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FREE VIBRATION RESPONSE OF MULTIPHASE MAGNETO-ELECTRO-ELASTIC PLATES BY FINITE ELEMENT METHOD

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Abstract

Free vibrations of simply supported multiphase magneto-electro-elastic plates have been studied by semi-analytical finite element method. Assumed shape functions are used in the plane of plate and one dimensional finite elements are used across the thickness of the plate. $BaTiO_3$-$CoFe_2O_4$ composite is used as magneto-electro-elastic material.

Keywords: Magneto-electro-elastic, Free Vibration, Finite element

1. INTRODUCTION

Recently the research on magneto-electro-elastic composite as smart materials has gained more importance. These materials have the capacity to convert one form of energy, viz., magnetic, electric and mechanical energy to another form of energy. They exhibit desirable coupling effect between electric and magnetic fields, which are useful in smart or intelligent structure applications and find applications in magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, etc. These smart materials seem to provide unique capabilities of sensing and reacting to external disturbances, thus helping to design based on performance, reliability and light weight requirements imposed in any modern structural applications.

Studies on static and dynamic behavior of these materials in the form of plates and shells have been carried out in literature. Pan[1] has derived exact solutions for three dimensional, anisotropic, linearly magneto-electro-elastic, simply supported and multilayered rectangular plates under static loadings. Pan and Heyliger [2] have studied the free vibration behaviour of magneto-electro-elastic plate. Sunar et al. [3] have studied the finite element modeling of thermopiezomagnetic medium.

Buchanan[4] has studied the behaviour of infinitely long magneto-electro-elastic cylindrical shell using semi analytical finite element method. Buchanan[5] has studied the vibration behaviour of an infinite plate consisting of layered versus multiphase magneto-electro-elastic composites. Bhangale and Ganesan[6] have studied the free vibration behaviour of simply supported functionally graded and layered magneto-electro-elastic plates by finite element method. Chen *et al.* [7] carried out free vibration analysis of non-homogeneous transversely isotropic magneto-electro-elastic plates. A free vibration study of clamped–clamped magneto-electro-elastic cylindrical shell was carried out by Annigeri *et al.* [8], also the authors[9] have studied the Free Vibrations of simply supported layered and multiphase Magneto-Electro-Elastic Cylindrical Shells. Aboudi[10] has carried out micromechanical analysis of fully coupled electro-magneto-thermo-elastic composites. In his study, a homogenization micromechanical method is employed for the prediction of the effective moduli of magneto-electro-elastic composites. His study includes determination of effective elastic, piezoelectric, piezomagnetic, dielectric, magnetic permeability and electromagnetic coupling moduli, as well as effective thermal expansion coefficients and the associated pyroelectric and pyromagnetic constants for magneto-electro-elastic composite.
From the literature survey it is found that there is no finite element formulation available for vibration studies on multiphase finite magneto-electro-elastic plate. Hence in present study, free vibration analysis of multiphase magneto-electro-elastic plates has been carried out by using series solution in conjunction with finite element approach as developed by Rajesh and Ganesan[6] for functionally graded magnet-electro-elastic plates, however in this paper it is extended for multiphase magneto-electro-elastic plates. The main aim of the study is to bring out the effect of piezoelectric, piezomagnetic and coupling terms on frequency behaviour through proposed five cases of free vibrations for $BaTiO_3$-$CoFe_2O_4$ composite plate.

2. BASIC EQUATIONS

The equilibrium equations for the magneto-electro-elastic solids for balance of body force, electric charge and electric current can be written as shown below. Pan [1]

$$\sigma_{ij,j} + f_b = 0\,, D_{ij,j} - f_e = 0\,, \quad B_{ij,j} - f_m = 0 \tag{1}$$

where f_b, f_e and f_m are the body force, electric charge density and electric current density respectively. The coupled constitutive equations for anisotropic and linearly MEE solids can be written as Buchanan [5].

