaped piping system of the absorber is fastened to the cross-section of the bridge to form a rigid frame with three DOF, Fig 10. The pipe is partially filled with fluid with its relative motion described by the displacement *u(t)* of its interface to gas, Fig. 10a. After choosing the liquid mass m_f, the TLCD design parameters are still the horizontal length of the liquid column *B*, the length of the liquid column in the inclined pipe section at rest *H*, the horizontal and inclined cross-sectional areas A_B and A_H, respectively, and the opening angle β of the inclined pipe section. If the piping system is sealed, the gas inside the air chamber is quasi-statically compressed by the liquid surface in relatively slow motion. Hence, the pressure difference, see again Fig. 1a, when properly linearized, changes the undamped circular natural frequency of the TLCD defined in Eq. (6). The absolute acceleration of the reference point *A* of the frame *(y', z')* in "prescribed rigid body motion", Fig. 10b, and thus the guiding acceleration are given by

$$\vec{a}_A = \ddot{v}\vec{e}_y + \ddot{w}\vec{e}_z - d_A\left(\ddot{\vartheta}\vec{e}_y^{\,'} + \dot{\vartheta}^2\vec{e}_z^{\,'}\right),\ \vec{a}_g = \vec{a}_A + \ddot{\vartheta}\hat{\vec{r}}^{\,'} - \dot{\vartheta}^2\vec{r}^{\,'},\quad \hat{\vec{r}}^{\,'} = \vec{e}_x \times \vec{r}^{\,'}$$

$$a_{y'} = \ddot{v} + \ddot{w}\vartheta - d_A\ddot{\vartheta},\quad a_{z'} = \ddot{w} - \ddot{v}\vartheta - d_A\dot{\vartheta}^2 \tag{26}$$

Performing the integration and substituting the guiding acceleration in Eq. (3) yields the second of Eq. (9) in its most general form. Rotational angles are assumed to be small, $|\vartheta| << 1$, thus implying linearizations in Eq. (26), turbulent damping is equivalently linearized,

$$\ddot{u} + 2\zeta_A\omega_A\dot{u} + \omega_A^2\left[1 - \kappa_1\frac{\ddot{w}}{H\,\omega_A^2} + \left(\kappa_1\frac{d_A}{H} - \kappa_2\right)\frac{\dot{\vartheta}^2}{\omega_A^2}\right]u = \kappa\left(\ddot{v} - g\vartheta\right) - \left(\kappa\,d_A + \kappa_1\frac{B}{2}\right)\ddot{\vartheta} \tag{27}$$

In Eq. (27), a parametric excitation is apparent, caused by both, the vertical and torsional motions. The geometry coefficients $\kappa, \kappa_1, \kappa_2$ and the effective length L_{eff} of the liquid column in Eq. (27) are defined by

$$\kappa = \frac{B + 2H\cos\beta}{L_{eff}},\ \kappa_1 = \frac{2H\sin\beta}{L_{eff}},\ \kappa_2 = \frac{B\cos\beta + 2H}{L_{eff}},\ L_{eff} = 2H + \frac{A_H}{A_B}\,B \tag{28}$$

Conservation of momentum and of moment of momentum, with respect to point *A*, of the fluid body render the resulting control force components in the moving frame and the resulting moment, with their nonlinear parts included, Fig. 10a, with additional geometry coefficients apparent,

$$F_{y'} = m_f\left[a_{y'} - \bar{\kappa}\left(\ddot{u} - u\dot{\vartheta}^2\right) + \frac{\bar{\kappa}_1}{2H}\left(\left(H^2+u^2\right)\ddot{\vartheta} + 4u\dot{u}\dot{\vartheta}\right)\right]$$

$$F_{z'} = m_f\left[a_{z'} - \bar{\kappa}\left(u\ddot{\vartheta} + 2\dot{u}\dot{\vartheta}\right) + \frac{\bar{\kappa}_1}{2H}\left(\left(H^2+u^2\right)\dot{\vartheta}^2 - 2\left(\dot{u}^2 + u\ddot{u}\right)\right)\right] \tag{29}$$

$$M_{Ax} = m_f\left[\bar{\kappa}_3 H^2\ddot{\vartheta} + \frac{\bar{\kappa}_1 B}{2}\ddot{u} - \bar{\kappa} u a_{z'} + \frac{\bar{\kappa}_1}{2H}\left(H^2+u^2\right)a_{y'} + \bar{\kappa}_2\left(u^2\ddot{\vartheta} + 2u\dot{u}\dot{\vartheta}\right)\right] + M_{AG}$$

$$M_{AG} = m_f g\left[\bar{\kappa} u + \frac{\bar{\kappa}_1}{2H}\left(H^2+u^2\right)\vartheta\right],$$

$$\bar{\kappa} = \frac{B + 2H\cos\beta}{L_1}, \quad \bar{\kappa}_1 = \frac{2H\sin\beta}{L_1}, \quad L_1 = 2H + \frac{A_B}{A_H}B, \quad m_f = \rho A_H L_1$$

$$\bar{\kappa}_2 = \frac{B\cos\beta + 2H}{L_1}, \quad \bar{\kappa}_3 = \frac{2H}{3L_1}\left(1 + \frac{3B^2}{4H^2} + \frac{3B}{2H}\cos\beta + \frac{A_B}{A_H}\frac{B^3}{8H^3}\right) \tag{30}$$

If the damping coefficient exceeds the cut-off value of parametric resonance, the influence of parametric excitation in Eq. (27) becomes negligible.

