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MULTIVARIABLE FREQUENCY-DOMAIN SYSTEM IDENTIFICATION ALGORITHMS FOR EXPERIMENTAL AND OPERATIONAL MODAL ANALYSIS

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Abstract
In this contribution state-of-the-art frequency-domain modal parameter estimators will be presented. In experimental modal analysis (EMA), mechanical systems with a few inputs and hundreds of outputs have to be identified. This requires adapted frequency-domain estimators designed to handle large amount of data in a reasonable amount of time. Next, attention will be paid to operational modal analysis, which is a complementary technique to traditional experimental modal analysis. In operational modal analysis unknown operational forces are present. It will be shown how the modal parameters can be estimated from output-only (OMA) as well as input/output (OMAX) measurements done at operating conditions.

INTRODUCTION

The majority of structures can be made to resonate, i.e. to vibrate with excessive oscillatory motion. Resonant vibration is mainly caused by an interaction between the inertial and elastic properties of the materials within a structure. To better understand any structural vibration problem, the resonant frequencies of a structure need to be identified and quantified. Today, modal analysis has become a widespread means of finding the modes of vibration of a machine or structure. In every development of a new or improved mechanical product, structural dynamics testing on product prototypes is used to assess its real dynamic behaviour. Starting from simple techniques for trouble shooting, it has evolved to a 'standard' approach in mechanical product development. Beginning from the modal model, design improvements can be predicted and the structure can be optimized. Based on the academic principles of system identification, experimental modal analysis helps the engineers to get more physical insight from the identified models. Continuously expanding its application base, modal analysis is today successfully applied in automotive engineering (engine, suspension, body-in-white, fully trimmed cars, ...), aircraft engineering (ground vibration test, landing gear, control surfaces, in-flight tests), spacecraft engineering (launchers, antennas, solid panels, satellites,...), industrial machinery (pumps, compressors, turbines, ...) and civil engineering (bridges, off-shore platforms, dams, ...).

Nevertheless, the current evolution in mechanical engineering towards the use of Computer Aided Design (CAD) like Finite Element Models (FEM) results in a changing role for testing [1], [2]. Today the optimization process in product development is under strong pressure because of the competitive market, increasing customers' demands and by consequence the design cycle becomes shorter in time. This results in an increasing use of simulations based on numerical models to reduce the number of prototypes and expensive experiments. Still, testing plays an important and evermore critical role, in every step of the development process for target setting, bench-marking and model updating. All this, together with the decreasing expertise of the users, since modal analysis have been transferred from the realm of the research experts to the product development workfloor [3], makes that the demands for modal parameter estimation algorithms still increase in terms of accuracy, speed, automation and physical interpretation.

In this paper attention will be paid to the design of such dedicated frequency-domain modal parameter estimators for experimental and operational modal analysis.

EXPERIMENTAL MODAL ANALYSIS

Experimental modal analysis (EMA) identifies a modal model from the measured forces applied to the test structure and the measured vibration responses. Typically the number of input forces, N_i, ranges from 1 to 5 while the measured output, N_o, can be larger that 1000. This results in the following modal model with dimension $N_o \times N_i$

$$\mathbf{H}(s) = \sum_{m=1}^{N_m} \frac{\mathbf{\Phi}_m \mathbf{L}_m^T}{s - \lambda_m} + \frac{\mathbf{\Phi}_m^* \mathbf{L}_m^{*T}}{s - \lambda_m^*} \tag{1}$$

with $\mathbf{\Phi}_m \in \mathbb{C}^{N_o}$ the mode shape vectors and $\mathbf{L}_m \in \mathbb{C}^{N_i}$ the modal participation factors [4, 5, 6]. Direct estimation of the modal parameters is not practical because the modal model is strongly nonlinear in the parameters. Therefore, matrix-fraction descriptions are usually preferred.

