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FULL-SCALE EXPERIMENTAL MODAL ANALYSIS OF AN OVERHEAD TRANSMISSION LINE TOWER CROSSING GUAMÁ RIVER

Sandro D. Rescinho Amador*[1], Luis A. Conte Mendes Veloso[2] and Remo Magalhães De Souza[2], Jean R. Ferreira Aguilera[3]

[1]Civil Engineering Post Graduation Programme, Federal University of Para, Brazil
Campus Universitário do Guamá, Prédio do Centro Tecnológico, Sala 232
Rua Augusto Côrrea, 01 CEP 66075-110, Belem, PA, Brasil
[2]Department of Civil Engineering Construction, Federal University of Para
Campus Universitário do Guamá, Prédio do Centro Tecnológico, Sala 232
Rua Augusto Côrrea, 01 CEP 66075-110, Belem, PA, Brasil
[3]Department of Civil Engineering, Pontifical Catholic University of Rio de Janeiro
Rua Marquês de São Vicente, 225, Gávea
Rio de Janeiro, RJ - Brasil - 22453-900
diord@ufpa.br

Abstract

This paper presents the results of a dynamic analysis performed on a steel overhead transmission line tower crossing Guamá River, located in Eastern Amazon Region, Brazil.
Lying in the left margin of the river, the steel latticed tower measures, approximately, 75 meters of height and supports six conductors which cross two spans of about 800 meters long. To investigate the dynamic behaviour of this structure, an experimental modal analysis was performed by using a set of low-frequency piezo-resistive (ICP) accelerometers suitably installed in two cross sections along the height of the tower.
Since it is very difficult to measure the magnitude of wind excitation, an output-only modal analysis method based on stochastic subspace identification (SSI) was employed, instead of conventional input-output modal analysis identification methods, to extract the eigenfrequencies, and corresponding damping ratios and mode shapes of the tower. This method showed to be efficient for identification of such structure as it was capable detect two close-spaced modes around 1.8 Hz. A further comparison between the experimental and the theoretical dynamic behaviour showed that, due to the simplifications in modelling the cables, they are in good agreement only for the first three from a total of eight analysed modes of vibrations.

Eds.: Av. Augusto Corrêa, 01, Campus do Guamá, Centro Tecnológico, Sala 236, Belém, PA, Brasil.

1 INTRODUCTION

The interest on the structural behaviour of the electrical transmission line system has been increased in the latest decades in Brazil, since their infrastructure has become older. In Amazon region, for instance, where these infrastructures overcome great obstacles such as the rainforest and large rives crossings, cases of collapse of the latticed towers have been occurred due to wind gusts. One of the most notorious of these cases was the collapse of one of the towers placed at Tapajós River crossing, located in the western of the state of Pará, Brazil. A 160 meters high tower became collapsed after the exceptionally violent wind gusts, leading to interruption of the energy supply.

2 DESCRIPTION OF THE STRUCTURE

Measuring 75 meters high, the free stand type tower is composed by steel bars with “L” shaped sections and lies at the left margin of Guamá River. Together with two neighbour towers, one of them standing in the middle and the other in the right margin of the river, that left margin tower supports six conductor cables that cross two spans of about 800 meters, figure 1. Other studies on the ambient induced vibration of latticed structures in Amazon region are found, for instance, in [6].

Figure 1 – Overhead electrical transmission line tower at the left margin of the Guamá River.

3 EXPERIMENTAL MODAL ANALYSIS

With six low frequencies accelerometers suitably installed in the structure as the setup showed in figure 2, an experimental modal analysis was performed. The time histories signals showed in figure 3 were processed in SISMEC (System for Output-only Modal Analysis of Civil Engineering Structures), which is a GUI toolbox developed in MATLAB platform for an output-only experimental modal analysis. The theory behind this application can be found in [1], [2] and [3], and was implemented in a M. Sc. project.