Modal (SDOF) substructure synthesis of a single TLCD to a bridge

Extending the, more or less coupled flexural and torsional, partial differential equations of forced vibrations of a continuous beam, recorded e.g. in Nowacki [52], to include oblique bending, yield, see [53] for details, when inserting the single-term Ritz approximation with Galerkin's procedure applied, ($m=\rho A=const$ is the mass per unit length and light modal structural damping has been added),

$$v(x,t) = Y(t)\chi(x), \quad w(x,t) = Y(t)\phi(x), \quad u_T(x,t) = e\vartheta(x,t) = Y(t)\psi(x),$$

$$\int_0^l m\left[\chi_i\chi_j + \phi_i\phi_j + \psi_i\psi_j + \frac{c}{e}\left(\psi_i\phi_j + \phi_i\psi_j\right) - \frac{d}{e}\left(\psi_i\chi_j + \chi_i\psi_j\right)\right]dx = \begin{bmatrix} 0 & \text{for } i \neq j \\ I_e/e^2 & \text{for } i = j \end{bmatrix}$$

$$\ddot{Y} + 2\zeta\Omega\dot{Y} + \Omega^2 Y = \left[-\frac{F_{y'}}{M}\left(\chi + \phi\psi\frac{Y}{e} - \frac{d_A}{e}\psi\right) - \frac{F_{z'}}{M}\left(\phi - \chi\psi\frac{Y}{e}\right) - \psi\frac{M_{Ax}}{eM}\right]_{x=\xi} + \frac{F(t)}{M}$$

$$F(t) = \int_0^l\left[\chi(x)p_y(x,t) + \phi(x)p_z(x,t) + \psi(x)\frac{m_x(x,t)}{e}\right]dx, \quad M = I_e/e^2 \tag{31}$$

Preparation of the (linear) tuning procedure requires the linearized equations of motion of the projected main system and of the absorber synthesized. Neglecting parametric forcing in Eq. (27) and substituting the linear dynamic parts of Eq. (28), yield the linear matrix equation of the coupled modal SDOF-main system with a single TLCD, Eq. (10) properly generalized,

$$\tilde{M}_S\begin{bmatrix}\ddot{Y}\\ \ddot{u}\end{bmatrix} + \tilde{C}_S\begin{bmatrix}\dot{Y}\\ \dot{u}\end{bmatrix} + \tilde{K}_S\begin{bmatrix}Y\\ u\end{bmatrix} = \begin{bmatrix}F(t)/M\\ 0\end{bmatrix}, \quad \tilde{C}_S = \begin{bmatrix}2\zeta\Omega & 0\\ 0 & 2\zeta_A\omega_A\end{bmatrix}, \tag{32}$$

$$\tilde{K}_S = \begin{bmatrix}\Omega^2 & \mu\psi\bar{\kappa}g/e\\ \psi\kappa g/e & \omega_A^2\end{bmatrix}_{x=\xi}, \quad \mu = m_f e^2/I_e,$$

$$\tilde{M}_S = \begin{bmatrix} 1+\mu\left(\chi^2+\phi^2+\left(\dfrac{\psi d_A}{e}\right)^2\left(1+\bar{\kappa}_3\dfrac{H^2}{d_A^2}\right)-\dfrac{2d_A}{e}\psi\chi\right) & \mu\left(-\chi\bar{\kappa}+\dfrac{B}{2e}\psi\bar{\kappa}_1+\dfrac{d_A}{e}\psi\bar{\kappa}\right) \\ -\chi\kappa+\dfrac{B}{2e}\psi\kappa_1+\dfrac{d_A}{e}\psi\kappa & 1 \end{bmatrix}_{x=\xi}$$

Extension, with several TLCD attached, produces the hyper matrix form of Eq. (32).

