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EFFECT OF CROSS-COUPLING ON THE INJECTION OF VIBRATORY POWER FROM SETS OF POINT FORCES INTO A PERIODIC SUPPORT STRUCTURE

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Abstract

In machine installations vibratory power is being transmitted as structureborne sound via the machine's feet and into its supporting structure. The paper examines this injection of power from sets of point forces that act on a periodic-type support structure, as is found in ship and aircraft foundations. A developed analytical model [1] of a *finite* periodic structure formed by beam-type elements is used for determining the injected power. The model is also used for examining what effect that spatial cross-coupling between excitation points has on the power injection, which often occurs in several transmission coordinates. The comb-type foundation being considered herein comprises of a waveguide with structural supports (side-branches) attached *asymmetrically* at regular intervals. By allowing for three motion-coordinates, this tri-coupled periodic structure experiences flexural-longitudinal wave coupling phenomena. This is taken into account when determining all the mobilities at driving force locations that govern the power injection into the periodic assembly. These results are used for examining the effect of spatial cross-coupling on the power injection for cases of 'distributed' excitation by sets of point forces with variable phases.

INTRODUCTION

Interior noise in transportation vehicles is often caused by audio-frequency vibration from installed machines. These vibrations are transmitted as structureborne sound via the mounts and into the foundation structure. Aircraft, ships, and offshore structures are often modularly built-up structures, which form so-called spatially *periodic* systems. Prediction of the vibratory power transfer from a machine and into this type of foundation and adjacent environments is important for interior noise predictions. Studies of multi-point mounted machine sources on *non-periodic* foundations have shown that the transfer of power can be predicted with an acceptable accuracy by a summation of *uncorrelated* local power contributions in all translational transmission terminals [2,3]. This prediction assumes that transmission in rotational coordinates can be ignored and that spatial cross-coupling between mounting points can be neglected; this last assumption generally holds, provided that the distance between mounting points *exceeds* one-half wavelength of flexural motion in both source and receiver, or for frequencies that exceed the fundamental anti-resonance (node) frequency of both source and receiver. Results from a recent Round Robin Test [4] clearly support these assumptions.

The purpose of this paper is to examine whether this also applies for line-type supporting structures in which forced responses in neighbouring mounting points more easily 'communicate' with one another, and hence influence the totally injected power. Examined herein is this injection of vibratory power from sets of point forces that act on a periodic-type support structure. These contact forces simulate the action on the foundation from a *resiliently* mounted machine source. Herein, we consider a simplified comb-like supporting structure in the form of a foundation beam supported asymmetrically at regular intervals by vertical beams (side-branches). The dynamics of this structure is based on an analytical model of a finite periodic structure developed in ref. [1]. Allowing for three motion-coordinates, this tri-coupled periodic structure experiences flexural-longitudinal wave coupling, and this phenomenon is taken into account when determining all mobilities at driving force locations that govern the power injection into the periodic foundation. These results are used for examining the effect of spatial cross-coupling on the power injection for cases of 'distributed' excitation by sets of point forces with variable phases.

OUTLINE OF THEORETICAL MODEL

Consider a two-dimensional source/isolator/receiver arrangement (Figure1) and the resulting action of contact forces on the foundation, causing velocities $v_i(t)$ in the motion coordinates u, w, φ. For harmonic vibration at frequency f, the column vector $\{v\}$ of these complex velocities is by definition related to the vector of contact forces $\{F\}$ via the receiver mobility matrix $[Y]$ as $\{v\}= [Y]\{F\}$. With forces acting simulta-