$$\sigma_j = c_{jk}S_k - e_{kj}E_k - q_{kj}H_k\,, D_j = e_{jk}S_k + \varepsilon_{jk}E_k + m_{jk}H_k \; B_j = q_{jk}S_k + m_{jk}E_k + \mu_{jk}H_k \tag{2}$$

where σ_j, D_j and B_j indicate the stress, electric displacement and magnetic induction.
S_k, E_k and H_k represent strain, electric field and magnetic field.
C_{jk}, ε_{jk} and μ_{jk} are the elastic, dielectric and magnetic permeability coefficients.
e_{kj} , q_{kj}, and m_{jk} are the piezo-electric, piezo-magnetic and magneto-electric material coefficients respectively.

The strain displacement relations are

$$S_{xx}=S_1=\frac{\partial u}{\partial x};\ S_{yy}=S_2=\frac{\partial v}{\partial y};\ S_{zz}=S_3=\frac{\partial w}{\partial z}$$

$$S_{yz}=S_4=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y};\ S_{xz}=S_5=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x};\ S_{xy}=S_6=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}; \tag{3}$$

where *u*, *v* and *w* are mechanical displacements in co-ordinate directions *x*, *y* and *z*. The electric field vector E_i is related to the electric potential ϕ as shown below.

$$E_x=E_1=-\frac{\partial\phi}{\partial x};E_y=E_2=-\frac{\partial\phi}{\partial y};\ \ E_z=E_3=-\frac{\partial\phi}{\partial z} \tag{4}$$

The magnetic field H_i is related to magnetic potential ψ as shown below.

$$H_x=H_1=-\frac{\partial\psi}{\partial x};H_y=H_2=-\frac{\partial\psi}{\partial y};\ \ H_z=H_3=-\frac{\partial\psi}{\partial z} \tag{5}$$

3.0 FINITE ELEMENT FORMULATION

The finite series solution has been assumed to satisfy the simply supported boundary conditions for plates. The finite element method is adopted in thickness direction of the plate. The nodal variables are u, v, w, ϕ and ψ. The shape functions are as follows;

$$u(x,y,z)=\sum_{n=1}^{N}\sum_{m=1}^{M}U_{nm}(z)\cos rx\sin sy\ , \qquad v(x,y,z)=\sum_{n=1}^{N}\sum_{m=1}^{M}V_{nm}(z)\sin rx\cos sy$$

$$w(x,y,z)=\sum_{n=1}^{N}\sum_{m=1}^{M}W_{nm}(z)\sin rx\sin sy\ , \qquad \varphi(x,y,z)=\sum_{n=1}^{N}\sum_{m=1}^{M}\Phi_{nm}(z)\sin rx\sin sy$$

$$\psi(x,y,z)=\sum_{n=1}^{N}\sum_{m=1}^{M}\Psi_{nm}(z)\sin rx\sin sy, \qquad \text{Here } r=\left(\frac{n\pi}{L_x}\right),s=\left(\frac{m\pi}{L_y}\right) \tag{6}$$

where *n* and *m* being two positive integers and *N* and *M* are the number of terms in the series to be accounted for the general loading. The present study has been carried out similar to the reported by Pan and Heyliger [2] for *m*=*n*=1. In the end the analysis has been reduced for finite element in thickness direction with three-dimensional dependence, the solution based on the choice of *n* and *m*. The two noded finite element and the assumed shape functions are

$$U=[N_u]\{U\};\quad \Phi=[N_\varphi]\{\Phi\};\quad \Psi=[N_\psi]\{\Psi\} \tag{7}$$

$$\text{where}\quad N_1=\left(1-\frac{z_i}{z_{i+1}-z_i}\right);\ N_2=\left(\frac{z_i}{z_{i+1}-z_i}\right)$$

