ICSV13 - Vienna

The Thirteenth International Congress on Sound and Vibration

Vienna, Austria, July 2-6, 2006

NUMERICAL SIMULATION OF ROTOR-STATOR INTERACTION NOISE GENERATION AND PROPAGATION IN AN ENGINE DUCT WITH SEGMENTED LINERS

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Abstract

By utilizing the acoustic mode matching between the source modes and the propagation modes, the theoretical prediction system of rotor-stator interaction noise generation and propagation and attenuation in a segmented annular duct is presented. That means the prediction of sound propagation and attenuation in the segmented ducts may no longer perfectly depend on the in-duct mode measurements, and the studies on the sound propagation and attenuation in ducts can be accomplished not only by modal measurements, but also by utilizing the source prediction to determine the source modes due to rotor-stator interaction. The numerical calculations show the effects of blade passing frequency (BPF), rotor blades/stator vanes number, axial distance between the rotor and stator rows on the in-duct sound attenuation and the resulting sound power levels. As for the given linings mounted in ducts on the fan/compressor facilities, there is an optimum operating condition when the in-duct sound attenuation due to acoustic treatments approaches the maximum. In addition, although increasing the axial distance between the rotor and stator will result in decrease of the in-duct generated noise, but the increase of the axial distance is often restricted by aerodynamic performance and weight of the engine. Perhaps in contrast to increasing the rotor-stator axial distance, choosing a proper combination of blade number and vane number in order to produce the cut-off condition for acoustic modes propagating in ducts would be more effective.

INTRODUCTION

No doubt, the in-duct sound field prediction system studies in aircraft engine can lay a theoretical basis for the practical engineering noise suppression design in an engine nacelle. As we know, one of main sources in turbofan engine locates on axial fan/compressor. In fact, noise generated by rotor-stator interaction is the main cause of discrete frequency noise emitted by fan/compressor. Although many efforts have been made in the theoretical prediction for rotor-stator interaction noise generation [1]-[4] [9], but for a long time, the prediction of rotor-stator interaction noise generation and the analysis of its propagation and attenuation in a duct with multi-segments has been investigated apart. Therefore, while sound propagation and attenuation in the segmented ducts is predicted by using the mode analysis method, the reliance on the in-duct acoustic mode measurements has existed to some extent. In order to fill this gap and omission of theoretical studies and to provide theoretical basis for the suppressing design of engine nacelle, based upon an exhaustive studies of the prediction model of rotor wakes-stator interaction noise[1][2][9][10] and the mode analysis model of sound propagation and attenuation in an annular duct with axial multi-treatments[6][7][11], this paper suggests the acoustical mode matching principle between the source modes and the propagation modes in an annular duct with multi-treatments, presents the prediction method for rotor-stator interaction noise generation and its propagation and attenuation in an annular duct with multi-treatments. This means that the prediction of sound propagation and attenuation in the segmented ducts may no longer completely depend on the in-duct mode measurements, and the investigation on the sound propagation and attenuation in ducts can be accomplished not only by acoustic mode measurement, but also by using the source prediction to determine the source modes excited by rotor-stator interaction, and in light of the mode analysis method, to investigate the sound field in an annular duct with linings. Therefore, the theoretical prediction for rotor-stator interaction noise generation and its propagation and attenuation in an annular duct with multi-treatments is developed by acoustic mode matching method between the source mode and propagation mode. By using this prediction method, the numerical calculations showing the effects of blade passage frequency, blade/vane numbers, rotor-stator axial spacing on in-duct sound attenuation and sound power level before and after ducts, are presented.

THEORETICAL MODEL AND MODE MATCHING METHODS

1. Rotor-stator interaction noise theoretical model

1.1 In-duct sound field model for fan/compressor

Supposing the duct is an infinite straight hard-wall duct and the coming flow is subsonic uniform flow, from Lighthill's equation, that considers the effects of moving medium on sound generation of fluid and propagation, only taking into account the influence of dipole source generated from the fluctuating forces exerted on the flow by the rotor, thus the solution of Lighthill's equation considering the effects of solid boundary can be obtained [5]. By simplifying, sound pressure emitted by fan or compressor in a straight annular duct containing a uniform flow is derived as follows.