The Least-Squares Complex Frequency-domain (LSCF) approach

A scalar matrix-fraction description - better known as a common-denominator transfer function - will be used in this section. The frequency response function (FRF) between output o and input i will be modeled as

$$H_{oi}(\Omega_k) = \frac{B_{oi}(\Omega_k)}{A(\Omega_k)} \tag{2}$$

with $B_{oi}(\Omega_k) = \sum_{j=0}^{n} b_{oij}\Omega_k^j$ the numerator polynomial between response o and input i and $A(\Omega_k) = \sum_{j=0}^{n} a_j\Omega_k^j$ the common-denominator polynomial. Several choices are possible for the polynomial basis functions Ω_k. A discrete-time model, $\Omega_k = \exp(i\omega_k T_s)$, will be used (with T_s the sampling period).

Frequency response functions (FRFs) are commonly used as primary data in modal analysis. Most often, modal tests take place in laboratory conditions, and so, the applied forces can be measured together with the response of the structure (e.g., accelerations). Starting from the measured FRFs, $\hat{H}_{oi}(\omega_k)$ (with $o = 1, \ldots, N_o$, $i = 1, \ldots, N_i$ and $k = 1, \ldots, N_f$), estimates of the transfer-function coefficients can be obtained by minimizing the following least-squares cost function

$$\ell = \sum_{o,i}\sum_{k} |E_{oi}(\omega_k)|^2 \tag{3}$$

with

$$E_{oi}(\omega_k) = A(\Omega_k)\hat{H}_{oi}(\omega_k) - B_{oi}(\Omega_k) \tag{4}$$

For generality, a frequency-dependent weighting function, $W_{oi}(\omega_k)$, will be added [7]

$$E_{oi}(\omega_k) = \frac{A(\Omega_k)\hat{H}_{oi}(\omega_k) - B_{oi}(\Omega_k)}{W_{oi}(\omega_k)} \tag{5}$$

The Jacobian matrix is defined as $\mathbf{J} = \partial\mathbf{E}/\partial\mathbf{\Theta}$ with $\mathbf{E} = (\mathbf{E}_{11}^T, \mathbf{E}_{12}^T, \ldots, \mathbf{E}_{N_oN_i}^T)^T$ and $\mathbf{E}_{oi} = (E_{oi}(\omega_1), \ldots, E_{oi}(\omega_{N_f}))^T$ and with parameter vector $\mathbf{\Theta} = (\mathbf{b}_{11}^T, \mathbf{b}_{12}^T, \ldots, \mathbf{b}_{N_0N_i}^T, \mathbf{a}^T)^T$. The dimension of the Jacobian matrix is $N_oN_iN_f \times N$ with $N = (N_oN_i+1)(n+1)$. As $N_oN_iN_f$ can be quite large in modal analysis applications, the normal equations, $\mathbf{N} = \mathbf{J}^H\mathbf{J}$, are implicitly computed to reduce the matrix dimensions. The dimension of the square matrix $\mathbf{N}$ equals $N = (N_oN_i + 1)(n + 1)$.

It can be proven that cost function (3) can be written as

$$\ell = \sum_{o,i} \left\{ \begin{array}{c} \mathbf{b}_{oi} \\ \mathbf{a} \end{array} \right\}^H \left[\begin{array}{cc} \mathbf{R}_{oi} & \mathbf{S}_{oi} \\ \mathbf{S}_{oi}^H & \mathbf{T}_{oi} \end{array} \right] \left\{ \begin{array}{c} \mathbf{b}_{oi} \\ \mathbf{a} \end{array} \right\} \tag{6}$$

with

$$\mathbf{R}_{oi} = \sum_k \frac{\mathbf{Z}_k^H\mathbf{Z}_k}{W_{oi}^2(\omega_k)}, \quad \mathbf{S}_{oi} = -\sum_k \frac{\mathbf{Z}_k^H\mathbf{Z}_k\hat{H}_{oi}(\omega_k)}{W_{oi}^2(\omega_k)}, \quad \mathbf{T}_{oi} = \sum_k \frac{\mathbf{Z}_k^H\mathbf{Z}_k|\hat{H}_{oi}(\omega_k)|^2}{W_{oi}^2(\omega_k)} \tag{7}$$

and $\mathbf{Z}_k = [1, \Omega_k, \Omega_k^2, \ldots, \Omega_k^n]$. In the minimum of the cost function, the derivative of (6) with respect to $\mathbf{b}_{oi}$ should be zero, i.e.,

$$\frac{\partial\ell}{\partial\mathbf{b}_{oi}} = 2(\mathbf{R}_{oi}\mathbf{b}_{oi} + \mathbf{S}_{oi}\mathbf{a}) = 0 \tag{8}$$