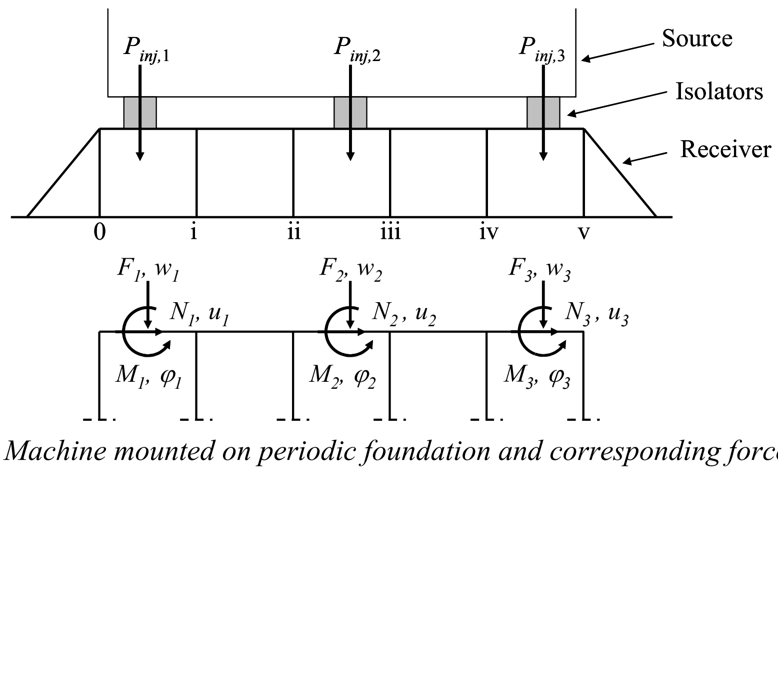


Figure 1 – Machine mounted on periodic foundation and corresponding force excitations.

neously in n coordinates, the time-averaged power P_i injected in the i'th coordinate is

$$P_i = \frac{1}{2}\mathrm{Re}\{F_i v_i^*\} = \frac{1}{2}\mathrm{Re}\{F_i \sum_{j=1}^{n} Y_{ij}^* F_j^*\}, \qquad [W] \qquad (1)$$

where * means the complex conjugate quantity. This shows that the i'th velocity results from the entire set of forces. Thus, the total injected power $P = \sum P_i$ is

$$P = \frac{1}{2}\mathrm{Re}\left\{\sum_{i}^{n} F_i \left(\sum_{j=1}^{n} Y_{ij}^* F_j^*\right)\right\} = \frac{1}{2}\mathrm{Re}\left\{\{F\}\left[Y^*\right]\{F^*\}^T\right\}, \qquad [W] \qquad (2)$$

where superscript T means a transposed vector.

System Mobilities

The mobilities appearing in eq. (2) are complicated functions of frequency, and point excitation in any single direction results in responses in all three motion coordinates, because of flexural-longitudinal wave coupling in the structure. Based on wave propagation characteristics of an *infinite* periodic structure, a transfer mobility was derived for a *finite* system [1] for the case of force or moment excitation applied at a beam intersection (junction). In the present investigation this mobility is generalized to include "in-bay" point excitation by using reciprocity as is outlined below.

Let an arbitrary position for point forcing on the foundation be denoted by p and the position of responses be denoted by q. And let the left-most junction position of the periodic structure be denoted by 0. The transfer mobility matrix $[Y_{p0}]$, which relates velocities at p to forces at 0 is already available [1]. The mobility matrix $[Y_{0p}]$ relating the vector of velocity responses $\{v_0\}$ at 0 to the vector of forces $\{F_p\}$ at p follows from reciprocity [5] as $[Y_{0p}] = [Y_{p0}]^{\mathrm{T}}$. The velocity vector can thus be written

$$\{v_0\} = [Y_{0p}]\,\{F_p\} = [Y_{p0}]^{\mathrm{T}}\{F_p\}. \qquad (3)$$

The very same response vector $\{v_0\}$ could be generated by an equivalent force vector $\{F_{0,eq}\}$ applied at position 0, giving $\{v_0\} = [Y_{00}]\{F_{0,eq}\}$, where $[Y_{00}]$ is the direct mobility matrix at junction 0. In a rearranged form this yields

