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BROADBAND PASSIVE SYNTHETIC APERTURE TOWED-ARRAY PROCESSING

Edmund J. Sullivan*[1], Jason D.Holmes[2], William M. Carey[2]

[1]EJS Consultants, 46 Lawton Brook Lane, Portsmouth Rhode Island 02871 USA
[2] College of Engineering, Boston University, Boston, MA 20015 USA
paddypriest@aol.com (e-mail address of lead author)

Abstract
Bearing estimation using an acoustically short towed array can be enhanced by incorporating the motion of the array into the signal model. It was previously theoretically shown (J. Acoust. Soc. Am., Vol. 115, No 4) that by casting the problem as a joint estimation problem, where the parameters to be estimated are the bearing and the fundamental frequency of the discrete Fourier transform of the broadband data, the bearing estimation performance of the synthetic aperture algorithm, as measured by the variance of the bearing estimate, can exceed that of the conventional array processor. This occurs because the processor is able to exploit the bearing information contained in the Doppler shift. A Kalman filter is utilized to carry out the joint estimation problem. The two unknowns, the fundamental source frequency and the bearing angle, constitute the state vector elements. The measurement model is the frequency averaged phases of the Fourier components. The data are preprocessed by computing a sequence of contiguous Fourier transforms, and averaging the phases of the frequency lines. This sequence of averaged phases then constitutes the time series which is the input to the Kalman filter. An auxiliary measurement equation is used to relate the observed fundamental frequency to that of the source frequency. After a short theoretical introduction, experimental results based on the radiated noise of a ferry under way in Nantucket Sound are shown. Results of successful bearing estimation with an array of acoustic length of approximately two wavelengths will be compared to results using the same data but with a conventional beamformer. The improvement in performance of the synthetic aperture algorithm over the conventional algorithm will be quantified.

INTRODUCTION

Passive synthetic aperture is a means of enhancing the performance of a moving array by exploiting the information in the Doppler. For example, in the case of bearing estimation, the forward motion of a towed array causes a difference in the spectrum of the received signal as compared to that of the signal at the source [1]. This can be seen from the Doppler equation as follows. Consider a plane wave signal arriving at a receiver that is moving with speed v, where the direction of propagation of the signal is at angle θ with respect to the normal to the direction of motion of the receiver. If the signal at the source is narrow-band with radian frequency ω_0, then the frequency at the receiver is given by

$$\omega = \omega_0\left(1 + (v/c)\sin\theta\right) \tag{1}$$

Here, c is the speed of sound in the water. Thus, if one has knowledge of the source frequency, the bearing can be found. Passive synthetic aperture bearing estimation uses this idea by casting the problem as a joint estimation of the source frequency and the bearing angle. In this paper, the problem is cast in the form of a Kalman filter [2]. More information on passive synthetic aperture and its history can be found in [3, 4] and references therein.

PROBLEM FORMULATION

STATE EQUATION

Consider first a narrow band version of the problem. Let a line array of N elements be moving in the $+x$ direction of an $x-y$ coordinate system. Let the plane-wave signal be arriving at angle θ with respect to the y axis, measured to be positive for clockwise rotation. The signal is then represented in complex form as

$$s_n = a_n e^{i(\omega_0/c)(x_n+vt)\sin\theta - i\omega_0 t} \tag{2}$$

ω_0, is the radian frequency of the signal at the source, x_n is the coordinate of the n^{th} receiver element, θ is the bearing angle measured from broadside, a_n is the signal amplitude and t is time. We choose to work in the phase domain, since this avoids the need to include the signal amplitude as a nuisance parameter, and more important, it allows the problem to be cast in a linear form. This leaves the two parameters ω_0 and θ to be estimated. The state equation of the Kalman filter is given by

$$\begin{bmatrix} z_1(t\,|\,t) \\ z_2(t\,|\,t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1(t\,|\,t-1) \\ z_2(t\,|\,t-1) \end{bmatrix} \tag{3}$$

with $z_1 = \omega_0$ and $z_2 = \omega_0 \sin\theta$. Note that there are no "dynamics" in the state equation. This is based on the assumption that the parameters are changing slowly in time. This is usually referred to as a "random walk" model.

MEASUREMENT EQUATION

Since we choose to work in the phase domain, the measurement equation is based on the exponent in Equation 1. Consider a discrete spectrum of the broadband signal. This will have a set of phases associated with it. Since these phases are related by the frequency index, an average phase can be computed as

$$\phi_n = (n/\Delta m)\sum_{Mlo}^{Mhi}\{m\omega_0(d/c)\sin\theta + \varphi_{mn}\}/m - i\omega_0 t \tag{4}$$

Here, Mlo and Mhi are the respective low frequency and high frequency indices of the DFT, Δm is the number of frequency components of the DFT and m is the frequency index. Note that this refers all of the phases to that of the lowest frequency line. Since only the phase *differences* are relevant to the problem, there are only $N-1$ receiver based measurements for the N receivers. These are given by

$$y_n = \phi_{n+1} - \phi_n \quad n = 1,2,\cdots,N-1 \tag{5}$$

There is an auxiliary measurement equation that is based on the observed frequency. This is basically the Doppler relation of Equation 1 and is given by

$$y_N = \omega = \omega_0\left(1 + (v/c)\sin\theta\right) \tag{6}$$

Thus, the measurement system has the linear form

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N-1} \\ y_N \end{bmatrix} = \begin{bmatrix} 0 & d/c \\ \vdots & \vdots \\ 0 & d/c \\ 1 & v/c \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{7}$$

Here, $d = x_{n+1} - x_n$, the inter-element spacing of the line array receiver elements.

