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ON THE DESIGN OF HELMHOLTZ RESONATORS FOR DAMPING PULSATIONS IN GAS TURBINE COMBUSTION CHAMBERS

Valter Bellucci*, **Dariusz Nowak, and Bruno Schuermans**
ALSTOM

ALSTOM (Switzerland) Ltd, Brown-Boveri-Strasse 7, CH-5400 Baden, Switzerland
valter.bellucci@power.alstom.com (e-mail address of lead author)

Abstract

In modern gas turbines operating with premixed combustion flames, the suppression of pressure pulsations is an important task related to the quality of the combustion process and to the structural integrity of engines. In this work, the use of Helmholtz resonators for damping thermoacoustic pulsations occurring in combustion systems is discussed. The dampers are modeled according the harmonic oscillator model and a detailed analysis of the physical mechanism responsible for sound attenuation is performed. The theory we present links the suppression of acoustic pressure to damper number, geometry and bias flow. The theory is validated by using impedance tube experiments and engine tests performed on an ALSTOM GT11N2 heavy-duty gas turbine.

INTRODUCTION

In modern gas turbines operating with premixed combustion flames, pressure pulsations may occur when the resonance frequencies of the system are excited by heat release fluctuations produced by flow fluctuations independent of the acoustic field (combustion noise). Heat release fluctuations may be also generated by acoustic oscillations in the premixed stream. The feedback mechanism inherent in such process may lead to combustion instabilities, the amplitude of pulsations being limited in this case by nonlinearities [1]. The suppression of these thermoacoustic pulsations is an important task that may be addressed by means of passive control methods like the use of Helmholtz resonators [2], [3], [4]. As shown in Fig. 1, a classical Helmholtz resonator consists of a volume with a neck through which the fluid inside the resonator communicates with an external medium. When a Helmholtz resonator is applied to

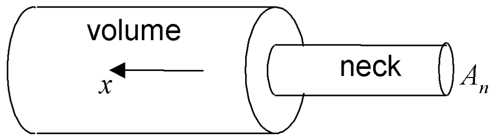

Figure 1: *Schematic of Helmholtz resonator.*

an enclosure, in correspondence of the neck mouth a frequency dependent boundary is realized. This boundary is acoustically described by the neck mouth impedance Z_n, i.e. the ratio in the frequency domain between acoustic pressure and acoustic velocity normal to the neck mouth. An important characteristic of the resonator is its (circular) resonance frequency ω_{res}, i.e. the frequency that corresponds to $Im(Z_n) = 0$. When a Helmholtz resonator with neck mouth area A_n is applied to an enclosure, on the surface A_n the infinite impedance boundary (corresponding to zero acoustic velocity) is replaced by the resonator mouth impedance Z_n. The effect of the damper on the j-resonance frequency ω_j of the enclosure without resonator has been studied by several authors [5], [6], [7]. Generally, at the frequency ω_j amplitude reduction may occur when $\omega_{res} = \omega_j$. However, the analysis also shows that if the excitation is not confined to ω_j, the application of resonators may generate new amplitude maxima at frequencies close to ω_j. The aim of this work is to investigate the effect of resonators applied to an enclosure like a gas turbine combustion chamber, where combustion noise is responsible for acoustic excitation within a large frequency range around ω_j. Therefore, first the pressure field in the enclosure is expressed by means of a series of acoustic modes. Then, the effect of the resonator impedance on the single term of the modal expansion is analyzed and a theory is derived to predict the sound suppression. Finally, the theory is validated using impedance tube experiments and engine test performed on an ALSTOM GT11N2 heavy-duty gas turbine.