$$\{F_{0,eq}\} = [Y_{00}]^{-1}\{v_0\}, \qquad (4)$$

where the superscript −1 means matrix inversion. Substituting eq. (3) into (4) gives an expression for the equivalent force vector in terms of known quantities, and this can also be used for obtaining the velocity vector $\{v_q\}$ at the desired position q :

$$\{F_{0,eq}\} = [Y_{00}]^{-1}\,[Y_{p0}]^{\mathrm{T}}\{F_p\} \quad \text{and} \quad \{v_q\} = [Y_{q0}]\{F_{0,eq}\} \qquad (5, 6)$$

Eliminating $\{F_{0,eq}\}$ from these finally gives the general transfer mobility matrix $[Y_{qp}]$, relating the vector of velocities at position q to the in-bay excitation force vector at p

$$\{v_q\} = [Y_{q0}]\,[Y_{00}]^{-1}[Y_{p0}]^{\mathrm{T}}\{F_p\} = [Y_{qp}]\,\{F_p\}. \qquad (7)$$

On Wave Propagation in Periodic Structures

Free harmonic waves in a *tri*-coupled, infinite periodic structure is governed by three independent wave types, each of which is characterized by a pair of "propagation constants" $\mu = \pm(\mu_R + i\mu_I)$, where the negative value corresponds to a positive-going wave and *visa versa*. Usually, μ_R is called the attenuation constant and μ_I the phase constant. So, if a single positive-going characteristic harmonic wave with propagation constant $\mu = -(\mu_R + i\mu_I)$ and frequency f travels through the system, then the complex velocities $v(\xi)$ and $v(\xi+l)$ at identical positions ξ in adjacent periodic elements of length l are related by $v(\xi+l)e^{i2\pi ft} = e^{\mu}\, v(\xi)e^{i2\pi ft}$. This shows that free wave motion is possible only in frequency bands where μ is purely *imaginary*. These bands are known as "propagation zones" or "pass bands". For negligible structural damping, the wave thus propagates throughout the system without a change in amplitude. The frequency bands in which μ is *real* are called "attenuation zones" or "stop bands", since no transport of vibrational energy is possible and the wave amplitude is attenuated (reduced) from element to element. Solutions for these propagation constants μ_i of an infinite periodic structure are the crucial (and controlling) functions for determining propagation properties and the dynamics of finite periodic structures [1].

NUMERICAL INVESTIGATION OF CROSS-COUPLING

The periodic foundation structures examined in the numerical simulations are similar to that in Figure 1b with five bays. The width of all beam components is 40 mm, and the six vertical support beams are of 180 mm length and 3 mm thickness, whereas the *foundation beam* thickness h is varied as 3 mm, 6 mm and 9 mm. The span length is 160 mm. The material is steel with Young's modulus of $200{\cdot}10^9$ N/m^2 and density of 7840 kg/m^3. Structural damping is modelled by a complex Young's modulus with a damping loss factor of 0.001 for the foundation beam and 0.003 for the vertical support beams. The termination mobility of the support beams is varied, from a pair of vertical and horizontal blocked springs of complex stiffness $\underline{s} = 12.5{\cdot}10^6(1+i0.3)$, to a similar pair of blocked springs of $\underline{s} = 4.0{\cdot}10^5(1+i0.3)$, each in parallel with a viscous damper of damping constant r = 890 kg/s, which is identical to the equivalent input properties of a 3 mm infinite plate of steel. This latter termination mobility is chosen to simulate the expected energy absorption from the remaining structural environment not specifically modelled, e.g., a complicated double-bottom of a ship.