3.1 Stochastic state-space model

Since input information is not available in an output-only vibration experiment, it is not possible to distinguish between the input u_k and noise, though these components are substituted by the stochastic components w_k and v_k, yielding the following stochastic state-space model in discrete time:

$$\begin{aligned} x_{k+1} &= A_d x_k + w_k \\ y_k &= C_d x_k + v_k \end{aligned} \tag{1}$$

For system identification purposes, a suitable way to gather the output data signals of a vibration experiment is to assemble a block Hankel matrix, scaled by $\sqrt{N}$, with $2i$ rows and N columns.

$$H^{ref} = \frac{1}{\sqrt{N}} \left[\begin{array}{cccc} y_0^{ref} & y_1^{ref} & \cdots & y_{N-1}^{ref} \\ y_1^{ref} & y_2^{ref} & \cdots & y_N^{ref} \\ \cdots & \cdots & \cdots & \cdots \\ y_{i-1}^{ref} & y_i^{ref} & \cdots & y_{i+N-2}^{ref} \\ \hline y_i & y_{i+1} & \cdots & y_{i+N-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+N} \\ \cdots & \cdots & \cdots & \cdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+N-2} \end{array} \right] = \left(\frac{Y_p^{ref}}{Y_f} \right) \begin{array}{l} \updownarrow ri \\ \updownarrow li \end{array} \quad \frac{"past"}{"future"} \tag{2}$$

This matrix is divided into two blocks rows Y_p^{ref} and Y_f. The first *ri* rows gathers past reference output data and second *li* rows holds the future output data. The index *r* refers to the number of reference sensors, and *l* denotes the number of all sensors used in the vibration experiment. An extended absorbability matrix is defined as:

$$O_i = \left\{ C^T \quad (CA)^T \quad (CA^2)^T \quad (CA^{i-1})^T \right\}^T \tag{3}$$

This is a *li x n* matrix where *A* is the state matrix and *C* is the output matrix. The index *n* denotes the model order, and the pair {*A, C*} is assumed to be observable, which means that the modal parameters are observed in the output data of a vibration experiment [1].

3.2 Data-driven stochastic subspace identification (SSI-DATA) method

The experimental modal analysis of the latticed structure crossing Guamá River was performed by using the SSI-DATA method. The advantages of this method lie upon the innovations proposed by [1] and [2], where projection of the future outputs into the row space of the past outputs is the key step. In this section, only a brief description of the SSI-DATA method is showed. Further details are found, for instance, in [7].

3.2.1 Kalman Filter States

Through the definitions of Kalman filter, a state input vector $\hat{x}_{k+1}$ is estimated from the observations of the output data up to time *k* [1]. More details about Kalman states can be found, for instance, in [1] and [2]. These estimates are gathered to assemble the Kalman filter estates sequence $\hat{X}_i$.

$$\hat{X}_i \equiv (\hat{x}_i \quad \hat{x}_{i+1} \quad \cdots \quad \hat{x}_{i+j-1}) \tag{4}$$

3.2.2 Implementation

The implementation of SSI-DATA method begins by factorizing the Hankel matrix (2) into a QR product.

$$H^T = \left(\frac{Y_p^{ref}}{Y_f} \right)^T = RQ^T; \qquad H = QR^T \tag{5}$$

Where Q is an orthonormal matrix, and R is a lower triangular matrix.

$$H = \begin{array}{cc} & \begin{array}{cccc} ri & r & l-r & l(i-1) \\ \leftrightarrow & \leftrightarrow & \leftrightarrow & \leftrightarrow \end{array} \\ \begin{array}{cc} ri & \updownarrow \\ r & \updownarrow \\ l-r & \updownarrow \\ l(i-1) & \updownarrow \end{array} & \begin{bmatrix} R_{11} & 0 & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \end{array} \begin{array}{c} j \to \infty \\ \leftrightarrow \\ \begin{Bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \\ Q_4^T \end{Bmatrix} \end{array} \begin{array}{cc} \updownarrow & ri \\ \updownarrow & r \\ \updownarrow & l-r \\ \updownarrow & l(i-1) \end{array} \tag{6}$$

