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Calibration of the two microphone transfer function method to measure acoustic impedance in a wide frequency range

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Abstract

In many acoustic simulations, particularly when using lumped parameter models or electrical analog circuits, the acoustic impedance of a component needs to be determined accurately. A widely used acoustic impedance measurement method is the "two microphone transfer function method", which is standardized in ISO-10534-2. When the acoustic impedance is needed over a wide frequency band, for example from 10Hz to 10kHz, this method faces some limitations. In this paper, a new calibration method will be proposed such that acoustic impedances can be measured with high accuracy over a wide frequency range. The estimation of the speed of sound has been eliminated. Using the measured transfer functions between the two sensors at two different reference sections, the sensor positions will be accurately calibrated. The calibration of the sensor mismatch has become superfluous, so interchanging positions of the sensors is not necessary. A recursive procedure has been proposed to maximize the microphone position accuracy. The resulting calibration procedure has been reduced to the accurate determination of the sensor positions.

INTRODUCTION

A proper design of acoustic systems involving components such as silencers, resonators, absorbing materials, horns, etc ..., requires an accurate acoustic characterization of the various components. Such systems can be simulated using lumped element models or electrical analog circuits. The accuracy of the simulation result depends on the accuracy of the acoustic impedance of each component. Therefore, accurate acoustic impedance measurement methods are mandatory.

Dalmont [1] presents an overview of several techniques for acoustic impedance measurement.

The most straight forward technique uses a pressure and a volume velocity sensor [1], whereby the impedance is calculated directly from the ratio between both measured quantities. The direct measurement of the volume velocity can be carried out using for example a hot wire anemometer. Another approach is using an excitation source with a known volume velocity.

Other methods are based on connecting known impedances to the impedance to be measured. These methods are mostly used to determine the internal acoustic impedance of a source [2, 3, 4]. To the impedance to be measured, known acoustic loads are connected and the response to the source is measured. A set of equations results, from which the unknown impedance can be quantified. However, the equation set is often ill-conditioned and provides inaccurate impedance results.

A common acoustic impedance measurement technique is the standing wave ratio (SWR) method (the classical Kundt duct). This method is described in the ISO-10534-1 standard. The ends of the Kundt duct are closed by an excitation source at one end and the unknown impedance at the other end. The source generates a sinusoidal signal which results in a standing wave pattern in the duct. A microphone is moved along the axis of the duct. The minimum and maximum pressure amplitude of the standing wave and the location where the minimum and maximum amplitude occur are determined. From these data, the reflection coefficient and the acoustic impedance are calculated.

Another common method is the ”two microphone transfer function method”, which is described in ISO-10534-2. This method uses the transfer function measured between two pressure sensors at two distinct positions in the measurement wave guide to determine the acoustic impedance attached at one side of the wave guide. This method is discussed in more detail in the next section.

When the acoustic impedance is needed over a wide frequency band, for example from 10Hz to 10kHz, all the previously described impedance measurement methods face some limitations. Therefore, a new calibration method is under investigation which allows acoustic impedances to be measured with high accuracy over a wide frequency range. In this method, the estimation of the speed of sound is no longer required. In the calibration of the sensor mismatch, the deviation in sensor position after interchanging the pressure sensors has been taken into account. A recursive procedure is proposed to refine the microphone position calibration. This refinement can be repeated until the maximum calibration accuracy is reached.

THE TWO MICROPHONE TRANSFER FUNCTION METHOD

In this section, the principle of the two microphone transfer function method is discussed. Figure 1 presents the setup for acoustic impedance measurement. The setup consists of a straight duct which is the measurement acoustic wave guide. At the left end, an excitation source, such as a loudspeaker, is connected. At the right end, the impedance to be measured is connected. This impedance includes everything present at the right side of the reference section. Two microphones at two distinct positions x_1 and x_2 measure the sound pressure inside the duct. From the transfer function between the two microphones, the reflection coefficient and consequently, the unknown connected impedance will be determined.