ACOUSTIC THEORY

In an uniform reacting low-Mach number mean flow, the superimposed acoustic field is described by the non homogeneous Helmholtz equation and the linearized momentum equation that read respectively as [8]

$$\nabla^2 \widehat{p} + k_P^2 \, \widehat{p} = -i \, \omega \, \frac{\gamma - 1}{\overline{c}^2} \, \widehat{Q} \, , \qquad \widehat{\mathbf{u}} = \frac{i}{\omega \, \overline{\rho}} \, \nabla \, \widehat{p} \tag{1}$$

where $\widehat{p}(\omega, \mathbf{x})$ and $\widehat{\mathbf{u}}(\omega, \mathbf{x})$ are the frequency domain acoustic pressure and velocity respectively (being $\mathbf{x}$ the space vector), k_P is the complex wave number, $\omega = 2\pi f$ the circular frequency with f frequency, $\overline{\rho}$ and $\overline{c}$ the mean flow density and speed of sound respectively, γ the specific heat ratio and $\widehat{Q}$ the frequency domain volumetric heat release related to the combustion process. In general, $\widehat{Q}$ is the sum of two contributions: the combustion noise term independent from the acoustic field and the amplifier term that is a function of acoustic pressure and/or acoustic velocity. The boundary conditions are expressed as $\widehat{p}/ \; \widehat{\mathbf{u}} \cdot \mathbf{n} = Z$ on

∂V where $Z(\omega,\mathbf{x})$ is the acoustic impedance on the boundary ∂V and $\mathbf{n}$ the outward normal. Using the Green functions, the pressure field in the volume V may be expressed as [8, p. 554]

$$\widehat{p}(\mathbf{x}) = \sum_{j=0}^{\infty} \frac{i\,\omega\,\overline{\rho}\,\psi_j(\mathbf{x})}{V\Lambda_j(k_j^2-k_P^2)}\left[\frac{\gamma-1}{\overline{\rho}\,\overline{c}^2}\int_V \widehat{Q}(\mathbf{x})\psi_j(\mathbf{x})dV - \oint_{\partial V}\frac{\widehat{p}(\mathbf{x})}{Z(\mathbf{x})}\psi_j(\mathbf{x})d\partial V\right] \quad (2)$$

where $\psi_j(\mathbf{x})$ are the eigenfunctions defined as the non trivial solutions of the problem

$$\nabla^2\psi_j + k_j^2\,\psi_j = 0\,, \qquad \nabla\psi_j\cdot\mathbf{n} = 0 \quad \text{on} \quad \partial V \quad (3)$$

where both the eigenfrequencies $\omega_j = k_j\,\overline{c}$ and eigenmodes ψ_j are real and frequency independent. Moreover, the eigenmodes are orthogonal, i.e. $\int_V \psi_j\psi_m dV = V\Lambda_j\delta_{j,m}$ with $\Lambda_j = 1/V\int_V \psi_j^2 dV$. Usually, the term $(k_j^2-k_P^2)$ is expressed as $(\omega_j^2+i\xi_j\omega_j\omega-\omega^2)/\,\overline{c}^2$, being ξ_j the modal damping typically of order 0.01 [8].

In the following we consider an enclosure equipped with N_R equal resonators that are acoustically "compact", i.e. both the mode values and the resonator impedance are constant on the k-resonator mouth located at $\mathbf{x}_k$ (low frequency hypothesis). Moreover, we concentrate on the acoustic mode ψ_j with eigenfrequency ω_j belonging to the low frequency range, where the average separation between eigenfrequencies is much larger than the average modal bandwidth. We start the analysis assuming heat release fluctuations independent from the acoustic field (combustion noise). Thus, for frequencies close to ω_j the modal coupling may be assumed negligible and Eq. (2) is approximated by the relation

$$\widehat{p}(\omega,\mathbf{x}) \approx \frac{i\,\omega}{\omega^2 - i\,\xi_j\,\omega_j\,\omega - \omega_j^2}\left[B(\omega)\sum_{k=1}^{N_R}\psi_j(\mathbf{x}_k)\;\widehat{p}(\omega,\mathbf{x}_k) - C(\omega)\right]\psi_j(\mathbf{x}) \quad (4)$$

$$B(\omega) = \frac{\overline{\rho}_E\;\overline{c}_E^2\,A_n}{V_E\,\Lambda_j\,Z_n(\omega)}\,, \qquad C(\omega) = \frac{\gamma_E-1}{V_E\,\Lambda_j}\int_{V_E}\widehat{Q}(\omega,\mathbf{x})\,\psi_j(\mathbf{x})\,dV$$

where the suffix E refers to enclosure conditions and V_E is the volume of the enclosure. To express the resonator impedance Z_n, the harmonic oscillator model is used. It gives [5]

