

OPTIMISATION OF AN ELECTROMAGNETIC FEEDBACK FOR ACTIVE OR PASSIVE DAMPING AND ENERGY SCAVENGING

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Abstract

In the fields of active control, active or passive damping, energy scavenging, most of the applications are based on the use of electro-mechanical transducers. For many reasons, piezoelectric devices are very often chosen to insure the conversion between the mechanical and the electrical quantities. Nevertheless, electromagnetic devices can be in some cases of great interest. This study is focused on the use of a contact-less electromagnetic actuator for active or passive damping and energy scavenging purpose. Its great advantage compared to piezoelectric transducers is the absence of purely mechanical coupling. A reliable multiphysic modelling can thus easily be obtained and optimisation studies can be performed. The studied structure is an academic cantilever beam around its first bending mode. This mono-dimensional case allows analytical developments and some interesting results are found concerning the energy balance between the structure and the feedback loop. For example, it is shown that the optimal feedback for passive damping is different from the one for energy scavenging.

INTRODUCTION

To reduce the level of structural dynamic responses, a solution consists in increasing damping. This can be done either passively or by active strategies [5] [6]. Passive techniques include both the use of dissipative materials and the use of electromechanical converters connected to passive electrical circuits. In both cases, the energy dissipation is produced by Joule effect. The idea to store the energy instead of dissipating it has recently appeared. This approach called "energy scavenging" or "energy harvesting" is very promising as it could continuously supply the electrical power for small mobile devices. To insure the energy conversion,

piezoelectric materials are used in most of the recent developments [1] [3] [4] whereas electromagnetic transducers are more rarely studied [2]. This paper is focused on the use of a contact-less electromagnetic actuator for active or passive damping and energy scavenging. The main interest of this kind of actuator is that there is no purely mechanical coupling, leading to the possibility of obtaining a simple but reliable multi-physic modelling. Both Finite Element (FE) and analytical studies can thus easily be performed.

EXPERIMENTAL SETUP

The chosen structure (Fig. 1) is a simple cantilever beam and only the first bending mode is studied. The beam ($525 \times 50 \times 9.6 \text{ mm}$) is made of steel, strongly fixed at one end on a 15 tons seismic mass and free at the other end. Two similar instrumentations are mounted at points 1 and 2 respectively located at 155 and 335 mm of the fixed end. Each one of these instrumentations is constituted by a "homemade" electromagnetic actuator, a force cell and an accelerometer. The actuator consists of a permanent magnet fixed to the ground and a coil placed in the induction field of the magnet and linked to the structure by a very thin aluminium cone. The force cells are located between the actuators and the structure, whereas the accelerometers are located on the other side of the beam. The actuator located at point 1 is connected to a power amplifier in order to provide the external excitation, the one located at point 2 is connected to the feedback loop. A Hall effect probe is used to measure the driving current in the coils.



Figure 1 – The experimental setup

MODELLING

A multi-physic modelling of the system has been built. Its general scheme is given figure 2. The beam is represented by a 2 input 2 output Linear Time Invariant (LTI) system linking the normal forces f_1 , f_2 applied by the actuators at points 1, 2 to the transverse displacements x_1 , x_2 at the same points. The feedback loop is modelled by a mono-dimensional transfer function G(s) linking x_2 to f_2 .



Figure 2 – General modelling of the coupled system

The cantilever beam

The Finite Element (FE) model of the cantilever beam is made of classical Euler-Bernoulli elements. The masses of the actuators and sensors are represented by lumped masses (52 grams) at the corresponding nodes. The measured material density is 7850 kg.m⁻³. In order to determine the material Young's modulus, an experimental modal analysis of the four first bending modes has been performed, leading to the frequencies and damping indicated in table 1. The initial Young's modulus was updated to the value of 201 600 MPa in order to have the best prediction of the first modal frequency. Damping is assumed to be modal and the experimental damping factors are used for all the numerical simulations.