In the frequency domain, the wave pattern in the wave guide is governed by the one-

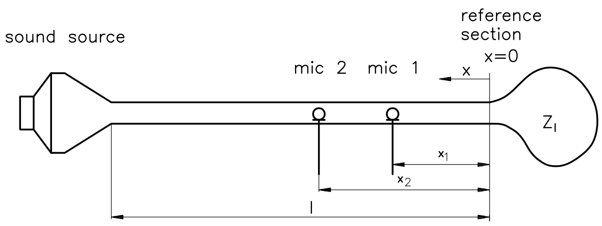

Figure 1: Wave guide with an unknown acoustic impedance Z_l.

dimensional Helmholtz wave equation, which describes the pressure distribution along the wave guide. At each position x, the pressure in terms of the wave number k in the wave guide equals [7]:

$$p(x,k) = \phi_g \frac{Z_0 Z_g}{Z_0 + Z_g} \cdot \frac{e^{-jkl}}{1 - \Gamma_l \Gamma_g e^{-j2kl}} \cdot (e^{jkx} + \Gamma_l e^{-jkx}) \tag{1}$$

wherein $j = \sqrt{-1}$, ϕ_g is the source volume velocity, Z_g the source internal impedance, Z_0 the wave guide characteristic impedance, l the distance between the exciting sound source and the reference section, Γ_l and Γ_g the reflection coefficients at the load side and source side respectively.

To measure the load impedance using the two microphone method, the transfer function T_{12} between the pressures at two distinct positions x_1 and x_2 is taken:

$$T_{12} = \frac{p(x_1,k)}{p(x_2,k)} = \frac{e^{jkx_1} + \Gamma_l e^{-jkx_1}}{e^{jkx_2} + \Gamma_l e^{-jkx_2}} \tag{2}$$

Notice that the source reflection coefficient drops out, the reflection coefficient at the load is the single unknown. Consequently, the choice of the source type is free. The load reflection coefficient Γ_l will then be isolated from equation (2) and the load impedance Z_l results from:

$$Z_l = Z_0 \frac{1+\Gamma_l}{1-\Gamma_l} = j Z_0 \frac{\sin kx_1 - T_{12} \sin kx_2}{\cos kx_1 - T_{12} \cos kx_2} \tag{3}$$

CALIBRATION OF THE SET-UP ACCORDING TO ISO 10534-2

Equation (3) becomes inaccurate when the reflection coefficient Γ_l approaches unity. This situation occurs when the unknown impedance deviates largely from the characteristic impedance of the wave guide. Therefore, prior calibration of the setup is necessary to obtain accurate results.

The ISO 10534-2 standard demands the following calibration actions:

- The velocity of sound needs to be determined accurately using measurements of ambient temperature and atmospheric pressure.
- The distance between the pressure sensors needs to be measured accurately.

- The mismatch between the amplitude and phase of the pressure sensors needs to be calibrated. In short, the procedure is to measure the transfer function T_{12} of the two pressures at position x_1 and x_2, then interchange the two pressure sensors from location x_1 to x_2 and from x_2 to x_1 respectively, measure the transfer function T_{21} and calculate the calibration factor δ such that:

$$\delta^2 T_{12} T_{21} = 1 + 0j \tag{4}$$

  This calibration factor δ is complex and frequency dependent.

In order to illustrate the accuracy of the ISO calibration procedure, a simulation with representative data for such setup is carried out.

Suppose the wave guide has a diameter of 40 mm, the characteristic impedance will be $Z_0 = 320\,\mathrm{k}\Omega$ ($1\,\Omega = 1\mathrm{Pa\,s/m^3}$). The distance between the nearest pressure sensor and the reference section is $x_1 = 0.2\,\mathrm{m}$ and between the farest pressure sensor and the reference section $x_2 = 0.5\,\mathrm{m}$. Suppose the impedance measurement range is limited to 120 dB (ref. Z_0), i.e. 60 dB above and below Z_0, the closed duct ("infinite") impedance Z_l will be limited to $1000\,Z_0$. When determining the reversed transfer function T_{21} after interchanging the sensors, position deviations of 0.5 mm have been introduced on the distances x_1 and x_2. These deviations are realistic, due to tolerances on the mounting hole and the position of the acoustic centre of the sensor.