Elimination of $\mathbf{b}_{oi}$ from (8) and substitution in (6) yields $\ell = \mathbf{a}^H\mathbf{D}\mathbf{a}$ with $\mathbf{D}$ an $(n + 1) \times (n + 1)$ matrix

$$\mathbf{D} = \sum_{o,i} \mathbf{T}_{oi} - \mathbf{S}_{oi}^H\mathbf{R}_{oi}^{-1}\mathbf{S}_{oi} \tag{9}$$

Assuming that the last coefficient of $\mathbf{\Theta}$ equals 1 (i.e. $a_n = 1$) yields the following least-squares estimates

$$\hat{\mathbf{a}} = \left\{ \begin{array}{c} -(\mathbf{D}(1:n,1:n))^{-1}\mathbf{D}(1:n,n+1) \\ 1 \end{array} \right\} \tag{10}$$

and $\hat{\mathbf{b}}_{oi} = -\mathbf{R}_{oi}^{-1}\mathbf{S}_{oi}\hat{\mathbf{a}}$.

The poles, λ_m in (1), are obtained by computing the roots of the estimated denominator polynomial, with coefficients $\hat{\mathbf{a}}$. The mode shape vectors, $\mathbf{\Phi}_m$, and the modal participation factors, $\mathbf{L}_m$, can be obtained from the estimated numerator coefficients, $\hat{\mathbf{b}}_{oi}$, or form the residue matrices, $\mathbf{R}_m \in \mathbb{C}^{N_o \times N_i}$, occuring in

$$\mathbf{H}(s) = \sum_{m=1}^{N_m} \frac{\mathbf{R}_m}{s-\lambda_m} + \frac{\mathbf{R}_m^*}{s-\lambda_m^*} \tag{11}$$

These resudue matrices can be estimated in least-squares sense once the poles are known. Next, the (rank one) residue matrices are decomposed into $\mathbf{R}_m = \mathbf{\Phi}_m\mathbf{L}_m^T$ by means of a singular value decomposion.

Remarks

- The dimensions of the reduced matrix (9) is much smaller than the original Jacobian matrix. Moreover, when a discrete-time model is used (i.e., $\Omega_k = \exp(i\omega_k T_s)$ with T_s the sampling period) the matrices $\mathbf{R}_{oi}$, $\mathbf{S}_{oi}$, and $\mathbf{T}_{oi}$ becomes Toeplitz. This means that only the first rows (and columns) of the matrices have to be computed. Further, when the frequencies ω_k, $k = 1,\ldots,N_f$, are uniformly distributed, a discrete Fourier transform can be used to compute these elements in a time efficient way [8, 9, 10].

- The above equations implicitly assume that the polynomial coefficients are complex valued. Hence, when a common-denominator model is used, the order of the denominator polynomial n equals the number of estimated modes ($n = N_m$). For real-valued coefficients the order n has to be doubled ($n = 2N_m$). An advantage of complex coefficients is that n is twice smaller that for real coefficients, and so, a better numerical conditioning of the normal equations is obtained. It can be shown that the least-squares normal equations for real-valued coefficients are obtain by replacing the complex-valued $\mathbf{R}$, $\mathbf{S}$, and $\mathbf{T}$ matrices by they real part, i.e., $\mathrm{Re}(\mathbf{R})$, $\mathrm{Re}(\mathbf{S})$, and $\mathrm{Re}(\mathbf{T})$.