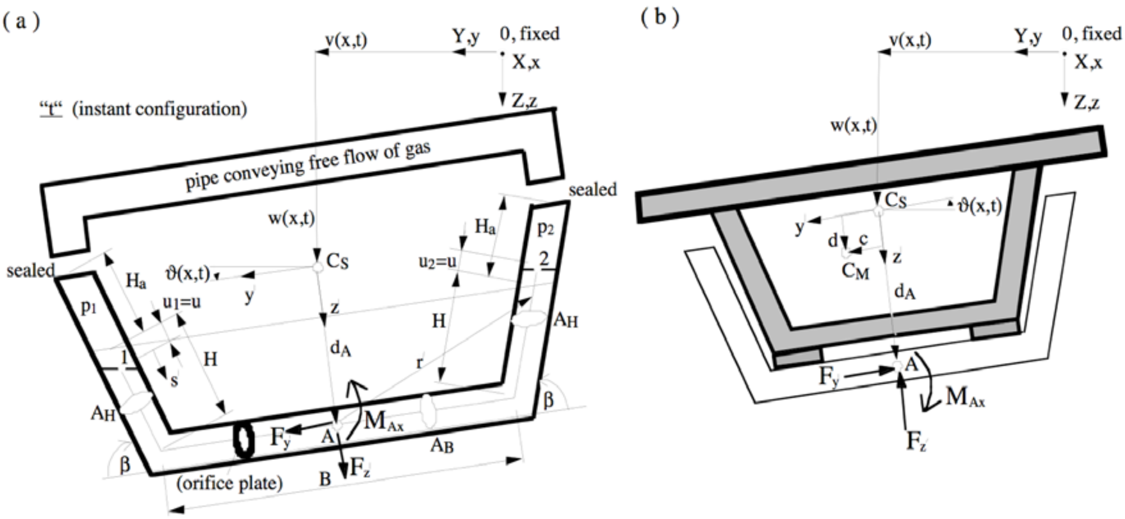

Figure 10. – Free-body-diagram (a) - Symmetrically designed TLCD (moving reference frame (y',z'), origin C_S). When sealed, gas volume at rest $A_H H_a$. (b) - Single cross section $x=\xi$ of the bridge (three D.O.F. in-plane motion). Absorber forces applied. Center of mass C_M. Symmetry in stiffness, center C_S.

Den Hartog tuning in case of dominating horizontal lateral vibrations

Consequently, the vertical and torsional mode shapes ϕ and ψ are set equal to zero in Eq. (32). The mass ratio of the equivalent TMD results and the optimal frequency ratio of the TLCD, e.g. Eq. (2), changes accordingly to,

$$\mu^* = \frac{m_A^*}{M^*} = \left.\frac{\mu\kappa\bar{\kappa}\chi^2}{1+\mu\chi^2(1-\kappa\bar{\kappa})}\right|_{x=\xi}, \qquad \mu = \frac{m_f}{M}, \tag{33}$$

$$\delta_{opt} = \frac{\omega_A}{\Omega} = \left.\frac{\delta_{opt}^*}{\sqrt{1+\mu\chi^2(1-\kappa\bar{\kappa})}}\right|_{x=\xi}, \ \zeta_{A,opt} = \zeta_{A,opt}^* \tag{34}$$

Parameter excitation of the TLCD by the vertical flexural vibrations of the bridge remains ineffective if the optimal linear damping coefficient is larger than the cut-off value of time-harmonic parametric resonance, [53],

$$\zeta_{A,opt} = \frac{4U_0\delta_L}{3\pi} > \zeta_{A,0}^{(w)} = \kappa_1\frac{\max\left|w(x=\xi_k)\right|}{H} \tag{35}$$

Den Hartog Tuning in case of dominating torsional vibrations

Consequently, the oblique flexural vibrations are neglected, thus the vertical and lateral mode shapes ϕ and χ are set equal to zero in Eq. (32). The mass ratio of the equivalent TMD turns out in a more elaborate fashion with its optimal frequency ratio changed to,

$$\mu = m_f\, e^2 / I_e\,,\ \mu^* = \left. \frac{\mu\left(\bar{\kappa}_1 B + 2\bar{\kappa} d_A\right)\left(\kappa_1 B + 2\kappa d_A\right)\psi^2}{4e^2\left[1+\mu\psi^2\left(\bar{\kappa}_3\frac{H^2}{e^2}+\frac{d_A^2}{e^2}-\frac{1}{4e^2}\left(\bar{\kappa}_1 B + 2\bar{\kappa} d_A\right)\left(\kappa_1 B + 2\kappa d_A\right)\right)\right]}\right|_{x=\xi}, \tag{36}$$

$$\delta_{opt} = \left. \frac{\delta_{opt}^*}{\sqrt{1+\mu\psi^2\left[\bar{\kappa}_3\frac{H^2}{e^2}+\frac{d_A^2}{e^2}-\frac{1}{4e^2}\left(\bar{\kappa}_1 B + 2\bar{\kappa} d_A\right)\left(\kappa_1 B + 2\kappa d_A\right)\right]}}\right|_{x=\xi}, \tag{37}$$

The optimal damping coefficient remains unaffected. Parameter resonance in the torsional mode (*w=0* in Eq. (27)) remains ineffective if the sufficient condition applies, -note the cut-off value of parametric resonance turns out differently to Eq. (35)- see again Fig. 10a and consider Eq. (28), Ref. [53],

$$\zeta_{A,opt} = \frac{4U_0\delta_L}{3\pi} > \zeta_{A,0}^{(\vartheta)} = \frac{1}{8}\left|\kappa_1\frac{d_A}{H}-\kappa_2\right|\vartheta_{0,\max}^2 \tag{38}$$