Different types of forcing configurations are examined comprising a set of transverse (vertical) forces applied at *in-bay* positions, and a similarly applied set of longitudinal (axial) forces. Excitations at *junctions* are examined for a set of transverse forces and for a set of moment excitations. Each forcing set is composed of three harmonic point forces (or moments) of complex amplitude $|F_m|\exp[i(2\pi ft - \varphi_m)]$, m=1,2,3, with $|F_m|$=1.41 N (or $|M_m|$= $70.7{\cdot}10^{-3}$ Nm). The mutual phase relation φ_m between forces takes values of $\{\varphi_1, \varphi_2, \varphi_3\}=\{0, \pi/2, \pi\}$; $\{0, \pi, 2\pi\}$; and $\{0, 2\pi, 4\pi\}$. Neighbouring forces are thus in quadrature, in anti-phase, and in phase, respectively.

Results

Figure 2 shows frequency variations of the propagation constants for two of the wave types that are present in a tri-coupled periodic structure, having a *thin* foundation beam of h=3 mm; not shown is the third wave type, which is a flexural near-field. Wave propagation zones, for which μ is almost purely *imaginary*, $\mu \approx \pm i\mu_I$, are clearly seen at lower frequencies where wave type A is predominantly flexural, for example, in the bands from 230 to 295 Hz and from 420 to 590 Hz.

Figure 3 shows the examined structures and two types of applied forcing, i.e., junction and in-bay excitations, and the two types of terminals for the support beams. Figure 4 shows an example of a direct mobility and transfer mobilities at in-bay positions 2 and 3, computed with all three propagation constants as integral parts. This clearly shows periodic structure behaviour with the natural frequencies occurring in groups of N=five in the propagation zones, where N is the number of periodic elements [6]. The transfer mobilities also reveal low wave transmission and hence low vibration in some distinct attenuation zones, for example below 220 Hz and in bands from 295 to 420 Hz and 590 to 900 Hz; one therefore expect little effect of cross-coupling in the total power injected by the three transverse forces applied at in-bay positions as illustrated in Figure 3. Results in Figure 5 also confirm this.

Figure 5 shows comparisons of 'exact' calculations of the totally injected power for the three cases of *phase* relations between the forces, and the corresponding but

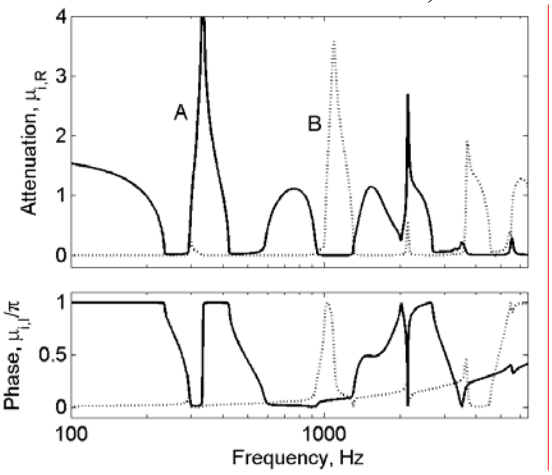


Figure 2 – Real and imaginary parts of propagation constants $\mu = (\mu_R + i\mu_I)$ for two wave types:——, flexural-longitudinal wave A; ----, flexural-longitudinal wave B.

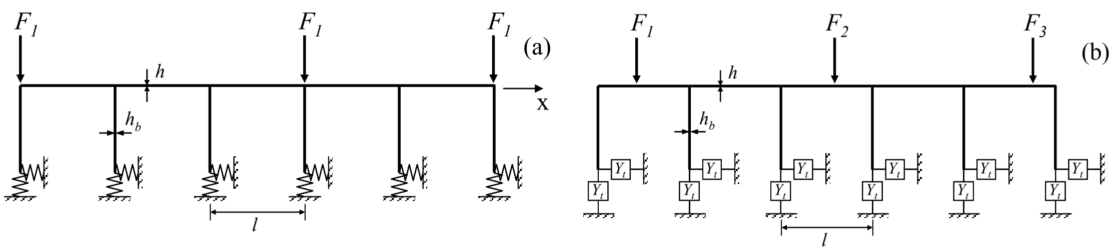
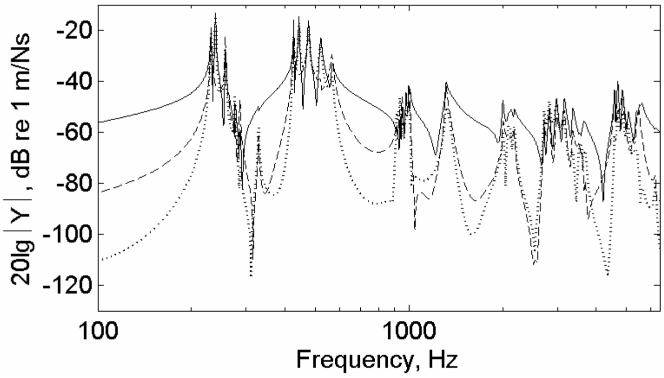