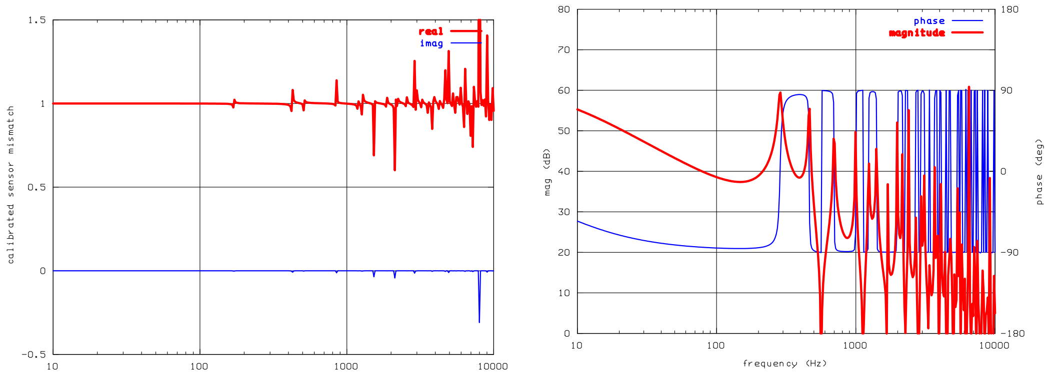

Figure 2: left: Calibration result of the pressure sensors according to ISO 10534-2. right: Resulting closed duct end impedance using the calibration data displayed in figure 2(left).

The calibration method described in the ISO 10534-2 standard is applied. The resulting calibration factor δ times the sensor mismatch is presented in figure 2(left). This product should equal $1 + 0j$. In the lower frequency range, the calibration has been successful. However, in the higher frequency range, starting from 150 Hz, the calibration is erroneous and even worse than the uncalibrated situation. The resulting impedance is presented in figure 2(right). This should be a straight line at 60 dB, but due to the erroneous calibration, large deviations results. In this case, the measurement range wherein the unknown impedance can be reliably measured is limited to 35 dB around Z_0 until 400 Hz. Above 400 Hz, reliable impedance measurements are not possible. This simulation demonstrates that it will be necessary to calibrate the pressure sensor positions accurately.

IMPROVED CALIBRATION METHOD

This paper proposes an improved calibration procedure. In the new method, it is sufficient to calibrate the sensor positions, expressed as the travelling times of the acoustic waves from the respective sensor positions to the reference section. The laboratory setup consists of a duct of 40 mm diameter and 2 m length equipped with a 60 W horn driver. The sensors are positioned at a distance $x_1 = 0.3$ m and $x_2 = 0.47$ m from the reference section. Two transfer functions are measured between the two sensor outputs. The first transfer function T_{12} is measured with the duct closed at the reference section, as presented in figure 3(left). An additional piece of duct of 46 mm length is added resulting in a shift of the reference section, as presented in figure 3(right). The second transfer function T_{34} is measured. The resulting transfer functions T_{12} and T_{34} are represented in figure 4(left) and figure 4(right) respectively. All calibration information will be extracted from these transfer functions. Finally, a recursive refinement procedure will maximize the accuracy of the position calibration.

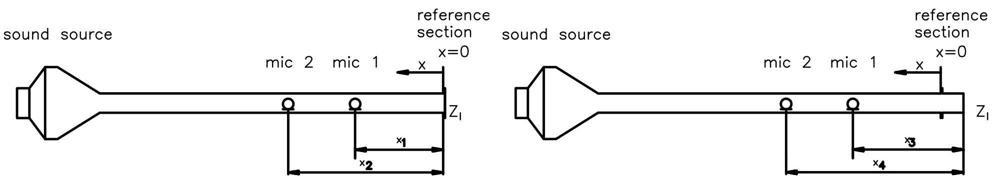

Figure 3: left: calibration setup with duct end closed at the reference section and right: closed at the shifted reference section.