NUMERICAL SIMULATION OF A MECHANICAL MODEL OF THE ORIGINAL TODA PARK BRIDGE

The detailed report by Nakamura and Fujino [8] provides all relevant data for tuning of TLCD. The pedestrian cable-stayed bridge, illustrated in Fig. 11 has a mass per unit of length $m=\rho A=4180$ *kg/m* and the first three natural frequencies and modal structural damping coefficients assigned: $f_{S1}=0.73$, $f_{S2}=0.93$ and $f_{S3}=2.04\,Hz$, $\zeta_{S1}=0.8$, $\zeta_{S2}=0.85$ and $\zeta_{S3}=0.90\%$. First and third mode are dominant vertical, the second mode is critical for a densly population of walking pedestrians being dominate lateral horizontal. Torsional stiffness is high and thus torsional vibrations become negligible. A single TLCD optimally positioned in the main span is tuned. The mode shape is taken for a double span beam, Blevins [54], $l=l_1+l_2$, Fig. 11, $\lambda_1=5$,

$$\chi_2(\xi) = \sin\lambda_1\xi + \eta_1\sinh\lambda_1\xi\,,\ \eta_1 = -\frac{\sin\lambda_1 l_1/L}{\sinh\lambda_1 l_1/L}\,,\ 0\le\xi\le l_1/L$$

$$\chi_2(\xi) = \eta_2\left(\sin\lambda_1(1-\xi)+\eta_3\sinh\lambda_1(1-\xi)\right),\ \ l_1/L\le\xi\le 1 \tag{39}$$

$$\eta_2 = \frac{\sin\lambda_i l_1/L}{\sin\lambda_i l_2/L}\,,\ \eta_3 = -\frac{\sin\lambda_i l_2/L}{\sinh\lambda_i l_2/L}\,.$$

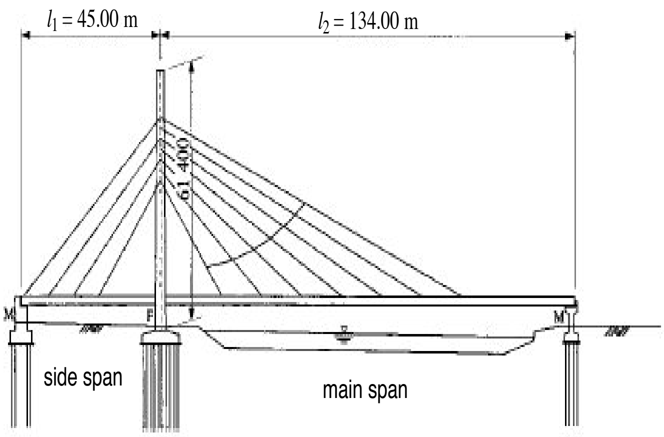

Figure 11. - Toda Park Bridge, Nakamura and Fujino [8]. Vibration prone in second mode.

The largest modal deflection is observed at ξ=0.60, thus defining the best position of the TLCD. Computing the kinetic energy defines the modal mass M_2 =259.5x10^3 *kg*. The TLCD parameter are selected, m_f=1500 *kg*, i.e. $\mu = m_f / M_2$ = 0.6% and B=2.0 *m*, H=2.0 *m*, A_B=A_H=0.25 m^2, $L_{eff}=2H+B$=6.00 *m*, $\beta = \pi / 4$, $\kappa = \bar{\kappa} = 0.80$ and $\bar{\kappa}_1 = 0.47$, Fig. 11. Den Hartog tuning yields $f_A = 0.92\,Hz$ and $\zeta_A = 0.038$. By means of the state vector of the bridge and a weighing matrix, $\bar{z} = \left[Y_1\, Y_2\, Y_3\, \dot{Y}_1\, \dot{Y}_2\, \dot{Y}_3\right]^T$ $\tilde{Q} = diag\left[10,10,10,1,1,1\right]$, fine tuning by minimizing the performance index $J = J(f_A, \zeta_A)$, Eq. (18), renders the final optimal parameters, $f_{A,opt} = 0.92\,Hz$ and $\zeta_{A,opt} = 0.032$.

The gas volume in the sealed TLCD is defined by H_a=1.56 *m*. For the Toda Park Bridge, the time periodic pedestrian excitation was found critical if assigned lateral and vertical, uniformly distributed over the bridge's main span, dense package, $n_P = 450$, see again [53a] for details. The condition to safely neglect parametric resonance of the TLCD, Eq. (35) with the maximum amplitude estimated, $\max|w_1| = 5 \times 10^{-3} m$ and substituted, yields $\zeta_{A,0} = 0.002 < \zeta_{A,opt} = 0.032$. The gain in effective structural damping of the second mode is demonstrated in Fig. 12 for the linear model. Estimating the maximum fluid displacement U_{max}=0.3 *m* and $\delta_L = 0.268$ allows for a fully nonlinear modeling in MATLAB. Fore results see Figs. 12 and 13. The critical resonant peak of mode number two is broken down by the action of the properly tuned TLCD. Inspection of Fig. 13 reveals the increase of the effective structural damping coefficient $\zeta_{S2,eff} = 2.6\%$ i.e. by a factor of three. Evaluating the cut-off damping coefficient $\zeta_{S,0} = 0.43\%$, Eq. (35), far below the effective structural damping, indicates prevention of the synchronization phenomenon.