PROCESSING ALGORITHM

Since the measurements are based on the relative phases of the receiver signals, some preprocessing must be done. The first step is to perform a discrete Fourier transform (DFT) over a selected time window. This time window must be no longer than the time of any significant change in the bearing. The phases of the complex amplitudes of the DFTs are then computed with the ‘angle’ function in MATLAB. These phases are then averaged over frequency and the pairwise elemental differences are then computed. That is, for the n^{th} phase difference, we must evaluate the following preprocessed measurement for each DFT.

$$y_n = (1/\Delta m)\sum_{Mlo}^{Mhi}\left\{\psi_{n+1}^{m} - \psi_{n}^{m}\right\}/m \qquad n = 1,2,\cdots,N-1 \tag{8}$$

In the above equation ψ_{n}^{m} is the measured phase of the m^{th} frequency component for the n^{th} receiver. The frequency measurement is taken to be the lowest frequency of the DFT of the received signal.

DESCRIPTION OF EXPERIMENT

An experiment conducted by Holmes et al, 2006, [5,6] with an autonomous vehicle and a six element towed array obtained experimental results that showed the ability of the system to obtain complex pressures suitable for synthetic aperture processing. The experiment was primarily conducted to form large synthetic apertures, O(4 Km), using narrowband manmade signals. During the experiment, a ferry from the mainland of Cape Cod on its way to the island of Nantucket passed through the area. The resulting radiated noise of this ferry provided the broadband data that is the basis for this work.

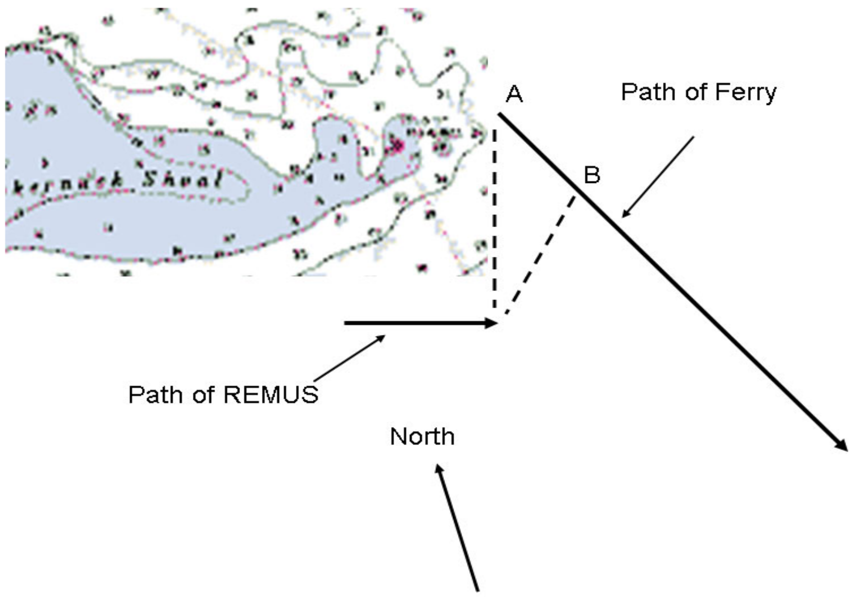

Figure 1 – Configuration of the experiment

The six element array, which had an element spacing of .75 meter, was towed at a speed of 1.5 meters/sec. The ferry appeared by emerging from a shallow region, known as Tuckernuck Shoal, at an angle very close to broadside (0^0) to the towed array, and the closest point of approach occurred at approximately $+20^0$. The array was moving in a straight line toward the course of the ferry, which was moving at approximately 20 kts, on a straight course from left to right with respect to forward endfire of the array. This configuration is depicted in Figure 1. The points A and B are the ferry positions for the respective beginning and end of the data used in this work. The distance between these two points is approximately 2 km. Although the radiated sound from the ferry was quite broadband, extending over a band from about 100 Hz to 1000 Hz, there was a particularly strong band of energy occurring between 890 Hz and 910 Hz. This energy was selected for the data in this paper. At this band of frequencies, the array has an acoustic length of approximately 2.3 λ .

A sequence of 200 one-second DFTs was generated in order to obtain the phase averages. The band was constrained to the 31 lines between 890 Hz and 910 Hz. These phase averages constituted the basis of the measurements used in Equation 8. The frequency measurement was taken directly as the lowest frequency of the DFT of the data. A Kalman filter, based on Equations 3 & 7 was then used to process the data.