$$p(x,t)=\frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_m(k_{rm,n}r)e^{im\varphi}}{\Gamma_{mn}}\int_{-\infty}^{\infty}\frac{e^{-i(\nu_{nm}^{\pm}x_1+\omega t)}}{k_{rn,m}}$$
$$\int_A \psi_m(k_{rm,n}r')e^{i(\nu_{nm}^{\pm}y_1-m\varphi')}\int_{-T}^{T}(\frac{m}{r'}\tilde{f}_D-\nu_{nm}^{\pm}\tilde{f}_T)e^{i(\omega-m\Omega)\tau}d\tau' dr' d\varphi' d\omega \qquad (1)$$

where $\Psi_m(k_{rm,n}r')=A_m J_m(k_{rm,n}r')+B_m Y_m(k_{rm,n}r')$

and
$$\begin{cases}\beta=\sqrt{1-M^2} & k_{rn,m}\equiv\sqrt{k_0^2-\beta^2 k_{rm,n}^2}\\ k_0=\omega/c & \nu_{nm}^{\pm}\equiv\dfrac{Mk_0}{\beta^2}\pm\dfrac{k_{rn,m}}{\beta^2}\\ f'_\alpha\equiv\dfrac{f_\alpha^{(1)}}{\left|n_1'^{(1)}\right|}+\dfrac{f_\alpha^{(2)}}{\left|n_1'^{(2)}\right|} & \alpha=T,D\end{cases}$$

1.2 Acoustic model for rotor-stator interaction noise

The effects of the viscosity of fluids and the thickness of blades are neglected. After taking the Fourier transformation of the Lighthill equation, and introducing the coordinates rotating with blades, $\varphi' \equiv\tilde{\varphi}-\Omega\tau$ and carrying out the integration with respect to τ, and identifying $\int_{-\infty}^{\infty}\Delta p(y',\tau)\exp(i\omega\tau)d\tau=\Delta\bar{p}(y',\omega)$, introducing an intrinsic chordwise coordinate z', z' varing from $z'=-b$ at the leading edge to $z'=+b$ at the trailing edge, thus, rotor-stator interaction noise can be determined.

$$\bar{p}(x,\omega)=\sum_m\sum_n\bar{p}_{mn}(\omega)\Psi_m(k_{rm,n}r)\exp(im\varphi-i\nu_{nm}^{\pm}x_1) \qquad (2)$$

$$\bar{p}_{mn}(\omega)=\frac{1}{2\Gamma k_{rn,m}}\int_{r_h}^{r_d}\Psi_m(k_{rm,n}r')(\frac{m}{r'}\cos\theta'+\nu_{nm}^{\pm}\sin\theta')\exp(i\nu_{nm}^{\pm}\delta_1'+\frac{m}{r'}\delta_2')$$
$$\int_{-b}^{+b}\left\{\sum_{E=0}^{V-1}\Delta\bar{p}_E(r',z',\omega)\exp\left(\frac{i2\pi mE}{V}\right)\right\}\exp\left[i(\nu_{nm}^{\pm}\cos\theta'-\frac{m}{r'}\sin\theta')z'\right]dz'dr' \qquad (3)$$

Before the prediction, the profile of rotor wake velocity and the unsteady forces on the stator vanes, which are induced by rotor wake, are also required to be determined.

(1) Rotor wake model

Based on the rotor wake model established by Kemp and Sears [4], and supposing that rotor wake is an inviscid shear perturbation of an otherwise uniform mean flow, Envia and Kerschen [3] used the Gaussian error profile to represent mathematical model of the rotor wake. Ignoring the effect of the lean of the wake, and considering the effects of the swept stator blades on the rotor wake-stator interaction noise, the disturbance component of the qth harmonic of rotor wake that is normal to stator vanes is given by

$$w_{X_2q}(X_1,X_2,X_3)=w_q\sin(\theta+\chi)e^{i\left[k_qz+\frac{2\pi EqB}{V}+qB\Omega t\right]} \qquad (4a)$$

$$w_q = w_c \sqrt{\frac{\pi}{\delta}} \frac{2l}{h\cos\chi} e^{-(qBl/2r\cos\chi)^2/\delta} \tag{4b}$$

$$k_q = \frac{qB}{r}(\sin\theta + \cos\theta\tan\chi) \tag{4c}$$

(2) Unsteady loading on blades

Assuming the blades are some plane cascades with zero camber, the pressure distribution on the rotor or stator blades can be calculated by means of 'strip-theory'[2] which divided the rotor/stator into a series of radial strip, i.e. thin plate linear cascades. From the linear Euler equations and it's corresponding boundary conditions, and associated with Eq.(4a)~Eq.(4c), yields

$$\begin{aligned} p_{mnq} = & \frac{\rho_0 U_r V}{2\Gamma k_{rn,mq}} \int_{r_h}^{r_d} \Psi(k_{rm,n} r') w_q(r') \sin(\theta+\chi)(\frac{m}{r'}\cos\theta + \nu_{nmq}^{\pm}\sin\theta) \\ & \exp(i\nu_{nmq}^{\pm}\delta_1' + \frac{m}{r'}\delta_2') \int_{-b}^{b} f_q(r',z) \exp\left[i(\nu_{nmq}^{\pm}\cos\theta - \frac{m}{r'}\sin\theta) z \right] dz dr' \end{aligned} \tag{5}$$

$$\bar{p}(r,\varphi,x,t) = \sum_m \sum_n p_{mnq} \Psi_m(k_{rm,n} r) \exp\left[i(m\varphi - \nu_{nm}^{\pm} x_1) \right] \cdot e^{-i\omega t} \tag{6}$$