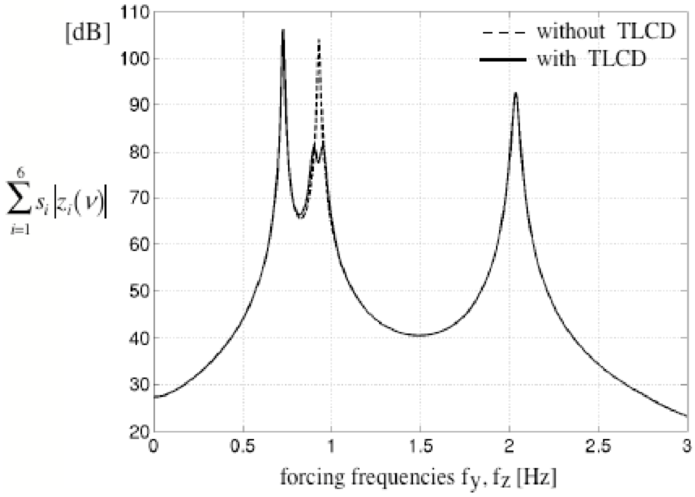

Figure 12. - Frequency response function (FRF) of the sum of weighted state space variables of the bridge. Both, lateral and vertical time harmonic forcing. Linearized analysis, [18}.

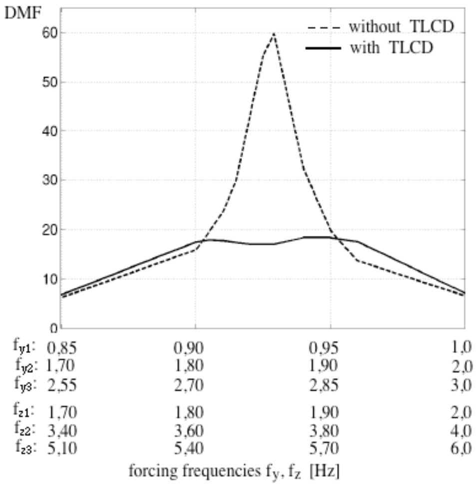

Figure 13. - Toda Park Bridge: DMF of the lateral displacement v(x=0.60) within the critical frequency window. Symmetric pedestrian excitation in lateral and vertical directions (frequencies listed). Turbulent damping. Refs. [18] and [53].

Application of TLCD during the cantilever method of bridge construction

Wind gusts are critical with respect to horizontal vibrations of the cantilevered bridge. Stepwise tuning during the advancing construction length of the tip-positioned TLCD increases the structural damping and thus allows longer overhangs. A scaled model tested in the laboratory of the Author in the Institute of Building Construction and Technology, Fig. 14, convincingly approve the simulation results in both cases, under time harmonic contact-less excitation, Fig. 15, measured and numerically simulated dynamic magnification factors are in good agreement in the critical frequency window, and by the decay rate of free vibrations, Fig. 16. Modal tuning is performed with respect to the basic mode of the cantilevered bridge, Achs [55].

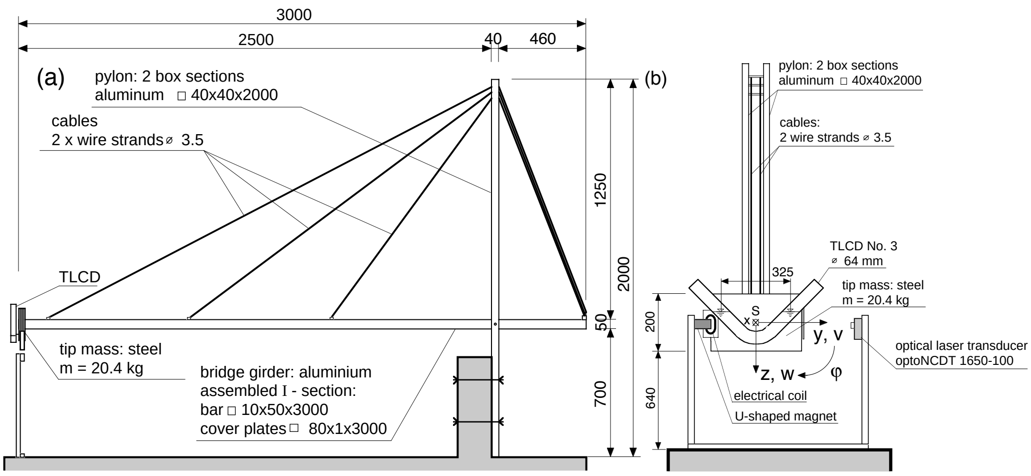

Figure 14. - Lab-model: cantilever method of bridge construction, dimensions in mm. (a) In side-view- (b) In front view, TLCD and electro-magnetic excitation shown. μ=6.2%, [55].