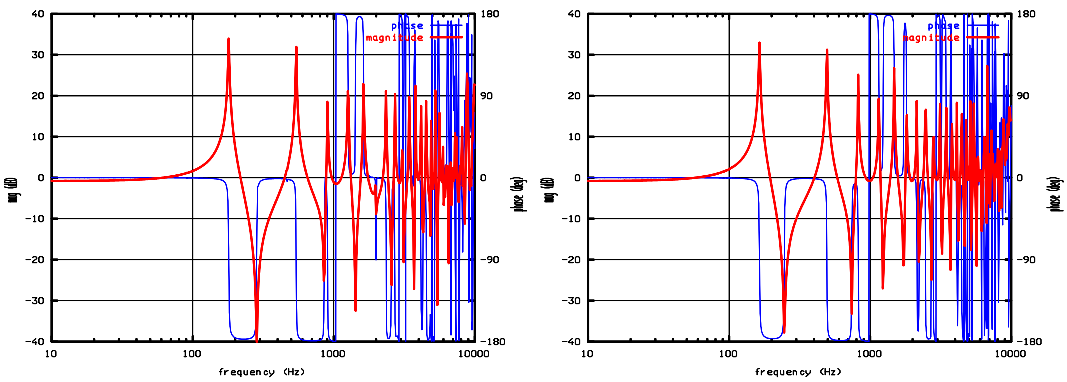

Figure 4: left: measured transfer function T_{12} between the sensors with the duct closed at the reference section and right: T_{34} with the duct closed at the shifted reference section.

Elimination of the speed of sound.

The speed of sound is eliminated by replacing kx_i by ωt_i, $(i = 1, 2, 3, 4)$, in expression (2), with t_1, t_2, t_3 and t_4 the travelling times needed for the wave to travel from the positions x_1, x_2, x_3 and x_4 to their reference sections respectively.

The travelling times t_1 and t_2 are estimated from the transfer function T_{12} (figure 4(left)),

t_3 and t_4 from the transfer function T_{34} (figure 4(right)). They correspond to the first pole for the farest microphone and the first zero for the nearest microphone position. The pole and the zero correspond to the first node of the pressure distribution of the standing wave appearing at the positions x_1, x_2, x_3 and x_4 respectively. These travelling times t_1, t_2, t_3 and t_4 equal:

$$t_1 = \frac{1}{4\,f_1} \quad , \quad t_2 = \frac{1}{4\,f_2} \quad , \quad t_3 = \frac{1}{4\,f_3} \quad \text{and} \quad t_4 = \frac{1}{4\,f_4} \tag{5}$$

in which the frequencies f_1, f_2, f_3 and f_4 are associated to the frequencies determined by the quarter wavelength between the reference section and the positions x_1, x_2, x_3 and x_4 respectively.

Determining the closed end impedance.

The next step considers the ratio between the two transfer functions T_{12} and T_{34}. The sensor mismatch is then removed from the resulting transfer function T_c, presented in figure 5(left). This results in the expression:

$$T_c = \frac{Z_c^2 \cos\omega t_1 \cos\omega t_4 - Z_0^2 \sin\omega t_1 \sin\omega t_4 - j\,Z_c\,Z_0\,sin\,\omega t_1 \cos\omega t_4 - j\,Z_c\,Z_0\,cos\,\omega t_1 \sin\omega t_4}{Z_c^2 \cos\omega t_2 \cos\omega t_3 - Z_0^2 \sin\omega t_2 \sin\omega t_3 - j\,Z_c\,Z_0\,sin\,\omega t_2 \cos\omega t_3 - j\,Z_c\,Z_0\,cos\,\omega t_2 \sin\omega t_3} \tag{6}$$

from which the closed end impedance Z_c will be calculated. This impedance (see figure 5right) determines the measurement range, and must be as large as possible. The expression (6) is quadratic in Z_c, resulting in two solutions. One solution corresponds to the direct reference section, the other to the shifted reference section. It is necessary to select the proper solution at each frequency point.