The sound power of rotor-stator interaction noise is given by [9]

$$W = \frac{\pi(r_d^2 - r_h^2)}{\rho_0 U} \sum_m \sum_n \sum_q G_{mn}(qB\Omega) \left| p_{mnq} \right|^2 \tag{7}$$

where $G_{mn}(qB\Omega) = \dfrac{\mp M^2 \beta^4 (qB\Omega/U) k_{rn,m}(qB\Omega)}{[qB\Omega/c_0 \pm M k_{rn.m}(qB\Omega)]^2}$

2. Sound propagation in an annular duct with multi-treatments

The mode analysis method [7] [11] which developed from the acoustic theory [6] in a duct with multi-treatments is used to calculate the sound propagation and it's attenuation due to axially segmented acoustic treatment in an annular duct. The detailed introduction for mathematical model and formula derivation are shown in Ref.[11]. It requires to be illustrated that source modes in mode analysis for sound propagation in ducts can be determined by calculating procedure of rotor-stator interaction noise. Calculating the amplitude of each propagating mode, by summation of each mode, sound power flux for forward and backward traveling wave at each interface plane in ducts can be obtained. The energy at the first and the last planes in the segmented duct can be used to calculate the total attenuation due to the treatments in the duct.

3 Mode matching principle

3.1 Acoustical matching between source modes and propagation modes

The prediction model for rotor-stator interaction source can be obtained from Eq.(6) .

$$\begin{aligned} p(r,\varphi,x,t) &= \sum_m \sum_n p_{mnq} \Psi_m(k_{rm,n} r) e^{i(m\varphi - \nu_{nm}^{\pm} x_1)} \cdot e^{-i\omega t} \\ &= \sum_m \sum_n p_{mnq} A_m \left[J_m(k_{rm,n} r) + C_m' Y_m(k_{rm,n} r) \right] \exp(im\varphi - i\omega t) \cdot \exp(-i\nu_{nm}^{\pm} x_1) \end{aligned} \tag{8}$$

where: $\nu_{nm}^{\pm} = \dfrac{M k_0 \pm \sqrt{k_0{}^2 - (1-M^2) k_{rm,n}^2}}{1-M^2}$ (9)

$$p(r,\theta,z,t)=\sum_m\sum_n A_{mn}p_r(k_{rm,n}r)e^{im\theta}\cdot e^{i(k_{zmn}z-\omega t)}$$
$$=\sum_m\sum_n A_{mn}\left[J_m(k_{rm,n}r)+C_mY_m(k_{rm,n}r)\right]\exp(im\theta-i\omega t)\cdot\exp(ik_{zmn}z) \qquad (10)$$

and: $$k_{zmn}=\frac{-Mk\pm\sqrt{k^2-(1-M^2)k_{rm,n}^2}}{1-M^2} \qquad (11)$$

where: $k=k_0=\omega/c$, $\omega=qB\Omega$, $\theta=\varphi_0$

According to the singularity of the acoustic pressure, and since the solution of the source prediction has the same eigenfunctions as that of the sound propagation prediction, and if the generalized Fourier series expansions have a series of the same fundamental function, then they must be in a singular form of decomposition. The following relationships can be derived from a comparison between Eq.(8)(9) and Eq.(10)(11).

$$k_{zmn}=-\nu_{nm}^{\pm} \qquad (12)$$

$$\{A_{mn}\}=\{p_{mnq}A_m\} \qquad (13)$$

Thus, the source modal vector Q_S , used to predict the sound propagation and suppression due to multi-treatments in an annular duct, may be obtained from the theoretical prediction of rotor-stator interaction noise.