Figure 3 – Five-bay periodic foundation structures with a set of transverse forces applied at: (a) junctions in spring terminated system and, (b) in-bay in spring-damper terminated system.



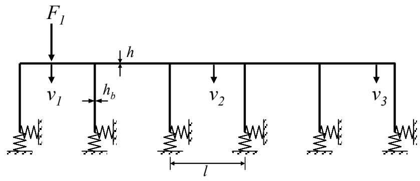


Figure 4 – Modulus of direct and transfer mobilities for transverse force excitation of five-bay periodic foundation structures of h=3mm; ——, Y_{11} *;------,* $Y_{21}= v_2/ F_1$ *;········,* $Y_{31}= v_3/ F_1$ *.*

simplified estimations of injected power determined as sums of the individual and uncorrelated contributions, that is, with cross-coupling being neglected. This means that only the diagonal terms of the foundation mobility matrix in eq. (2) are used. In the *h*=3mm-case the cross-coupling has relatively little effect overall, apart from the bands of mode-groups where the simple calculation over-predicts the injected power.

In the case of a *thick* foundation beam of 9mm the effect of cross-coupling is relatively small, and for 1/3-octave band-averaged values, say, the influence is less than ±3 dB. Moreover, because of the increased bending stiffness of the foundation

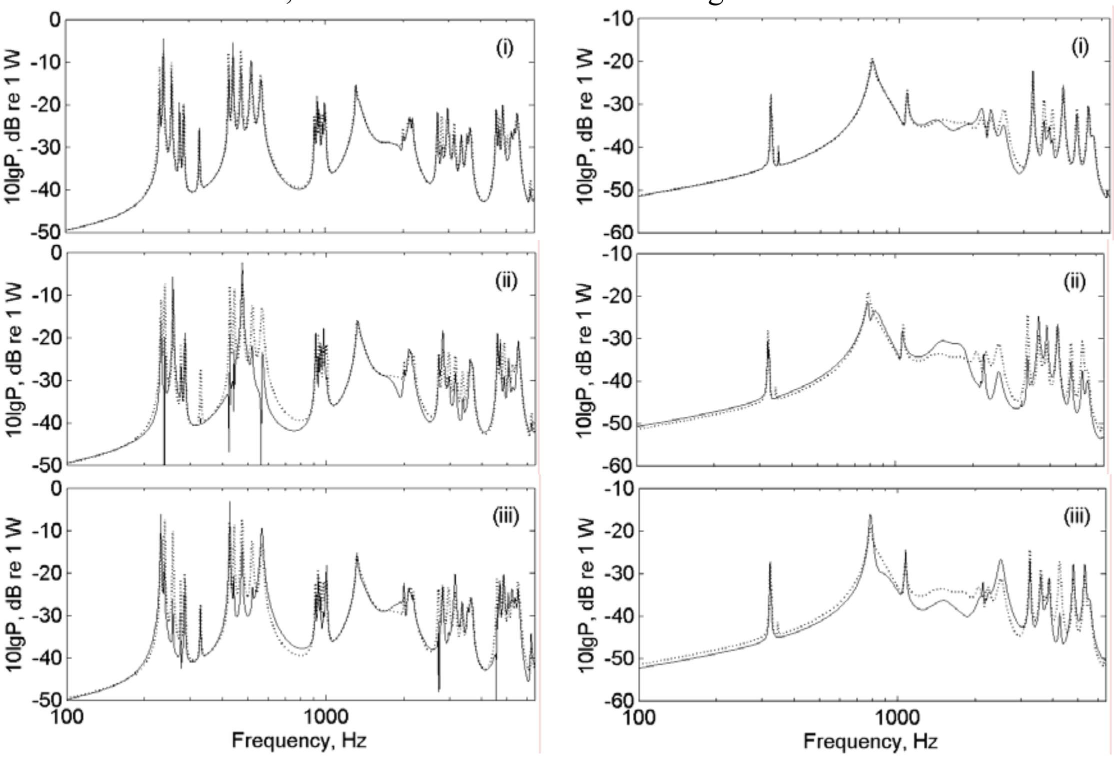


Figure 5 – Transmitted power from three in-bay transverse forces with phase relations $\{\varphi_1, \varphi_2, \varphi_3\}=\{0, \pi/2, \pi\}_{\mathrm{i}}$*;* $\{0, \pi, 2\pi\}_{\mathrm{ii}}$*; and* $\{0, 2\pi, 4\pi\}_{\mathrm{iii}}$*; ——, exact calculation; ----, estimate neglecting cross-coupling. Results at left for: h=3mm; at right for: h=9mm.*