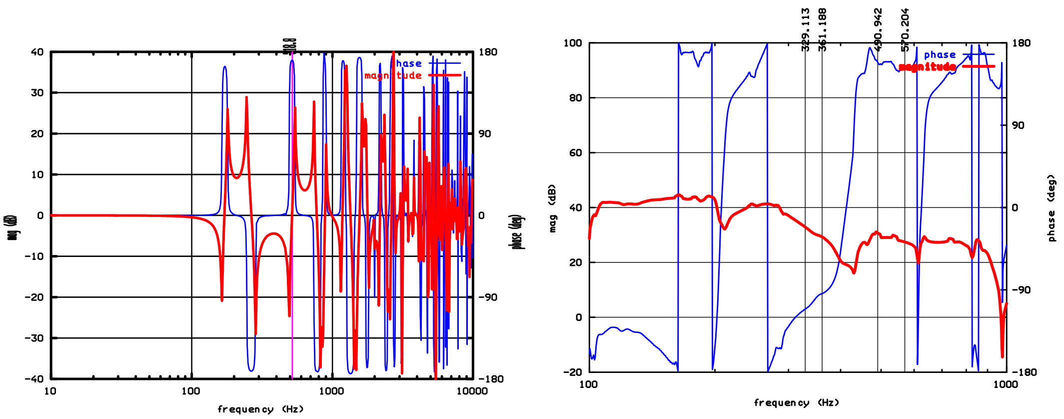

Figure 5: left: division of the two transfer functions $T_c = T_{12}/T_{34}$ right: resulting closed end impedance Z_c (dB(refZ_0)).

Refining the sensor positions.

Often, the travelling times t_1, t_2, t_3 and t_4 are still not sufficiently accurate when determined from the equations (5). The impedance is extremely sensitive to errors in the travelling times.

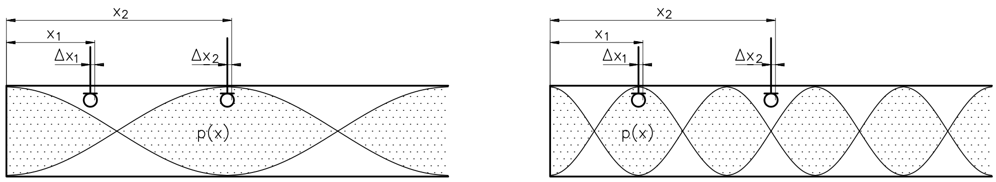

Figure 6: Refining situations for x_1 and x_2.

Therefore, the acoustic impedance Z_c of the closed duct itself will be used to refine these travelling times.

Figure 6 presents the situations wherein, respectively, the travelling times t_1 and t_2 will be corrected. When a pressure maximum occurs at one of the microphone positions, the gradient of the pressure is zero and a small deviation of the microphone position does not affect the accuracy of the measured pressure. This phenomenon occurs at distinct frequencies, when a half wave length stands between the sensor and the closed reference section. Consequently, the correction of t_1 will be executed at $f_2 = \frac{1}{2t_2}$ and the correction of t_2 at $f_1 = \frac{1}{2t_1}$. At these distinct frequencies, the distance between the two microphones is virtually correct. Consequently, at the reference section appears an impedance Z_c which is composed of the hard wall impedance and a short piece wave guide with a travelling time τ. The travelling time corrections τ_1 and τ_2 are:

$$\tau_l = \frac{t_2}{\pi} \arctan \frac{Z_0}{j\,Z_{c2}} \quad \text{and} \quad \tau_2 = \frac{t_1}{\pi} \arctan \frac{Z_0}{j\,Z_{c1}} \tag{7}$$

in which Z_{c2} is the measured closed end impedance Z_c evaluated at f_2 and Z_{c1} evaluated at f_1. In a same way, the travelling times t_3 and t_4 are corrected. The closed end impedance Z_c can be recalculated using these corrected travelling times. The correction of the travelling times can be repeated until the maximum accuracy has been reached. Figure 7(left) demonstrates how the closed end impedance rises towards 60 dB, when compared to figure 5(right).