The polyreference LSCF approach

Recently, a new non-iterative frequency-domain parameter estimation method was proposed. This so-called polyreference least-squares complex frequency domain method (also known as the PolyMAX estimator) can be implemented in a very similar way as the industry standard polyreference (time-domain) least-squares complex exponential method: in a first step a stabilisation diagram is constructed containing frequency, damping and participation information. Next, the mode shapes are found in a second least-squares step, based on the user selection of stable poles. The polyreference implementation is based on the (weighted) least-squares approach given in the previous section, but uses a right matrix-fraction description instead of a common denominator transfer function model. The poles, λ_m, together with the modal participation factors, $\mathbf{L}_m$, are obtained by reformulation $\det(\mathbf{A}(\Omega_k)) = 0$ into a generalized eigenvalue problem, resulting in nN_i eigenvalues (poles) and corresponding left eigenvectors (modal participation factors).

One of the specific advantages of this technique lies in the very stable identification of the system poles and participation factors as a function of the specified system order, leading to easy-to-interpret stabilisation diagrams [11]. This implies a potential for automating the method and to apply it to ‘difficult’ estimation cases such as high-order and/or highly damped systems with large modal overlap [12]. The mode shape vectors can be obtained from the $\mathbf{b}_j$ coefficients or are found in a second least-squares step, based on the user selected stable poles and corresponding modal participation factors. Indeed, when the poles and modal participation factors are known, the remaining parameters, i.e., the mode shape vectors $\mathbf{\Phi}_m$, appears linearly in (1).

OPERATIONAL MODAL ANALYSIS

Cases exist where it is rather difficult to apply an artificial force and where one has to rely upon available ambient excitation sources. In such cases, it is practically impossible to measure this ambient excitation, and consequently, the responses are the only signals that can be measured. Traditionally, one assumes that the outputs are the results of a stochastic process with white noise sources as inputs. The need to perform output-only modal analysis probably emerged first in civil engineering, where it is very difficult and expensive to excite constructions like bridges and buildings by an artificial excitation that exceeds the natural vibrations due to traffic and wind. Also in mechanical engineering, operational modal analysis proved to be useful: for instance to obtain modal parameters of a car during road testing or an airplane in flight conditions. Often operational test conditions differ from laboratory test conditions, because during the in-operation tests the real loading conditions on the structure are present (suspension pre-strains of a car on the road, aero-elastic interactions during flight conditions, ...). For example, an airplane during flight conditions has a (totally) different dynamical behaviour than an airplane tested in laboratory conditions. Another benefit of output-only modal analysis is the fact that a linear model of the system is obtained around the real working point of operation.

Output-only data

Starting from the input/output relation

$$\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega) \tag{12}$$

and assuming the input forces to be a stationary stochastic process, the power spectral matrix of the outputs, $\mathbf{S_Y}(\omega) \in \mathbb{C}^{N_o \times N_o}$, are given by

$$\mathbf{S_Y}(\omega) = \mathbf{H}(\omega)\mathbf{S_F}(\omega)\mathbf{H}^H(\omega) \tag{13}$$

with

$$\mathbf{H}(\omega) = \sum_{m=1}^{N_m} \frac{\mathbf{\Phi}_m \mathbf{L}_m^T}{i\omega - \lambda_m} + \frac{\mathbf{\Phi}_m^* \mathbf{L}_m^{*T}}{i\omega - \lambda_m^*} \tag{14}$$

Under the assumption that the operational forces are white input noise sources (i.e., $\mathbf{S_F}$ is a constant frequency-independent matrix), $\mathbf{S_Y}(\omega)$ can be modally decomposed as [13, 14]

$$\mathbf{S_Y}(\omega) = \sum_{m=1}^{N_m} \frac{\mathbf{\Phi}_m \mathbf{K}_m^T}{i\omega - \lambda_m} + \frac{\mathbf{\Phi}_m^* \mathbf{K}_m^{*T}}{i\omega - \lambda_m^*} + \frac{\mathbf{K}_m \mathbf{\Phi}_m^T}{-i\omega - \lambda_m} + \frac{\mathbf{K}_m^* \mathbf{\Phi}_m^{*T}}{-i\omega - \lambda_m^*} \tag{15}$$

where $\mathbf{\Phi}_m \in \mathbb{C}^{N_m}$ is the mode shape vector of mode m, and $\mathbf{K}_m$ the operational reference vector. This operational reference vector is a function of the modal parameters and the (frequency-independent) power spectral matrix of the unknown random input forces. As the input forces are unknown, it is impossible to recover the modal participation factors, $\mathbf{L}_m$, from $\mathbf{K}_m$. All other modal parameters are still identifyable.