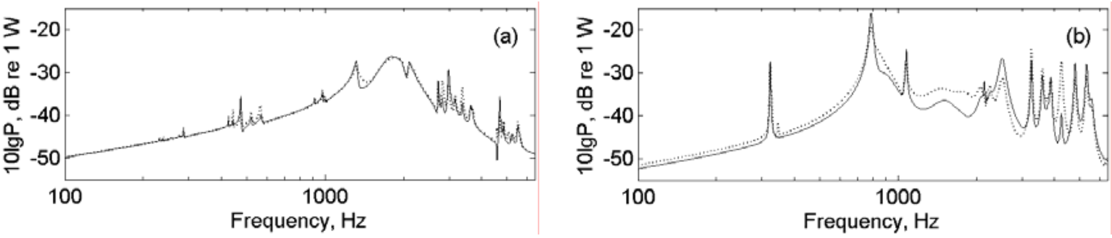


Figure 6 – Transmitted power from three forces acting: (a) at junctions of 3mm foundation beam; and (b) at in-bay positions of 9mm foundation beam. ——, exact calculation; ----, estimate neglecting cross-coupling. Phase relations $\{\varphi_1, \varphi_2, \varphi_3\}_{\mathrm{iii}} = \{0, 2\pi, 4\pi\}$.

beam, the spectral pattern is changed and the mode groups are 'pressed' towards much higher frequencies (beginning with 795 Hz): the injected power is seen to be much reduced in the mid-frequency range, especially in the previously mentioned two lowest mode groups where the power now is reduced by up to 35 dB, and on average by about 5 dB for frequencies exceeding approx. 900 Hz.

Designing the mounting of a machine so that its contact points are *aligned* with the supporting beams of the foundation as in Figure 3a is very important. This is illustrated for a *thin* foundation beam of 3mm in Figure 6a, which shows that the injected power becomes very small, and it is even less than that of a smooth broad envelope of the lowest values in Figure 5a at all frequencies. Or for that sake, mostly lower than the power injected by in-bay forces into a *thick* 9 mm foundation beam, see Figure 6b, which is taken from Figure 5b (iii). Results in Figure 6a also reveal that the exact calculation and the simplified estimate of total power are almost identical for $\{0, 2\pi, 4\pi\}$; this also applies for the other phase relations.