$$Z_n = i\,\frac{\overline{\rho}_R\;\overline{c}_R^2\,A_n}{\omega\,\omega_{res}^2\,V_R}\left(\omega^2 - i\,\frac{\omega_{res}}{q_R}\,\omega - \omega_{res}^2\right) \quad (5)$$

where the suffix R refers to resonator conditions, V_R is the resonator volume and q_R the resonator quality-factor defined as $q_R = \omega_{res}\,\overline{\rho}_R L'_n/R_n = \overline{\rho}_R\,\overline{c}_R^2 A_n/\omega_{res}V_R R_n$ with L'_n effective length of the damper neck and $R_n = Im(Z_n)$ resonator resistance. In combustion applications, a cooling flow must be maintained through the resonator in order to prevent overheating. When the acoustic velocity in the damper neck is much smaller than the mean flow velocity, the resonator resistance may be expressed as $R_n = \zeta\,\overline{\rho}_R\,\overline{u}_n$ where ζ is the pressure loss coefficient in the neck and $\overline{u}_n$ the average mean flow velocity in the neck [2]. Next, we introduce the quality-factor of the enclosure without resonators $q_E = \omega_j/\Delta\omega_j$, where $\Delta\omega_j$ is defined so that at $\omega_j \pm \Delta\omega_j/2$ the pressure amplitude is $1/\sqrt{2}$ times the peak amplitude at ω_j. Hence, using Eq. (4) with $Z_n = \infty$ and the assumption $\xi_j \ll 1$, one has

$q_E = 1/\xi_j$. Finally, by using Eqs. (4) and (5) with $\overline{\rho}_E \, \overline{c}_E^2 = \gamma_E \, \overline{p}_E \approx \overline{\rho}_R \, \overline{c}_R^2 = \gamma_R \, \overline{p}_R$, one has

$$\frac{\widehat{p}}{\widehat{p}_0}(\omega) = \frac{i/\varepsilon q_E}{(\widetilde{\omega}_{res} - i/\varepsilon q_R)^{-1} \, \omega_{res}/\omega_j \, - (\widetilde{\omega} - i/\varepsilon q_E)} \tag{6}$$

where we have introduced

$$\widetilde{\omega} = \frac{\omega^2 - \omega_j^2}{\omega \, \omega_j \, \varepsilon}, \qquad \widetilde{\omega}_{res} = \frac{\omega^2 - \omega_{res}^2}{\omega \, \omega_{res} \, \varepsilon}, \qquad \varepsilon = \sqrt{\frac{V_R \sum_{k=1}^{N_R} \psi_j^2(\mathbf{x}_k)}{V_E \, \Lambda_j}} \tag{7}$$

$$\widehat{p}_0(\mathbf{x}) = -C(\omega_j)\psi_j(\mathbf{x}) = \frac{\psi_j(\mathbf{x})}{V_E \Lambda_j \xi_j \omega_j}(\gamma_E - 1) \int_{V_E} \widehat{Q}(\omega, \mathbf{x}) \, \psi_j(\mathbf{x}) \, dV \tag{8}$$

In the following we assume white noise, i.e. $\widehat{Q}$ is frequency independent. Then, $\widehat{p}_0$ is the pressure at $\omega = \omega_j$ in the enclosure without resonators. When $1/q_R \to 0$ (slightly damped resonator), the pressure reduction $|\,\widehat{p}/\,\widehat{p}_0|$ at $\omega = \omega_j$ is maximized provided that $\omega_{res} = \omega_j$. In this case Eq. (6) reads as $[\,\widehat{p}/\,\widehat{p}_0]_{\omega=\omega_{res}=\omega_j} = 1/(1 + \varepsilon^2 \, q_R \, q_E)$, showing that when the excitation is confined to ω_j, the maximum pressure reduction is obtained by maximizing ε and q_R, i.e. by locating large volume lightly damped resonators close to mode antinodes.