The introduction of projection of the row space of future outputs onto the row space of the past reference outputs in equation (7) is the key step towards SSI-DATA method.

$$\mathfrak{P}_i \equiv Y_f / Y_p^{ref} \equiv Y_f Y_p^{ref^T} (Y_p^{ref} Y_p^{ref^T})^{\dagger} Y_p^{ref} \tag{7}$$

The main theorem of stochastic subspace identification [3] shows that this projection can be factorized into the observability matrix (3) and the Kalman filter state sequence (4).

$$\mathfrak{P}_i^{ref} = O_i \hat{X}_i = \begin{pmatrix} R_{21} \\ R_{31} \\ R_{41} \end{pmatrix} Q_1^T \tag{8}$$

These factors can also be determined through the employment of SVD on the projection.

$$\mathfrak{P}_i^{ref} = U_1 S_1 V_1^T, \quad O_i = U_1 S_1^{1/2}, \quad \hat{X}_i = O_i^{\dagger}\, \mathfrak{P}_i^{ref} \tag{9}$$

The definition of another projection from the shifted past and future outputs is needed for calculation of the system matrices A and C.

$$\mathfrak{P}_{i-1}^{ref} \equiv Y_f^- / Y_p^{ref+} = (R_{41} \quad R_{42}) \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix} \tag{10}$$

Similar to equation (8), this new projection can be factorized as:

$$\mathfrak{P}_{i-1}^{ref} = O_{i-1} \hat{X}_{i+1} \tag{11}$$

Where O_{i-1} is determined by deleting the last l rows of O_i. The shifted Kalman filter state sequence $\hat{X}_{i+1}$ is computed as:

$$\hat{X}_{i+1} = O_{i-1}^{\dagger} \mathfrak{P}_{i-1}^{ref} \tag{12}$$

Where $(\bullet)^{\dagger}$ denotes de Penrose inverse of a matrix.

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} \hat{X}_i + \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix} \tag{13}$$

Where $Y_{i|i}$ matrix is calculated by extracting the intermediate l block rows from the Hankel matrix (6).

$$Y_{i|i} = \begin{pmatrix} R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{pmatrix} \tag{14}$$

As ρ_w and ρ_v are the residuals uncorrelated with $\hat{X}_i$, the system matrices A and C are calculated by solving the over determined system of equations (15) in a least-squares sense.

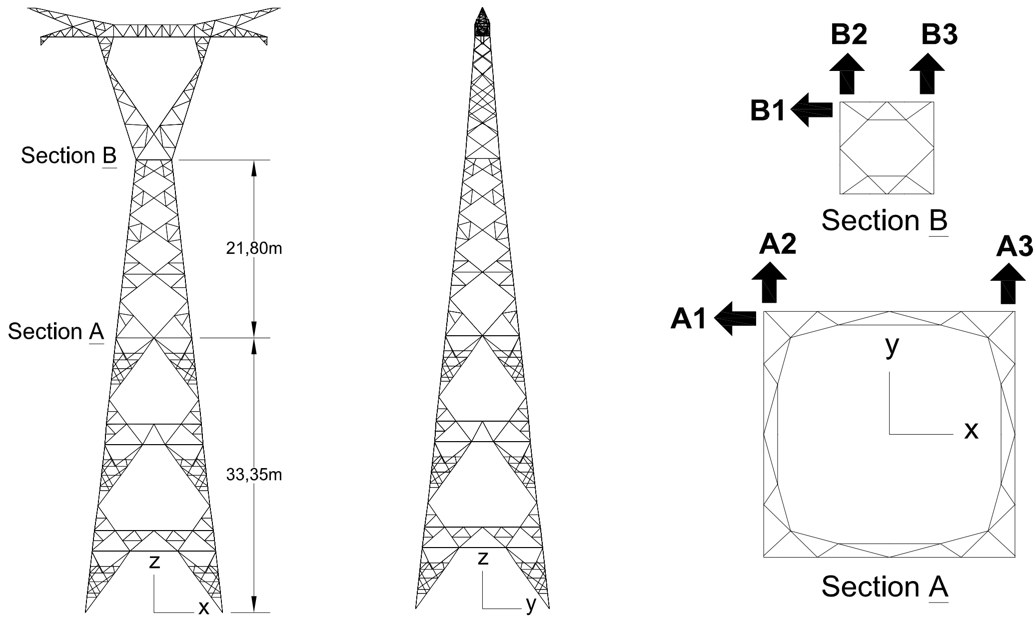
$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} \hat{X}_i^{\dagger} \tag{15}$$