For a coupled problem the finite element equations are as follows

$$\begin{gathered}\left[\left[K_{uu}\right]-\omega^2\left[M\right]\right]\{U\}+\left[K_{u\phi}\right]\{\phi\}+\left[K_{u\psi}\right]\{\psi\}=0\\ \left[K_{u\phi}\right]^T\{U\}-\left[K_{\phi\phi}\right]\{\phi\}-\left[K_{\phi\psi}\right]\{\psi\}=0,\\ \left[K_{u\psi}\right]^T\{U\}-\left[K_{\phi\psi}\right]^T\{\phi\}-\left[K_{\psi\psi}\right]\{\psi\}=0\end{gathered} \tag{8}$$

Various stiffness matrices are defined as shown below.

$$
\begin{aligned}
&[K_{uu}] = c\int [B_u]^T [C][B_u]dz, && [K_{u\phi}] = c\int [B_u]^T [e][B_\phi]dz; \\
&[K_{u\psi}] = c\int [B_u]^T [q][B_\psi]dz, && [K_{\phi\phi}] = c\int [B_\phi]^T [\varepsilon][B_\phi]dz \\
&[K_{\psi\psi}] = c\int [B_\psi]^T [\mu][B_\psi]dz, && [M] = c\int_v [N]^T [\rho][N]dv \\
&[K_{\phi\psi}] = c\int [B_\phi]^T [m][B_\psi]dz
\end{aligned} \tag{9}
$$

where $c = 0.25L_xL_y$

$[B_u],[B_\phi],[B_\psi]$ represents the strain-displacement, electric field-electric potential and magnetic field- magnetic potential relations respectively.

The FE formulation is similar to Rajesh and Ganesan[6] hence it is given briefly here. In equation (8), eliminating electric and magnetic potential terms by condensation techniques to get K_{eq}. The equation of motion for the system can be written as

$$[M]\{\ddot{U}\} + [K_{eq}]\{U\} = 0 \tag{10}$$

Where $[K_{eq}] = [K_{uu}] + [K_{u\phi}][K_{II}]^{-1}[K_I] + [K_{u\psi}][K_{IV}]^{-1}[K_{III}]$ (11)

The component matrices for equation (11) are shown below.

$$
\begin{aligned}
&[K_I] = [K_{u\phi}]^T - [K_{\phi\psi}][K_{\psi\psi}]^{-1}[K_{u\psi}]^T \\
&[K_{II}] = [K_{\phi\phi}] - [K_{\phi\psi}][K_{\psi\psi}]^{-1}[K_{\phi\psi}]^T \\
&[K_{III}] = [K_{u\psi}]^T - [K_{\phi\psi}]^T[K_{\phi\phi}]^{-1}[K_{u\phi}]^T \quad [K_{IV}] = [K_{\psi\psi}] - [K_{\phi\psi}]^T[K_{\phi\phi}]^{-1}[K_{\phi\psi}]
\end{aligned} \tag{12}
$$

The distribution of $\{\phi\}$ and $\{\psi\}$ can be as shown below.

$$\phi = [K_{II}]^{-1}[K_I]\{U\}\ ,\ \ \psi = [K_{IV}]^{-1}[K_{III}]\{U\} \tag{13}$$

To study the effect of magnetoelectric constant (m) on the system frequencies, equivalent stiffness matrix [$K_{eq\_reduced}$] is derived by neglecting the coupling between piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ materials. The magnetoelectric material coefficient (m) is zero for single phase $BaTiO_3$ and $CoFe_2O_4$[10].

$$[K_{\text{eq\_reduced}}] = [K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{u\phi}]^T + [K_{u\psi}][K_{\psi\psi}]^{-1}[K_{u\psi}]^T \tag{14}$$

To study the piezoelectric effect on frequency due to $BaTiO_3$ material, the stiffness matrix $[K_{\text{eq\_}\phi\phi}]$ is derived and is given by

$$[K_{\text{eq\_}\phi\phi}] = [K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{u\phi}]^T \tag{15}$$

To study the magnetic effect on frequency due to magnetic $CoFe_2O_4$ material $[K_{\text{eq\_}\psi\psi}]$ is used as stiffness matrix and is shown below.

$$\left[K_{\mathrm{eq}\_\psi\psi}\right]=\left[K_{uu}\right]+\left[K_{u\psi}\right]\left[K_{\psi\psi}\right]^{-1}\left[K_{u\psi}\right]^{T} \quad (16)$$

Material properties of $BaTiO_3$ and $CoFe_2O_4$ are given in Appendix. The Gaussian integration scheme is implemented to evaluate integrals of stiffness matrices. For the study, 150 two noded linear elements across the thickness direction are used with good convergence in results.