When considering frequencies close to ω_j, Eq. (6) reads as

$$\left[\frac{\widehat{p}}{\widehat{p}_0}(\omega)\right]_{\omega_{res}=\omega_j} = \frac{i/\varepsilon q_E}{(\widetilde{\omega} - i/\varepsilon q_R)^{-1} - (\widetilde{\omega} - i/\varepsilon q_E)} \tag{9}$$

Eq. (9) indicates that the pressure reduction is a function of the three nondimensional numbers $\widetilde{\omega}$, $(\varepsilon \, q_R)$ and $(\varepsilon \, q_E)$. Fig. 2 reports the H_∞ norm of the pressure ratio $[\,\widehat{p}/\,\widehat{p}_0]_{\omega_{res}=\omega_j}$ as a function of $1/(\varepsilon q_R)$ and $1/(\varepsilon q_E)$ [1]. Fig. 2 indicates that the pressure reduction is larger when $1/(\varepsilon q_E)$ is lower, that means the parameter ε should be always maximized. Furthermore, for a given value of (εq_E) the curve $(\varepsilon q_R)_\infty$ also reported in Fig. 2 provides the maximum pressure reduction. The $(\varepsilon q_R)_\infty$ curve is well interpolated by the fit

$$(\varepsilon q_R)_\infty = \frac{1}{1.2208 + 0.1271/(\varepsilon q_E)} \tag{10}$$

The criterion (10) indicates that when maximizing the pressure reduction with white noise excitation, besides ω_j other frequencies may be excited [6], [7]. Fig. 3 illustrates typical frequency responses for a fixed value of (εq_E) when varying (εq_R). Fig. 3 shows that when $(\varepsilon q_R) > (\varepsilon q_R)_\infty$, the amplitude at ω_j is minimized and two pulsation peaks appear at $\omega_\pm$. On the contrary, when $(\varepsilon q_R) < (\varepsilon q_R)_\infty$ the pressure spectrum has a maximum at $\omega = \omega_j$. In both cases, peak amplitudes are lower than in the case without resonators. The maximum pressure reduction is achieved with $(\varepsilon q_R)_\infty$.

In case of heat release fluctuations dependent on the acoustic field, a negative flame damping may be introduced to account for the amplification behavior of the flame [4]. Therefore, the global damping ξ_j is the sum of the flame damping and the internal positive damping representing the dissipative processes. When the total damping is negative, the mode

[1] $||\,\widehat{p}/\,\widehat{p}_0||_\infty = \max\{|(\widehat{p}/\,\widehat{p}_0)|\}$ for $\widetilde{\omega} \in [-\infty, \infty]$.

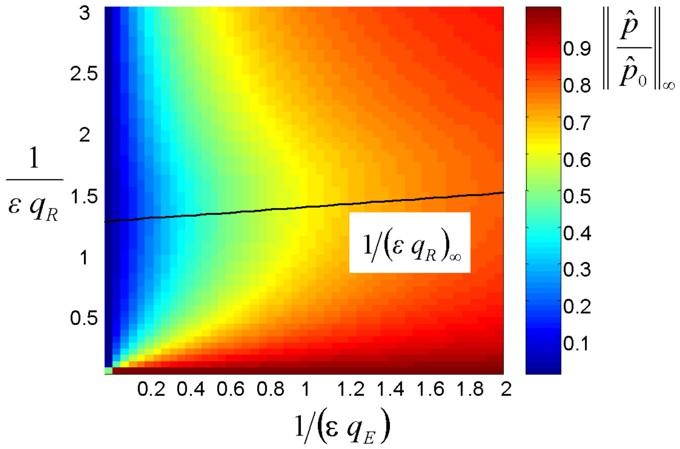

Figure 2: *H_∞-norm of pressure ratio $\widehat{p}/\widehat{p}_0$ with $\omega_{res} = \omega_j$.*

becomes unstable. The poles of the combustor equipped with dampers $\omega_\pm + i\nu_\pm$ are obtained by setting to zero the denominator of Eq. (6). By assuming small damping, one has $\omega_\pm \approx \omega_j(1 \pm \varepsilon/2)$ [6]. The system growth rates $\nu_\pm$ are then given by $\nu_\pm \approx \omega_j(\xi_j + 1/q_R)/4$ where the approximations $\nu_\pm \ll \omega_j$ and $\varepsilon \ll 1$ have been used. Hence, when $1/q_R$ is sufficiently large, the application of the resonators may force the sign of the modal damping $\xi_{j\pm}$ to be always positive, i.e. the system is unconditionally stable.

