

# VIBRATION ENERGY FLOW IN ASSEMBLED STRUCTURES

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#### Abstract

The work is concerned with energy transmission in plate-like structures, which undergo the different frequency point force excitation. The method for structural intensity computation is illustrated and formulas of structural intensity for plates and their relationship with internal force and strain are given in works [1, 2, 3, 4]. The formulas were used in the program algorithm for structural intensity calculations. The calculations were done with use the FE method to obtain harmonic response solution. Numerical examples of assembled structures are presented. The relationship between structural intensity and structural mode shapes as well as the changes of energy flow in plate for the excitation frequency change are discussed.

# **INTRODUCTION**

The structures of vehicles are subjected to external dynamic loading with various excitation frequencies, from slowly varying wave loads to high frequency interaction of engines, road and suspension induced forces. When the frequencies of the external forces are close to one of the natural frequencies of the structural components, the permissible vibration levels may be exceeded, which results in fatigue failure in the structure, or very high noise level inside. Since plates are most commonly used built-up structural elements in vehicles, the damage of plates or their connections will result in disintegration of overall system.

The quantity of the structural intensity is the power flow due to structural vibration per unit cross-sectional area in elastic medium. The interest on investigation of structural intensity arises for practical reasons. Structural intensity field indicates the magnitude and direction of vibrational energy flow at any point of a structure. Energy flow distribution offers information of energy transmission paths of mechanical energy. Dissipative elements, mechanical modification and active vibration control can be used for an alteration of energy flow paths within the structure. Of practical concerns are complex built-up structures, which can be successfully treated only by numerical computation when prediction of structural behavior in various operating conditions is needed. For these reasons, the investigation of energy flow paths in plate connections is important to the damage detection for vehicles.

The work presents formulations on structural intensity calculations. The method for structural intensity computation is illustrated and formulas of structural intensity for plates and their relationship with internal force and strain. The modelling and computation is done for one and two dimensional structures: plates and shells considered here as constructional elements. The numerical method of intensity evaluation was based on complex modal analysis and superposition of modes with use of finite elements method. There are presented results of the calculations which lead to the assessment of distribution of structural intensity (vector field) on the surface of simply supported rectangular steel plates connected by the different types of joints. The models included the source of vibrations (linear force excitation) and sink of energy in form of linear configuration of damping elements. The changes of finite elements grid density enabled detailed analysis of total vibration energy flow in analysed plates through the place of joint. Solved problem was intended to show the usability of structure surface intensity method in diagnostics of joints and role of stiffeners in mechanical constructions specially those typical for the vehicles as means of personal transport.

#### ASSESMENT OF ENERGY FLOW

For linear flexural vibration in thin plates, the total active structural intensity vectors consist of components due to shear waves, bending waves and twisting waves. Following the conventions of References [1, 2] the orthogonal components of structural intensity, in  $W/m^2$ , for thin-walled two dimensional structures may be written as:

$$\bar{I} = I_x \hat{i} + I_y \hat{j} \tag{1}$$

The components  $I_x$  and  $I_y$  are computed from the internal shears and moments, which for thin plates are proportional to the spatial derivatives of the transverse plate velocity [2, 3, 4]:

$$I_{x} = \frac{\langle Q_{x}\dot{w}\rangle_{t} + \langle M_{y}\dot{\theta}_{y}\rangle - \langle M_{xy}\dot{\theta}_{x}\rangle}{h}$$
(2)

$$I_{y} = \frac{\left\langle Q_{y} \dot{w} \right\rangle_{t} - \left\langle M_{y} \dot{\theta}_{x} \right\rangle + \left\langle M_{xy} \dot{\theta}_{y} \right\rangle}{h}$$
(3)

The spatial derivatives can be calculated analytically with the assumptions of plate theory or computed through finite differencing. Equations (2) and (3) are used to form formulas for structural intensity components [2, 3].

## **ANALYSED MODEL OF WELDED PLATES**

The main target of the analysis of the rectangular plate was the investigating the structural intensity fields for different frequency force excitation. In the calculations was applied elaborated own program for calculations of complex modal model, allowing the consideration of additional localised damping in the system.