5.0 RESULT AND DISCUSSION

Densities of both materials are assumed to be same. Five different cases of vibration are discussed and corresponding terminology is shown in Table-1.

Table 1 Cases of free vibrations in magneto-electro-elastic plate

Case	**Stiffness matrix used**	**Frequency**
I	$\left[K_{uu}\right]$	Structural frequency with elastic property only
II	$\left[K_{eq}\right]$	System frequency magneto-electro-elastic coupling
III	$\left[K_{eq\_reduced}\right]$	System frequency by neglecting magneto-electric coupling
IV	$\left[K_{eq\_\psi\psi}\right]$	System frequency with piezomagnetic phase
V	$\left[K_{eq\_\phi\phi}\right]$	System frequency with piezoelectric phase

5.1 Validation

A square plate of $(h/L_x) = (h/L_y)=1$ studied by Pan and Heyliger [2] is considered here for comparison of results. Table 2 shows the frequency parameter $\omega^* = \omega L_x\sqrt{C_{\max}/\rho_{\max}}$ for the first case and second case of free vibration. An excellent correlation has been observed for mode 1. It is seen that present finite element gives an excellent correlation with analytical results given in Ref. [2] for the plate made of either fully piezoelectric $BaTiO_3$ and or piezomagnetic $CoFe_2O_4$ material.

Table 2: Validation for the frequency studies

	$BaTiO_3$ (B)		**$CoFe_2O_4$ (F)**	
Mode	Ref [2]	present	Ref [2]	present
1	2.1091	2.1091	1.5403	1.5403
2	2.8153	2.8153	2.3373	2.3372
3	3.9614	3.9614	3.1866	3.1866
4	4.3888	4.3888	3.7914	3.7913
5	5.5071	5.5071	4.5343	4.5342

The material properties for vf=0% , 60% and 100% of $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$ composite are referred from for Aboudi[10].

The frequency is given in Hz for first ten modes. The following notations are used in Table 3, 4 and 5 in subsequent discussion.

f_{uu} = Structural frequency in Hz by using $[K_{uu}]$ as stiffness matrix. (Case- I)

f_{eq} = System frequency in Hz by using $[K_{eq}]$ as stiffness matrix. (Case-II)

$f_{eq\_reduced}$ = Frequency in Hz by using $[K_{eq\_reduced}]$ as stiffness matrix. (Case-III)

$f_{\psi\psi}$ = Structural frequency in Hz by using $[K_{\psi\psi}]$ as stiffness matrix. (Case-IV)

$f_{\phi\phi}$= Structural frequency in Hz by using $[K_{\phi\phi}]$ as stiffness matrix. (Case-V)

Table-3 gives the frequencies for MEE plate for vf =0% of $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$ composite. It can be observed that frequencies show increasing trend as the mode number increase. The frequency due to $[K_{eq}]$ is marginally higher than the conventional structural frequency f_{uu}, no difference between columns f_{keq} and $f_{keq\_reduced}$ is noticed as magnetoelectric coupling is absent in pure piezomagnetic $CoFe_2O_4$. The $f_{\phi\phi}$ values coincide with the structural frequency, since piezoelectric phase is absent for vf=0%.