Finally, the computation of the discrete poles Λ_d and the corresponding observed mode shapes is accomplished through the eigenvalue decomposition of the matrix A.

$$A = \Psi \Lambda_d \Psi; \quad V = C\Psi \tag{16}$$

3.3 Data Acquisition

In the vibration experiment of the transmission line tower six piezo-resistive accelerometers were used to measure acceleration. The sensors were suitably installed in two cross sections along the height of the tower. These sections are denoted as **A** and **B** as showed in figure 2.



(a) Elevation (front) (b) Elevation (side) (c) Cross sections of the tower

Figure 2 – Location of the accelerometers along the height of the tower

The structure was continuously monitored for five hours during a windy day at a sampling rate of 100 Hz. The collected data was filtered out by a digital Chebyshev type low-pass filter [7] with a cutoff frequency of 5 Hz, and re-sampled at a lower frequency of 12.5 Hz. A typical filtered time data signal and its corresponding spectrum are showed in figure 3.

A stability diagram [4] was constructed by ranging the model order from 2 to 80, and using the sensors B1 and B2 as references. The natural frequencies, damping ratios and corresponding mode shapes were easily determined by moving the mouse cursor over the stable poles. An illustration of the identification procedure and a detail of two close-spaced modes around 1.8 Hz are showed in figure 4.

The mean values of eigenfrequencies and damping ratios, and their corresponding standard deviations are showed in Table 1. These values are estimated for each mode from a sample of 10 stable poles selected from the stability diagram.