#### Simply supported plate under force excitation

Each intensity component was computed at all points of the grid at several tens resonance frequencies of thin, rectangular plate, simply supported on two shorter sides. The plate dimensions were 1 m x 3.3 m x 0,01 m. It was composed of two plates connected at the shorter side (model of line welding). The length of composing plates are 1 m x 1,5 m x 0.01 m. The material of plates was the constructional steel. The material properties of steel were Young's modulus:  $E = 2.11 \times 10^{11}$  Pa, Poissons ratio: v = 0.3 and density:  $\rho = 7860 \text{ kg/m}^3$ . The plate was excited by perpendicular force. A point damping device was applied to the plate to absorb energy as an energy sink. The FE method model was arranged using the NASTRAN software. Model consisted of equal square shell elements of QUAD4 type. The excitation forces and damping elements were introduced to the model at the special point locations. The harmonic excitation force was attached to the plate in place indicated on Figures 1, 2, 3, 4 by the star. The amplitude of the excitation was set to  $10^3$  N. The frequency of excitation was changed in the range from 25 to 1000 Hz.

## **Results of calculations**

Figures 1, 2, 3, 4 and 5 show the predicted structural intensity patterns. Each plot contains a set of structural intensity values shown in form of vector field. The vector lengths are proportional to the magnitude of the intensity at the vector's tail. The red star shown in each plot in the left part of first consisting plate to indicates the location of the excitation. The red triangle in the right part of the second consisting plate indicates the location of applied damping element. Vectors are proportional to the intensity magnitude and shown in coloured scale (left side of plot). Each plot shows clearly energy flow from the excitation on left part of the plate toward the damper placed on the right part of the second plate. Beyond the damper positions the intensity vectors show lower magnitudes, indicating energy dissipation at the damper. Figure 4 shows the predicted intensity pattern which breaks into two distinct circulations: one on the lower part (counter clockwise circulation) and the one on the upper part of the left plate (clockwise circulation). The plots of vectors go to zero at the shorter plate edges due to the increased stiffness.

The calculation has been done for 100 first mode shapes. In cases of low number of mode shapes there were observed significant changes in distribution of vectors for the same density of net of elements. For the some frequencies there were observed abrupt changes of vectors distribution and what is more significant the maximum value of vectors magnitude.



Figure 1 – Structural intensity field for two simply supported welded rectangular plates. Excitation force frequency 5 Hz.



Figure 2 – Structural intensity field for two simply supported welded rectangular plates. Excitation force frequency 35 Hz.



*Figure 3 – Structural intensity field for two simply supported welded rectangular plates. Excitation force frequency 48 Hz.* 



*Figure 4 – Structural intensity field for two simply supported welded rectangular plates. Excitation force frequency 53 Hz.* 

The change in value of vectors magnitude was of 7 orders varying from 1 till 2,0x  $10^7$  W/m<sup>2</sup>. The results for higher excitation frequency show intensity fields with many vortices difficult to interpretation.

The magnitude of structural intensity vectors decease with increase of number of finite elements. This means that in energy flow analysis it is not sufficient to observe only the intensity vectors. The better measure of energy flow is the total energy flow through the closed area around the places of excitation and damping or through the whole width of the plate.

#### **SUMMARY**

The distribution of structural intensity vectors gives the qualitative characteristic of vibration energy transportation in mechanical systems. Introduction of an additional measure in form of integral of magnitude structural intensity vector component perpendicular to the certain closed surface enables the quantitative assessment of energy transfer paths and its balance in the structure.

The method of analysis of structural intensity distribution enables the investigation in the regions of high concentration of vibration energy flow which consequently is exposed to the risk of damage or is propagating the sound waves to the environment. It can be also considered as the identification of the regions for application of additional damping in purpose of lowering of vibration level and resulting noise radiation.

The structural intensity pattern can be used to identify power transfer paths as it presents a vectorial nature of vibrational energy flow in structures. Presented method of structural intensity vector calculation enables its evaluation for chosen frequency range and mode shapes [3]. Theoretical dependencies connect the structural intensity with linear and angular displacements, forces and moments exerted to the structures like plates and shells. The calculations are done with the application of complex modal parameters - modal analysis based on finite element method.

From the calculated results, we can find that despite the change of the excitation force frequency acting on the plate, the structural intensity fields can clearly indicate the source, the sink and the transmission of energy flow from source of excitation to the sink through plate. The patterns of structural intensity in plate will be changed with many factors, such as loading characteristics, mode shapes of coupled structures, number and geometry shape of stiffeners attached to the plate, and many others. All them are have the influence upon the field distribution and magnitude of structural intensity vectors. This means also that the nature of structural intensity is frequency dependent. It has been proved by the numerical experiment that it is very sensitive in the range close to the mode frequency. It can be observed the abrupt changes of vector field distribution but not similar in shape to the suitable mode shape.

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