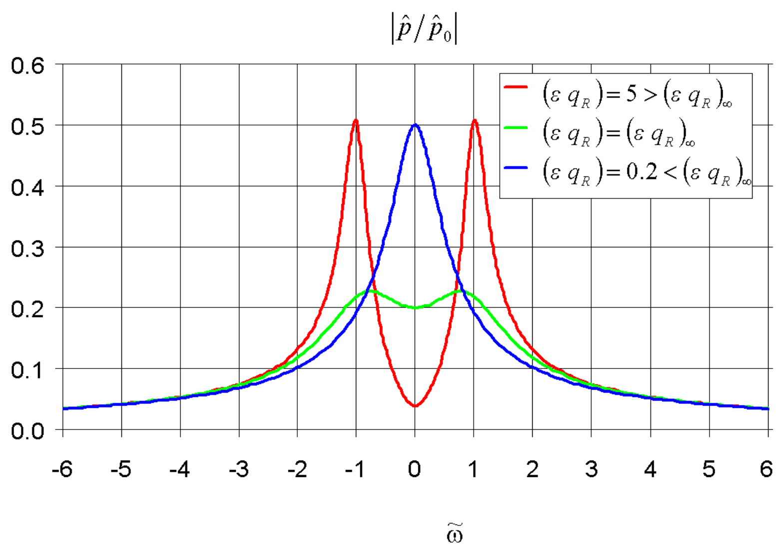

Figure 3: *Pressure ratio for $1/(\varepsilon q_E) = 0.2$. Effect of (εq_R) variation.*

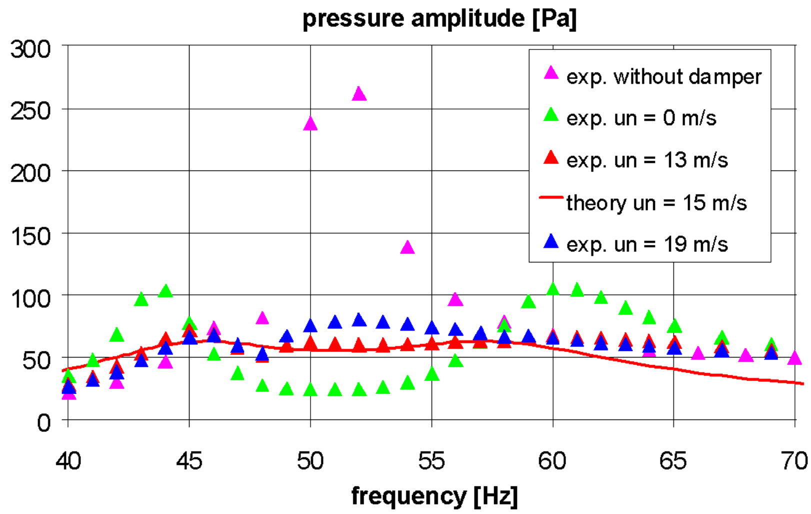

Figure 4: *Pressure spectra of impedance tube with and without dampers. Neck velocity variation.*

VALIDATION OF THE THEORY

The validation of the acoustic theory for damper design has been performed first using experiments conducted in an atmospheric impedance tube. The test rig at ALSTOM technology center in Baden, Switzerland, consists of a hollow steel tube where plane wave propagation occurs for frequencies below $620\ Hz$ (cut-off frequency). Four loudspeakers emitting pure toned frequency signals at various frequency intervals are used for the acoustic forcing. Using the multi-microphone method, pressure spectra may be measured at different axial positions [9]. In the impedance tube, acoustic pulsations have been suppressed by means of a resonator mounted on a flange located on one end of the rig. The resonator neck had a diameter of $35\ mm$ and was $162\ mm$ long. The resonator volume was composed of a $150\ mm$ diameter cylinder whose length was varied by means of a movable piston in order tune the resonator resonance frequency to the frequency of the first axial mode of the impedance tube [10]. Furthermore, air was injected inside the resonator volume in order to tune the resonator resistance. The quality factor of the enclosure was determined by using the impedance tube spectrum measured without resonator. Fig. 4 reports pressure spectra measured without and with resonator. In the latter case, the neck mean flow velocity $\overline{u}_n$ was varied from $0\ m/s$ to $19\ m/s$. In particular, the velocity $\overline{u}_n = 13\ m/s$ was found to give the maximum damping $||\widehat{p}/\widehat{p}_0||_\infty = 0.27$. According to Eq. (10), the minimum achievable pressure ratio was $||\widehat{p}/\widehat{p}_0||_\infty = 0.24$ in correspondence of a neck velocity $\overline{u}_n = 15\ m/s$. Pulsation amplitudes computed with Eq. (6) for $\overline{u}_n = 15\ m/s$ have been also reported in Fig. 4, showing good agreement with experiments.