One notice that the modal decomposition (15) contains both stable (λ_m) and unstable poles ($-\lambda_m$). It is however possible to get rid of the unstable poles by considering the so-called 'half' (or positive) power spectra, $\mathbf{S_Y^+}(\omega)$, which are obtained by Fourier transforming the positive lags of the correlation functions.

$$\mathbf{S_Y^+}(\omega) = \sum_{m=1}^{N_m} \frac{\mathbf{\Phi}_m \mathbf{K}_m^T}{i\omega - \lambda_m} + \frac{\mathbf{\Phi}_m^* \mathbf{K}_m^{*T}}{i\omega - \lambda_m^*} \tag{16}$$

Consequently, all FRF-based estimators can be used to obtain the poles and mode shape vectors by simply replacing the FRF measurements by the half power spectra.

Input/output data: The OMAX approach

OMAX stands for Operational Modal Analysis in presence of eXogenous inputs. In some in-operational testing applications, exciters are used to inject more energy in the system. This is for instance done during flight flutter tests, where artificial forces are applied on the wings of the airplane using special equipment. In that case, input signals are available and it is again possible to use classical EMA identification techniques to estimate the modal parameters from the input/output (or FRF) measurements. However, by doing so, the effect on the natural forces will be treated as ambient 'noise'. Traditional EMA techniques will remove this 'noise' by averaging the measurements. This is in contradiction with the output-only approach where the modal parameters are estimated using response data due to the ambient excitation only. Clearly, the ambient noise is not just 'noise' but it contains useful information about the system. To make an optimal use of the operational data, both measured (artificial) and unmeasured (natural) forces should be taken into account. By doing so, all the available information in the measured data can be optimally used. One can write that output o consists of two contributions: a forced part due to the applied (and measured) forces $F_i(\omega_k)$ and a second unknown part due to the (unknown) ambient excitation $E_o(\omega_k)$; a third term, $\frac{T_o(\Omega_k)}{A(\Omega_k)}$, models all transients effects,

$$Y_o(\omega_k) = \sum_i \frac{B_{oi}(\Omega_k)}{A(\Omega_k)} F_i(\omega_k) + \frac{C_o(\Omega_k)}{A(\Omega_k)} E_o(\omega_k) + \frac{T_o(\Omega_k)}{A(\Omega_k)} \tag{17}$$

This results in the following cost function [15]

$$\ell = \sum_{o,k} \frac{|A(\Omega_k)Y_o(\omega_k) - \sum_i B_{oi}(\Omega_k)F_i(\omega_k) - T_o(\Omega_k)|^2}{|C_o(\Omega_k)|^2} \tag{18}$$

In [10] a combined deterministic-stochastic frequency domain subspace system identification algorithm is proposed. This alternative 'OMAX' approach can also take both measured (artificial) and unmeasured (natural) forces into account. OMAX estimators are able to identify modes that are weakly excited by the applied forces, $F_i(\omega_k)$, as long as they are excited by the natural forces.

CONCLUSIONS

In this contribution, a multivariable frequency-domain modal estimator, based on a common denominator transfer function model, has been given. Next, a so-called 'polyreference' frequency domain least squares estimator has been proposed, which — in general — yields very clean stabilisation diagrams, easing dramatically the problem of selecting the structural system modes. Finally, attention has been paid to operational modal analysis. During operational tests unknown ambient (stochastic) excitations are present. To model systems in operational conditions, stochastic (OMA) and combined deterministic-stochastic frequency domain estimators (OMAX) were proposed.

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