For the foundation with strongly *absorbing* terminations of springs and viscous dampers in parallel (Figure 3b), the overall trends in total power injection are found to be the same at high frequencies, albeit with much lower and smoothened peaks. At low and mid-frequencies, however, the injected power is much *increased* by up to 17 dB. This is seen from the results in Figure 7, which correspond to those in Figure 6. Results for a set of *longitudinal* forces applied at in-bay positions of a foundation beam of 3mm are shown in Figure 8a. Here, deviations of about ±5 dB are noted between exact and simple estimates of power, which is controlled by distinct peaks; the strong peaks at 320 and 930 Hz also appear in some of the previous figures, where they result from flexural-longitudinal wave coupling. The component at 3000 Hz corresponds to the fundamental free-free mode of the foundation beam. Figure 8b shows results for a set of *moments* applied at junctions of a 9 mm foundation beam.

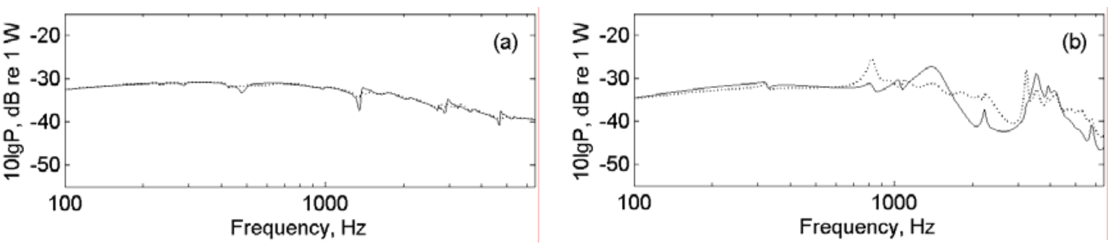


Figure 7 – As in Figure 6, but for high absorbing terminations; $\{\varphi_1, \varphi_2, \varphi_3\}_{\mathrm{ii}} = \{0, \pi, 2\pi\}$.

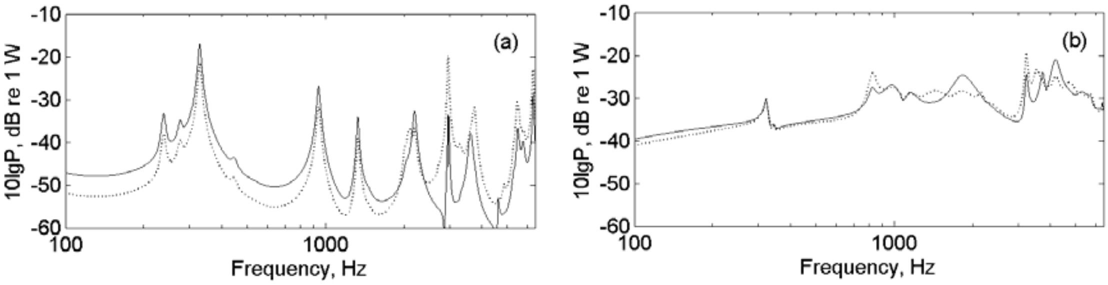


Figure 8 – As in Figure 7, but for: (a) longitudinal force excitation at in-bay positions and for h=3mm; and (b) moment excitation at junctions and for h=9 mm.

CONCLUSIONS

Exact formulations [1] are used for calculating the power being injected into a beam-type *periodic* foundation structure from sets of excitation point forces with variable phases. From these results and from comparisons with a simplified prediction, which neglects spatial *cross-coupling* between the excitation points, it is concluded that:

- Structural design can minimize power transfer considerably by aligning machine mounting points with the supporting beams of the foundation.
- Flexural cross-coupling can be neglected for the examined terminations since its effect is relatively small, with band-averaged deviations of less than ±3 dB for excitation at junctions, and likewise for in-bay excitation of a thick foundation.
- With deviations of about ±5 dB for longitudinal excitations the effect of cross-coupling has to be considered if such long wavelength excitation is significant.

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