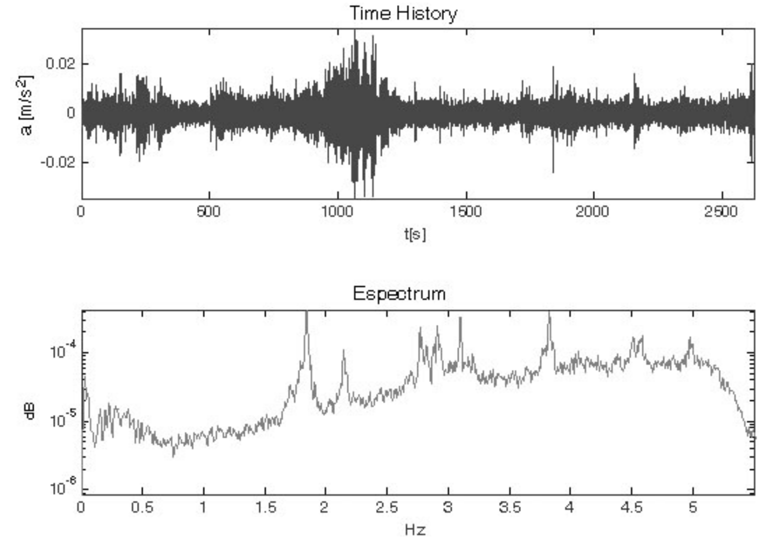


Figure 3 – Example of a typical measured signal and its corresponding spectrum

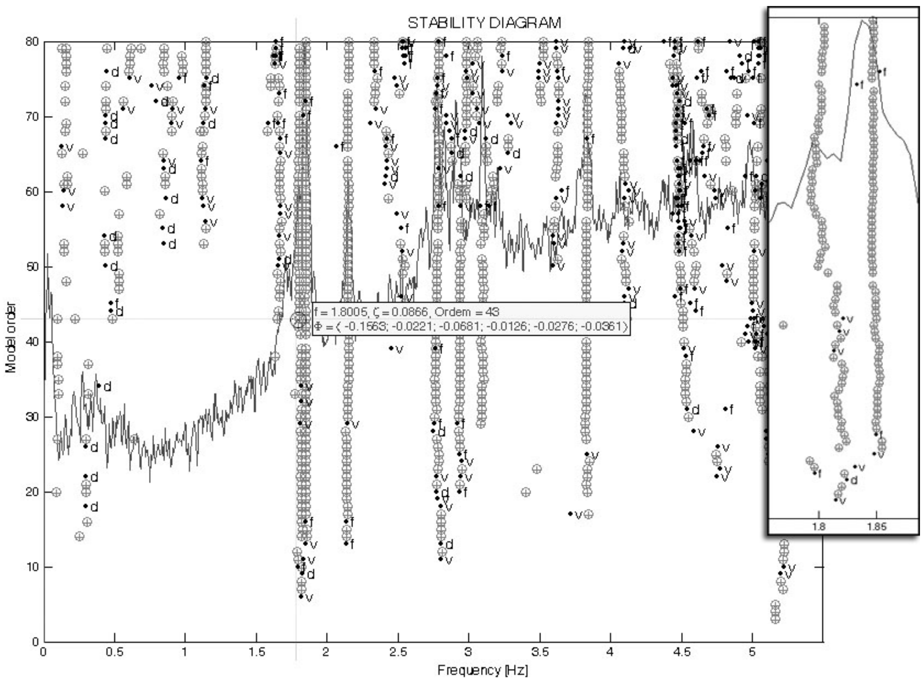


Figure 4 – Stabilization plot. The criterias are: 1% for frequencies, 2% for damping ratio, 1% for vectors (MAC). The used symbols are: ⊕ *- stable pole;* .v *- stable frequency and vector;* .d *– stable frequency and damping;* .f *– stable frequency.*

The estimated mean values of eigenfrequencies and damping raios, and their respective standard deviations for the first eight modes are showed table 1, as well as the theoretical eigenfrequencies from the FE analysis.

Table 1 – Mean values of the eigenfrequencies, damping ratios and standard deviations

Mode	SSI-DATA results				FE results
	Eigenfrequencies		Damping ratios		Eigenfrequencies
	$\bar{f}(Hz)$	$\sigma_f(Hz)$	$\bar{\zeta}(\%)$	$\sigma_\zeta(\%)$	
1	1,666	0,007	0,1801	1,9707	1,693
2	1,798	0,005	0,0820	0,1611	1,781
3	1,847	0,001	0,0435	0,0755	1,828
4	2,147	0,003	0,0741	0,2682	3,244
5	2,774	0,002	0,0685	0,0881	3,519
6	2,936	0,003	0,0743	0,1269	3,634
7	3,094	0,006	0,1223	0,3554	3,987
8	3,829	0,001	0,0335	0,0320	4,146

4 CONCLUSIONS AND FURTHER INVETIGATIONS

Although the experimental and FE modal parameters did not match for the last five modes, the structure showed to have similar dynamic behaviour in both analysis for the first three modes at which these parameters were in good agreement. An important aspect concerning these discrepancies is that only the estimated mass of the conductor cables was introduced in this FE model at their supporting joints, whereas the corresponding stiffness was considered in this primary model. A new FE model considering the stiffness and large displacements of the cables by using the co-rotational formulation [5] is under creation for a more accurate investigation.

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