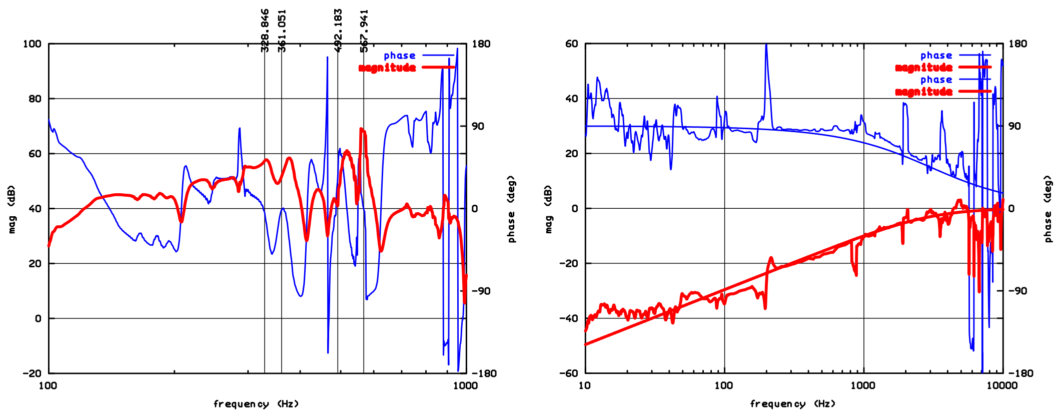

Figure 7: left: resulting closed end impedance after refining travelling times; right: resulting open end impedance after improved calibration.

DETERMINING UNKNOWN IMPEDANCE

After calibration, the unknown impedance needs to be determined. For that purpose, the unknown impedance is connected on the wave guide at the reference section and the transfer function T_l between the microphones is measured. The following data resulting from the calibration procedure are needed: T_{34}, Z_c, t_1, t_2, t_3 and t_4. The unknown impedance Z_l is then obtained from:

$$Z_l = Z_0 \frac{Z_0 \left(\sin\omega t_1 \, \sin\omega t_4 - T \, \sin\omega t_2 \, \sin\omega t_3\right) + j Z_c \left(T \, \sin\omega t_2 \, \cos\omega t_3 - \sin\omega t_1 \, \cos\omega t_4\right)}{Z_c \left(T \, \cos\omega t_2 \, \cos\omega t_3 - \cos\omega t_1 \, \cos\omega t_4\right) + j Z_0 \left(\cos\omega t_1 \, \sin\omega t_4 - T \, \cos\omega t_2 \, \sin\omega t_3\right)} \tag{8}$$

wherein $T = T_l / T_{34}$.

As an example, figure 7(right) presents the open end impedance, calibrated using the new method. The smooth lines represent the analytical spherical radiator impedance [6].

To further improve the measurement method, other error sources will be investigated, such as finite sensor dimensions, wave guide damping and transfer function amplitude variations.

CONCLUSION

A new calibration method is proposed to measure acoustic impedances with high accuracy over a wide frequency range. The estimation of the speed of sound is no longer required. Consequently, the temperature and ambient pressure measurements are not necessary. Using the measured transfer functions between the two sensors for two different reference sections, the sensor positions can be accurately calibrated. These sensor positions are further refined using a recursive procedure. The calibration of the sensor mismatch has become superfluous, so exchanging positions of the sensors is no longer needed. The resulting calibration procedure has been reduced to the accurate determination of the sensor positions.

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