	Mode 1	Mode 2	Mode 3	Mode 4
Exp. Frequency (Hz)	28.09	174.2	478.7	942.0
Exp. Damping (%)	0.205	0.195	0.098	0.158
Num. Frequency (Hz)	28.11	174.1	477.6	970.6

Table 1 – Results of the experimental and numerical modal analysis of the beam

As the present study only concerns the first bending mode, a condensation of the FE model is then performed where only the first mode is considered. Finally, by considering only the two degrees of freedom associated to the transversal displacement at points 1 and 2, the three transfer functions of the beam LTI system are obtained:

$$\begin{cases} h_{11}(s) = \frac{a_{11}}{s^2 + 2\xi_1 \omega_1 s + \omega_1^2} + k_{11}^0 \\ h_{12}(s) = h_{21}(s) = \frac{a_{12}}{s^2 + 2\xi_1 \omega_1 s + \omega_1^2} + k_{12}^0 \\ h_{22}(s) = \frac{a_{22}}{s^2 + 2\xi_1 \omega_1 s + \omega_1^2} + k_{22}^0 \end{cases}$$
(1)

where the scalar numerators a_{pq} are related to the normalised real eigenmodes and the additive terms k_{pq}^0 are scalar residues which represent the static contribution of the out-of-band modes. The forced responses are then given by:

$$x_1 = h_{11} f_1 + h_{12} f_2 \qquad \qquad x_2 = h_{21} f_1 + h_{22} f_2 \tag{2}$$

The validity of the initial and condensed FE models is illustrated figure 3 where the experimental and simulated Frequency Response Functions (FRF) h_{11} are presented. In the considered frequency band, the difference between the initial and the condensed model is negligible and the difference between these models and the experimental FRF is acceptable.



Figure 3 – *FRF* h_{11} : *initial FEM* (•), *condensed FEM* (**O**), *experimental* (+)

The electromagnetic feedback

The electromagnetic actuator and the feedback loop are modelled by the circuit given Fig.4. R_b and L_b are respectively the resistance and inductance of the actuator, *Fem* is the electromotive force, *i* and *V* are respectively the electrical current and voltage in the loop, Z_f is the feedback impedance.



Figure 4 – Electric modelling of the actuator and feedback impedance

The governing equations (3) describe the electrical circuit (left side) and the

electro-mechanical coupling (middle) where C_e is the coupling coefficient of the actuator. The transfer function of the feedback loop is then easily obtained (right side). The physical parameters have been identified to the following values: $R_b = 0.70 \Omega$, $L_b = 0.070 \text{ mH}$, $C_e = 2.1 \text{ N.A}^{-1}$.

$$\begin{cases} V = -R_b i - L_b \frac{di}{dt} + Fem \\ V = Z_f i \end{cases} \begin{cases} f_2 = C_e \times i \\ Fem = C_e \times \frac{dx_2}{dt} \end{cases} \quad G(s) = \frac{C_e^2 s}{Z + R_b + L_b s} \end{cases}$$
(3)

The coupled system

Reporting (4) into (2) gives the output x_1 , x_2 of the closed loop system:

$$x_{1} = \left(h_{11} - G\frac{h_{12}^{2}}{1 + Gh_{22}}\right)f_{1} \qquad \qquad x_{2} = \frac{h_{12}}{1 + Gh_{22}}f_{1} \qquad (4)$$

QUANTIFYING ENERGIES

For both passive or active damping and energy scavenging applications, it is interesting to quantify the energies involved in the two actuators and their evolution versus the feedback impedance. Numerical simulations will first be presented, then experimental results will be exposed and finally a simplified analytic model will be used to justify the observed properties.

Numerical simulations

The previously described multi-physic modelling is used to predict the behaviour of the coupled system for different values of the feedback impedance Z_f . For the interpretations to be more convenient, only resistive impedance is studied here, i.e. Z_f is purely real, and the total resistive impedance $Z = Z_f + R_b$ is considered in the following developments. Let us consider a harmonic excitation force f_1 . The average mechanical energies E_1 , E_2 involved in the two actuators are given by:

$$E_1 = \frac{1}{2} \operatorname{Re}\left(\overline{s \, x_1} \, f_1\right) \qquad \qquad E_2 = \frac{1}{2} \operatorname{Re}\left(\overline{s \, x_2} \, f_2\right) \tag{5}$$

where the notation \overline{X} is used for the complex conjugate of X. Figure 5 shows the evolutions of these two energies versus the excitation frequency and the value of the resistive feedback. The energy in the feedback actuator E_2 exhibits two ridges which may be particularly interesting for an energy scavenging application. Figure 6-a represents the root locus of the closed loop system when decreasing the resistive

impedance Z from $+\infty$ to 0. The two ridges of figure 5 correspond to the two extremities of the root locus: point A is the open loop system ($Z = +\infty$) and corresponds to the cantilever beam without feedback; point E is the short circuit system (Z = 0) and represents a conservative system where the negative resistance Z_f would compensate the physical resistance of the coil R_b .