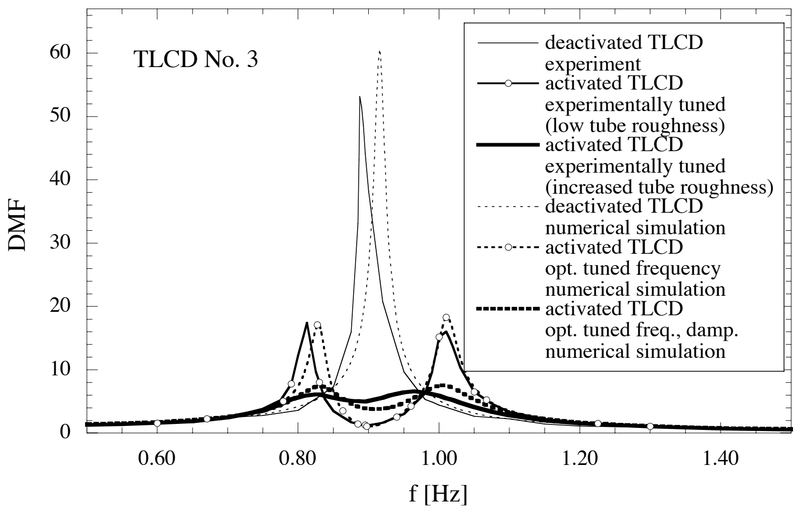

Figure 15. – Measured and calculated dynamic magnification factor, critical window, [55].

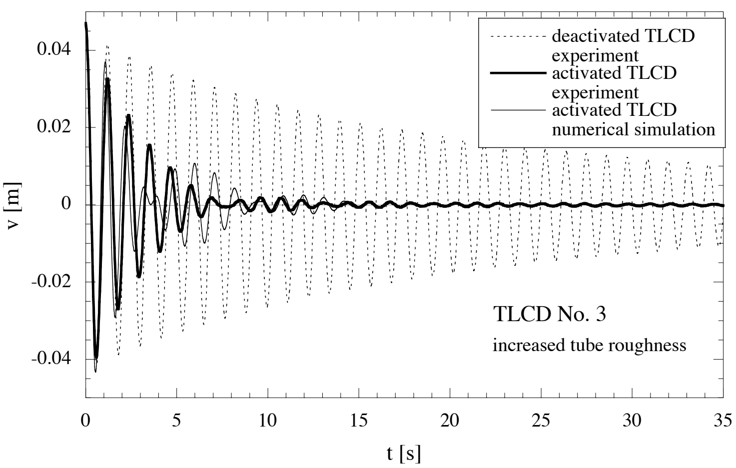

Figure 16. Decay rates of free vibrations, TLCD attached, [55].

THE VERTICALLY ACTING TUNED LIQUID COLUMN DAMPER

The U-shaped tuned liquid column damper (TLCD) increases the effective structural damping of horizontal vibrations similar to the classical tuned mechanical pendulum type damper (TMD). The pipe-in-pipe TLCD, Fig. 17 applies to vertical vibrations, likewise to the spring-mass-dashpot TMD, [5]. Since one air chamber must be sealed for sake of static over-pressure, the gas-spring effect is inherent in this design. The geometric analogy between the redesigned TLCD and the TMD still exists, making the first step in the tuning procedure ‘classical’. Subsequent fine tuning in state space when the TLCD is split into smaller ones in parallel action, renders an even more robust passive action. The experimentally observed averaged turbulent damping of the relative fluid flow and the weakly nonlinear gas-spring render the TLCD insensitive to overloads and parametric forcing due to the vertical motion.

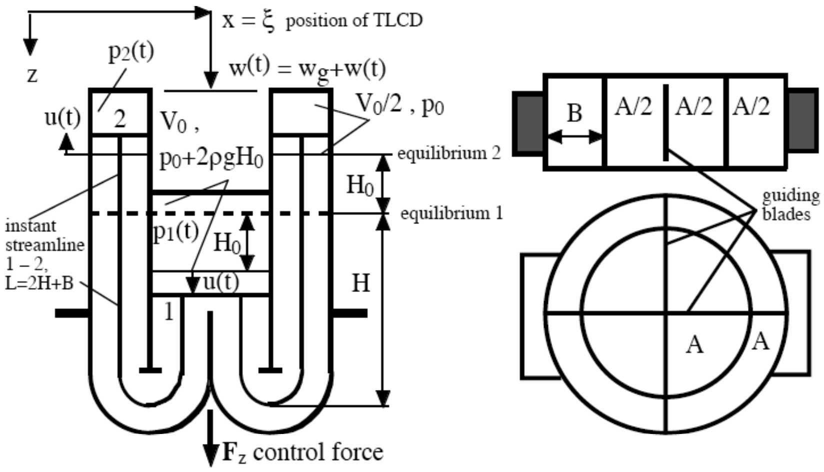

Figure 17. - Novel design of sealed TLCD for vertical vibration damping. Flat or curved bottom. Static over-pressure head H_0. Earlier design of Ref. [56] is optimized, [57]. Circular cylindrical (axi-symmetric) design needs guiding blades to circumvent rotational flow.