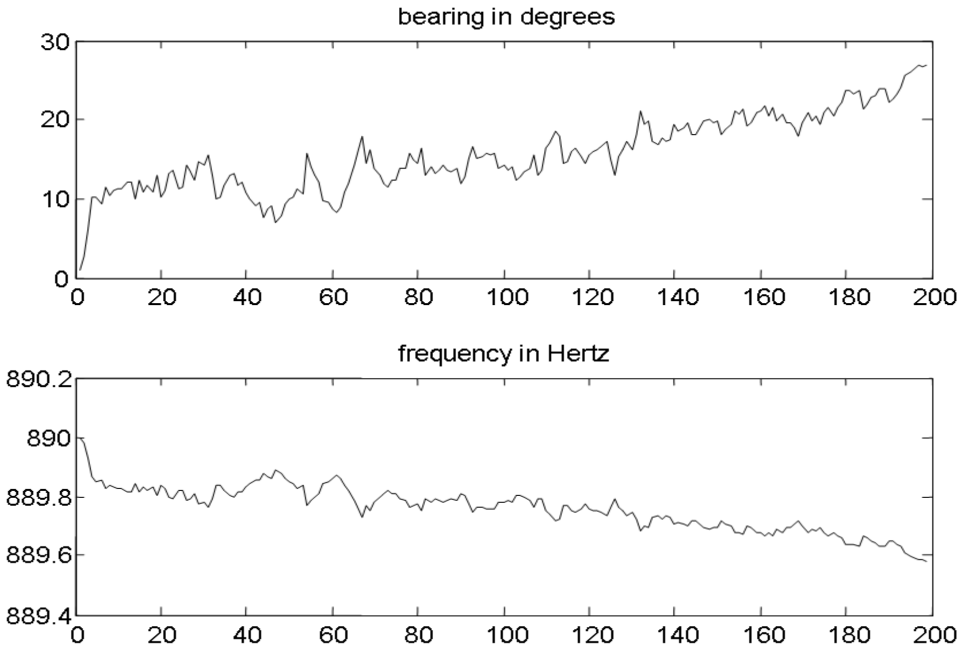

Figure 2 – Results of joint estimation of bearing and frequency

A more conventional synthetic aperture beamformer output used for a basis of comparison is given by

$$b(\theta,t)=\sum_{i=1}^{N}\sum_{j=1}^{M}P_{ij}(\omega)e^{\left[\frac{\omega}{c}x_{ij}\sin(\theta)-\omega t_i\right]}W(x_{ij}) \tag{9}$$

where P_{ij} is the complex pressure of the jth sensor at the ith time segment and $W(x_{ij})$ is an appropriate windowing function.

DISCUSSION

The results are shown in Figure 2. Also shown in this figure is the estimate of the source frequency. This can be thought of as the lowest frequency of a *virtual* DFT of the energy at the source. The value of the bearing estimate is corrupted between 0 and 40 seconds. The reason can be seen in Figure 3, which is a bearing frequency plot of the same data used to obtain the estimates in Figure 2. Here, the ambient noise, centered about 30^0 is the dominant energy. The dark red portion of the plot is the output of the conventional beamformer, given by Equation 9, and the yellow line is the Bearing estimate shown in Figure 2. It can be seen that the synthetic aperture estimator clearly

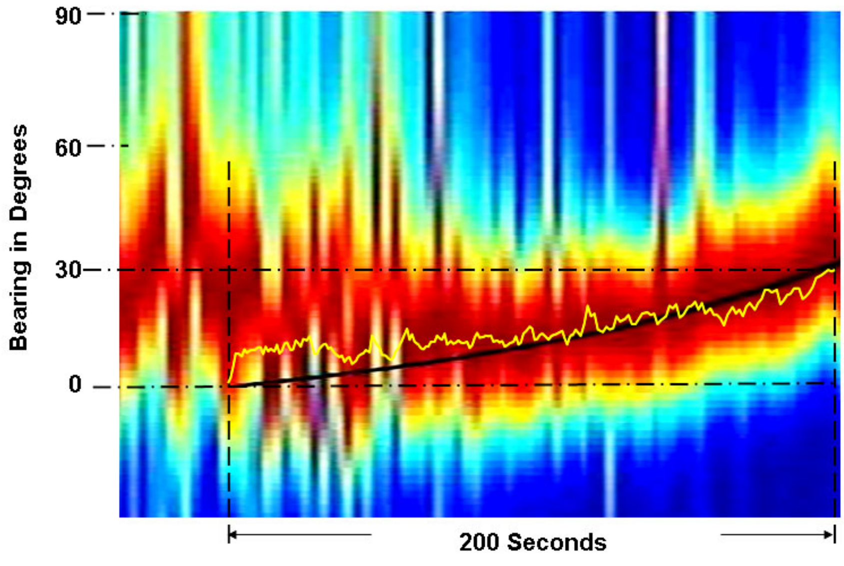

Figure 3 – Comparison with conventional beamformer

outperforms the conventional beamformer. The black line is the actual bearing of the ferry. The results of both bearing estimators are biased by the ambient noise at the earlier times*, i.e.,* those estimates below about 100 seconds. Also, there is not good agreement between the two estimators above 160 seconds. The reason for this is not clear.

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