Figure 5 – Evolution of the energies in actuator 1 (a) and 2 (b) versus frequency and total feedback resistance



Figure 6 – Root locus of the closed loop system (a); maximal energies in the actuators $E_1(+)$ and $E_2(\bullet)$ versus total feedback resistance Z (b)

It is also interesting to observe the maximal value of the energies for each value of the resistance Z (figure 6-b). The two peaks $(B_2, D+2)$ of the energy E_2 are the best

points for energy scavenging. A minimum value of both E_1 and E_2 can be found in between those peaks at point C. It corresponds in fact to the higher damping which can be produced as illustrated on the root locus.

Experimental results

Only a part of the previous results could be verified experimentally as no negative resistance was used for Z_{f} . As a consequence, the minimum value of the total resistance Z was the resistance of the coil $R_b = 0.7 \Omega$. Nevertheless, the figure 7 shows a very good agreement between the numerical predictions and the experimental points.



Figure 7 – Maximal energies in the actuators versus total feedback resistance Z: simulated $E_1(\mathbf{+})$; simulated $E_2(\mathbf{\bullet})$; experimental $E_1(\mathbf{\Box})$; experimental $E2(\mathbf{O})$

Analytical study

To justify the previous observation with an analytical study, the model of the beam is once again simplified to a 1 degree of freedom oscillator:

$$(ms^{2} + cs + k)x = f_{1} - f_{2}$$
(6)

where the parameters m, c, k are chosen to fit the first bending mode of the beam. This simplification corresponds to the situation where the points 1 and 2 would be coincident, i.e. the two actuators would be at the same location.

Developing the closed loop transfer function and reporting it into (5) leads to the expressions of the average energies E_1 , E_2 . As it can be seen in figure 6, the frequencies of the two points *B* and *D* are very close to the open loop and short circuit frequencies respectively. Taking into account this approximation allows the calculation of the two total resistive impedances which maximise the energy E_2 (7) and lead to the very simple properties (8) where ξ_B and ξ_D are the modal damping of the closed loop system for the impedances Z_B and Z_D respectively.

$$\omega_B \approx \sqrt{\frac{k}{m}} \quad \Rightarrow \quad Z_B \approx \frac{C_e^2}{c} \quad ; \quad \omega_D \approx \sqrt{\frac{k}{m} + \frac{C_e^2}{mL_b}} \quad \Rightarrow \quad Z_D \approx \frac{c L_b}{m} \left(1 + \frac{k L_b}{C_e^2}\right) \tag{7}$$

$$E_2/E_1 \approx 1/2$$
 $\xi_B \approx \xi_D \approx 2\xi$ (8)

CONCLUSIONS

This paper deals with the optimisation of an electromagnetic feedback for passive damping and energy scavenging applications. Some interesting results are found: (i) for a given external excitation force, two values of the feedback impedance maximise the harvested energy; (ii) when applying these values, the damping of the closed loop system is approximately twice the structural (or open loop) damping and the harvested energy is approximately half of the excitation energy; (iii) the impedance value which leads to the highest damping for the closed loop system also leads to a very low harvested energy although the harvested energy is nearly equal to the excitation one.

These first results are demonstrated numerically, experimentally and analytically. Future works will include the active damping situation and the optimisation of more complex feedback impedances. In particular, nonlinear feedback used for energy harvesting [4] will be simulated, experimentally implemented and hopefully optimised.

REFERENCES

[1] Cavallier B., Berthelot P., Nouira H., Foltête E., Hirsinger L., Ballandras S., "Energy harvesting using vibrating structures excited by shock", IEEE International Ultrasonics Symposium, Rotterdam, The Netherlands (2005).

[2] Glynne-Jones P., Tudor M.J., Beeby S.P., White N.M., "An electromagnetic, vibration-powered generator for intelligent sensor system", Sensors and Actuators A, **110**, 344-349 (2004).

[3] Lefeuvre E., Badel A., Richard C., Petit L., Guyomar D., "A comparison between several vibration-powered piezoelectric generators for standalone systems", Sensors and Actuators A, **126**, Issue 2, 405-416 (2006).

[4] Lesieutre G.A., Ottman G.K., Hofmann H.F., "Damping as a result of piezoelectric energy harvesting", Journal of Sound and Vibration, **269**, 991-1001 (2004).

[5] Monnier P., Collet M., "Definition of the mechanical design parameters to optimize efficiency of integral force feedback active damping strategy", Journal of Structural Control, **12**, 65-89 (2005).

[6] A. Preumont, *Vibration Control of Active Structures: An Introduction*. (Kluwer, 2nd edition, 2002).