When considering a single-degree-of-freedom (SDOF-) main system with such an alternative TLCD attached, see again Fig. 17, renders, upon equivalent linearization of the turbulent damping $\delta_L \dot{u}|\dot{u}|$, resulting in the relation $\delta_L = 3\pi\zeta_A/4 \max|u|$, and after substitution of the first, linear part of the control force $F_z = F_1 + F_2$, the coupled system of equations of motion,

$$\begin{bmatrix} 1+\mu & -\mu\kappa_0 \\ -\kappa_0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{w} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} 2\zeta_S\Omega_S & 0 \\ 0 & 2\zeta_A\omega_A \end{bmatrix}\begin{bmatrix} \dot{w} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} \Omega_S^2 & 0 \\ 0 & \omega_A^2 \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix} = -\begin{bmatrix} 1+\mu \\ -\kappa_0 \end{bmatrix}\ddot{w}_g + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{F(t)}{M} \tag{40}$$

$$\omega_{A,opt} = \sqrt{\frac{2g}{L}\left(1+n\frac{(h_0+H_0)}{H_a}\right)}\ ,\ F_1 = m_f\left(\ddot{w}^{(t)} - \kappa_0\,\ddot{u}\right),\ F_2 = -m_f\,\kappa_1\left[u\,\ddot{u}+\dot{u}^2\right]\Big/H\ ,$$

$$\mu = m_F/M_S\ ,\ H_a = V_0/A\ ,\ \kappa_0 = 2H_0/L\ ,\ \kappa_1 = (2H/L)$$

$-\left[\kappa_1 \ddot{w}^{(t)}\, u/H\right]$, the parametric forcing term omitted in Eq. (40): $\zeta_A > \zeta_{A,0}^{(w)} = \kappa_1 \max\left|w^{(t)}/H\right| < 1$

The well-known Den Hartog formula for the equivalent TMD (all parameters are denoted by a star) apply, where the mass ratio and, from Eq. (40) the transformed optimal parameter, become

$$\mu^* = m_A^* \big/ M^* = \kappa_0^2 \mu \Big/ \left[1 + \left(1 - \kappa_0^2\right)\mu\right], \; \delta_{A,\,opt} = \omega_A / \Omega_S = \delta_{A,opt}^* \Big/ \sqrt{1 + \left(1 - \kappa_0^2\right)\mu} \qquad (41)$$

Damping coefficient remains unchanged as above. Eq. (40) takes on a hyper matrix form for an MDOF- main system with modal coordinates Y_i, $i = 1,2,...\mathrm{N}$, with several TLCD, attached at positions $x = \xi_k$, $k = 1,2,...\mathrm{r}$ and possibly split into n_k smaller TLCD in parallel action at one and the same location. Fine tuning in state space is again recommended.

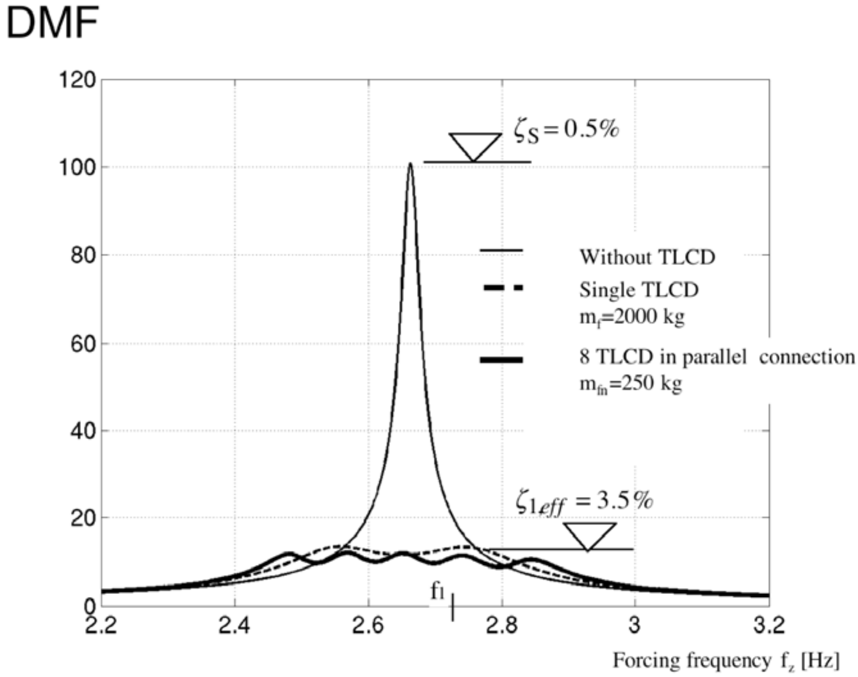

Figure 18. Dynamic magnification factor of the vertical deflection at mid-span of a standard 50m-span SS-bridge in the critical frequency window. Single force excitation, Ref. [57].