An additional validation has been performed by applying the theory to an ALSTOM GT11N2 gas turbine combustor. In this engine, resonators applied to the top of the silo com-

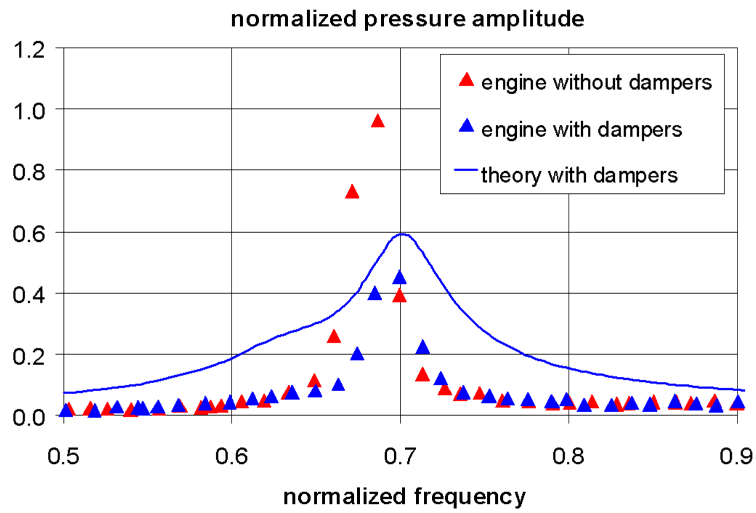

Figure 5: *Pressure spectra of GT11N2 gas turbine with and without dampers.*

bustor resulted in a very efficient measure to suppress thermoacoustic pulsations [2], [11]. Fig. 5 illustrates the spectra measured in the engine without and with 7 resonators designed to address the baseload peak frequency. In Fig. 5 the spectrum with dampers has been also computed using Eq. (6), with the combustor eigenmode calculated using Finite Element acoustics [11]. The predicted pressure ratio at ω_j was $|\widehat{p}/\widehat{p}_0| = 0.58$, whereas the engine tests showed a larger damping corresponding to a pressure reduction of 0.47. However, note that in this case thermoacoustic simulations have demonstrated that the pulsation peak was generated by combustion instability [11].

CONCLUSIONS

In this work, the acoustic damping effect of resonators applied to enclosures has been analyzed. The theory we have derived is suitable for gas turbine combustion chambers, where the temperature inside the damper (related to the cooling flow purging the resonator) differs from the combustion chamber temperature. The sound excitation has been assumed independent from the acoustic field (combustion noise). In agreement with the classical resonator design rules, we have found that the maximum pressure reduction is obtained by tuning the resonator resonance frequency to the pulsation peak frequency and by locating large volume lightly damped resonators close to mode antinodes. When the excitation is not confined to ω_j, the application of resonators may lead to the excitation of additional pulsation peaks. In the present work, we have obtained a theoretical expression providing the maximum suppression of acoustic amplitude for all the possible pulsation peaks occurring around ω_j, this condition being achieved when a specific value is assigned to the damper resistance. The theory has been validated by means of experiments performed in an impedance tube, where the resonator resistance was controlled by varying the mean flow throughout the damper neck. The experiments have confirmed the accuracy of the theory, with a 11% error on the maximum achievable damping. Finally, the theory has been also applied to tests performed on an ALSTOM GT11N2 heavy-duty gas turbine, where pressure spectra have been measured with and without resonators designed to address a specific pulsation frequency. Note that in this case the theory (developed for combustion noise) has been applied to a combustion instability pulsation peak, the computed pressure reduction showing an error of 23% with respect to engine tests.

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