Table 3: Frequencies for MEE plate for vf =0% of $BaTiO_3$

Mode	Case of free vibration				
	I	II	III	IV	V
1	5794.97	5767.19	5767.19	5767.19	5794.97
2	7394.62	7394.62	7394.62	7394.62	7394.62
3	8467.34	8462.41	8462.41	8462.41	8467.34
4	8753.51	8753.51	8753.51	8753.51	8753.51
5	11935.84	11935.84	11935.84	11935.84	11935.84
6	12060.38	12057.28	12057.28	12057.28	12060.38
7	14180.66	14198.29	14198.29	14198.29	14180.66
8	15881.76	15881.76	15881.76	15881.76	15881.76
9	16985.25	16980.49	16980.49	16980.49	16985.25
10	18938.49	18954.33	18954.33	18954.33	18938.49

Table-4 gives the frequencies for MEE plate for vf =100% of $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$ composite. This corresponds to the plate made of pure piezo-magnetic $CoFe_2O_4$ material. Here also the frequencies show increasing trend as the mode number increase. The system frequency due to $[K_{eq}]$ is higher than the conventional structural frequency f_{uu}, no difference between columns f_{keq} and $f_{keq\_reduced}$ is noticed as magnetoelectric (m) constant is absent in pure piezoelectric $BaTiO_3$. The $f_{\phi\phi}$ values coincide with the system frequency, since piezoelectric phase is absent for vf=100%.

Table 4: Frequencies for MEE plate for vf =100% of $BaTiO_3$

Mode	Case of free vibration				
	I	II	III	IV	V
1	5471.12	6016.26	6016.26	5471.12	6016.26
2	6562.53	6562.53	6562.53	6562.53	6562.53
3	7562.51	7993.51	7993.51	7562.51	7993.51
4	7993.51	8033.24	8033.24	7993.51	8033.24
5	10777.69	11242.50	11242.50	10777.69	11242.50
6	11242.50	11303.24	11303.24	11242.50	11303.24
7	12167.15	12521.33	12521.33	12167.15	12521.33
8	15185.08	15185.08	15185.08	15185.08	15185.08
9	15228.08	15713.64	15713.64	15228.08	15713.64
10	15347.17	15849.18	15849.18	15347.17	15849.18

Table-5 gives the frequencies for MEE plate for vf =60% of $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$ composite. This corresponds to the MEE composite plate made of 40% piezo-magnetic $CoFe_2O_4$ material and 60% of $BaTiO_3$. Apart from the increasing trend in frequencies the effect of magnetoelectric constant can be observed. The system frequency due to [K_{eq}] is higher than the conventional structural frequency f_{uu}, as some no difference in frequencies is observed between columns f_{keq} and $f_{keq\_reduced}$ is noticed. The $f_{\phi\phi}$ values are more than system frequency and $f_{\psi\psi}$ shows lower values than f_{keq}.

Table 5: Frequencies for MEE plate for vf =60% of $BaTiO_3$

Mode	Case of free vibration				
	I	II	III	IV	V
1	5624.92	5659.42	5674.55	5640.67	5662.54
2	6599.30	6599.30	6599.30	6599.30	6599.30
3	7545.08	7995.95	8026.56	7541.95	8029.41
4	8083.87	8083.87	8083.87	8083.87	8083.87
5	10781.54	10804.46	11434.74	10780.52	11434.74
6	11434.74	11434.74	11475.54	11434.74	11456.16
7	12629.08	13118.98	13100.61	12629.60	13098.50
8	15485.23	15485.23	15485.23	15485.23	15485.23
9	15634.03	16164.85	16152.75	15631.85	16155.21
10	16454.16	16643.84	16641.92	16434.34	16663.73

CONCLUSIONS

In this article, finite element procedure is adopted for the vibration of three-dimensional, anisotropic, simply supported magneto-electro-elastic plate. A series solution is assumed in the plane of the plate and finite element procedure is adopted across the thickness direction. The model is derived based on constitutive equation of magneto-electro-elastic material. Coupling between elasticity, electric and magnetic effects are included in the analysis. The effect of piezoelectric and piezomagnetic phases on frequency is computed. The magnetoelectric effect on system frequency is absent in pure $BaTiO_3$ or $CoFe_2O_4$ plates. The piezoelectric phase increases the frequency where as piezomagnetic phase decreases the system frequency.

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