Effective damping of the vertical flexural vibrations of a standard SS-bridge span

A simply supported bridge of span l=50m is considered with basic natural frequency f_1=2.73Hz. All modal masses are equal to $M=ml/2$=35.72x10^{3}kg. Since the low structural damping $\zeta_S = 0.5\%$ must be increased, a TLCD with fluid mass m_f=2000kg is placed at mid-span and modally tuned. Its dimensions, Fig. 17, are B=0.5m, as small as possible, H=1.00m, A=0.80m^2 and H_0=0.50m. Hence, $\kappa_0 = 0{,}40$ and with $\mu = m_f / M \approx 5.6\%$ the TMD equivalent is $\mu^* = 0.9\%$. The Den Hartog parameter result: $f_{A,opt} = 2.64\,\mathrm{Hz}$ and $\zeta_{A,opt} = 5.6\%$. Selecting the atmospheric reference pressure head h_0=10m and, since the natural frequency is assigned the gas volume results, H_a=380mm. The force $F(t) = F_0 \cos 2\pi f_z t$ with $F_0 = 10\mathrm{kN}$ (producing a static deflection at mid-span of $w_{stat} = F_0 / k_1 = 0.95\mathrm{mm}$) is considered next, rendering

$\max\left|w\left(f_z = 2f_A\right)\right| = 1\text{mm}$. Since the inequality holds true, $\zeta_{A,opt} = 5.6\% > \zeta_{A,0}^{(w)} = 0.08\%$, no effect of the parametric forcing is to be expected. Subsequently, the pipe-in-pipe TLCD is split into eight smaller units. Fine tuning in state space yields, $\tilde{Q} = diag[10\ 1]$, $f_{A1,8,opt} = 2.80$, $f_{A2,7,opt} = 2.51$, $f_{A3,6,opt} = 2{,}70\,\text{Hz}$ and $\zeta_{A1,8,opt} = 1.85$, $\zeta_{A2,7,opt} = \zeta_{A3,6,opt} = 1.73\%$. The gas volumes are accordingly adjusted to $H_{a1,8} = 0.32$, $H_{a2,7} = 0{,}40$, $H_{a3,6} = 0{,}34$ and $H_{a4,5} = 0{,}37\text{m}$.

Both, the dramatic increase of the effective structural damping by a factor of seven and the improvement of robustness are observed in the critical frequency window of the dynamic magnification factor of the vertical mid-span deflection, Fig. 18.

CONCLUSION

Design and tuning procedures for sealed tuned liquid column dampers (TLCD) are analyzed in detail and the substitution for classical tuned mechanical dampers is emphasized. Especially the worked analogy between TLCD and TMD action and their optimal parameter make tuning available at the fingertips. TLCD in their passive mode with the gas-spring effect taken into account are robust, self-controlling against overload and insensitive to any vertical excitation. The frequency range of application can be extended to about five Hertz.

In tall buildings all TLCD can be positioned on the top floor. However, those tuned with respect to higher modes can be positioned at the story with the largest modal displacement. In situ fine tuning is simple and can follow any structural degradation of aging buildings. Substitution of the isolation damping of base isolated buildings by TLCD in the base slab is proposed. Finally, the hybrid action of an actively controlled TLCD is proposed to counteract single peaks in (seismic) loading. Bang-bang controlled pressure supply to the air chambers above the fluid is suggested. A high pressure gas storage and reduction valves are the required additional hardware. However, valve control must guarantee constant gas mass after each stroke, a practically yet unsolved design problem.

Considering a laboratory model of a bridge and simulating vibration prone bridges forced by dense populations of pedestrian prove experimentally and in computational modeling the effective damping property of tuned liquid column dampers when placed at properly selected locations. Lateral and torsional vibrations can be reduced by U-shaped TLCD. A sufficient condition based on the cut-off damping coefficient of parametric resonance of the TLCD allows neglecting the vertical motion. Since the synchronization effect of the walking pace of people brakes down if the effective structural damping is larger than a cut-off value, applications of TLCD make bridges safe and useful without restrictions. Similarly, the Auckland-Harbour-Bridge in New Zealand that carries the flowing traffic without problems but was found vibration prone to runners, could be stabilized by TLCD to allow the unlimited crossing during the City-marathon-race. Vibrations in the same, low frequency band, when excited by wind gusts are reduced too. In the course of the cantilever method of bridge construction, critical lateral and rotational vibrations are controlled by properly moving a TLCD with easily adjustable tuning along side. Smaller units of TLCD in parallel action increase robustness however, the price are increased amplitudes of the (relative) fluid flow, damping coefficients are decreased in the course of fine tuning.

The novel pipe-in-pipe design by Reiterer and Ziegler [57] of the TLCD allows applications in structures with a main vertical vibration component. However, its efficiency is somewhat lower when compared to the U-shaped TLCD. Main advantages of all TLCD in passive action are simple construction, no moving mechanical parts and easy in-situ fine tuning.

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