3.3 Mode matching procedures

First, utilizing the parameters such as rotor-stator spacing, inter-blade phase angle, chord of rotor and stator and rotor setting angle and airflow Mach number, frequency qBΩ as well as rotor wake velocity profile and etc., the total sound power flux due to the interaction of rotor blade wakes with stator vanes is calculated by Eq.(7). The sound power flux for each spinning mode of source is calculated by

$$W_{S,m}=\frac{\pi(r_d^2-r_h^2)}{\rho_0U}\sum_n\sum_q G_{mn}(qB\Omega)\left|p_{mnq}\right|^2 \qquad (14)$$

Second, the frequency $f=qB\Omega/2\pi$ and flow Mach number, spinning mode order m and radial mode order n determined by the source prediction program as well as the hub/tip radius ratio taken as the known parameters are input into the mode analysis program for the sound propagation in an annular duct with multi-treatments. According to the analysis mentioned above, for each spinning, setting the source mode vector$\{Q_S\}=\{p_{mnq}/A_m\}$, the in-duct sound power level suppression $\Delta L_{W,m}$ due to the segmented liners for each spinning mode is calculated by the mode analysis codes.

Finally, the total sound power level at termination in duct, L_{WT}, which has been attenuated by the treatments， is evaluated. Thus, the theoretical prediction procedures for rotor-stator interaction noise generation and its propagation in an annular duct with multi-treatments is presented.

RESULTS AND ANALYSIS

By using the source prediction model for rotor-stator interaction noise and the mode analysis method for sound propagation in an annular duct with multi-treatments and the geometric and aerodynamic performance parameters of NASA Rotor 11 and Stator 4 provided by Ref.[8][10], the rotor-stator interaction noise's generation and it's propagation and suppression in a three-segment annular duct with the treated segment placed between two hard wall segments in the presence of uniform flow are calculated. The characteristic design parameters for fan stage 11-4 are listed as below. Rotor inlet tip diameter 0.50m, stage total pressure ratio 1.574, rotor blade number 44, stator vane number 48, rotor inlet hub/tip radius ratio 0.5, rotor-stator tip spacing is 1.27 rotor chords, rotor rotating velocity 16100 rpm, rotor tip speed 424.9 m/s, rotor tip inlet relative Mach number 1.394, rotor chord(mid-span) 46.2 mm, stator chord(mid-span) 40.6 mm, rotor aspect ratio 2.5, stator aspect ratio 2.3, rotor tip solidity 1.298, stator tip solidity 1.433, inlet weight flow is 29.5 Kg/s, air density 1.225 kg/m^3. The other geometric and performance parameter of NASA rotor 11 and stator 4 see Ref. [8][10]. The parameters of three-segmented annular ducts are given as follows. The inner radius is 0.1270 m, the outer radius is 0.2540m. The length of 3 segments is 0.2483, 0.2286, 0.534 m, respectively. The first and end segments in three ducts are hard-wall, the middle segment is acoustically treated. The specific acoustic impedance of the treated segment is (0.51,-1.82). Here, the termination reflection is not taken into account.

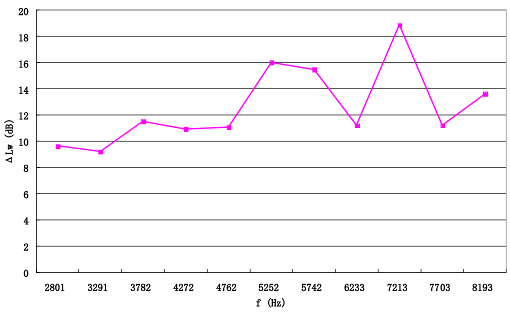

Figure.1 Effects of BPF on the in-duct sound attenuation (B=44, V=48, s/c=0.26)

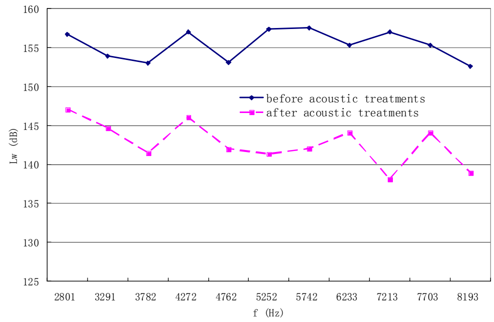

Figure.2 Effects of BPF on sound power level before and after acoustic treatments (B=44, V=48, s/c=0.26)

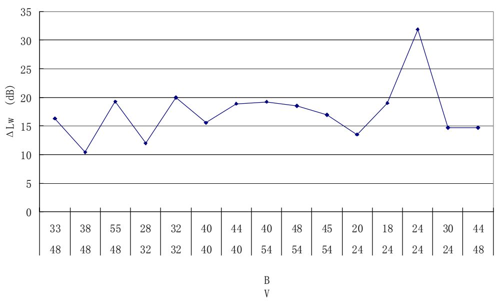

Figure.3 Effects of the combination of B and V on the in-duct sound attenuation

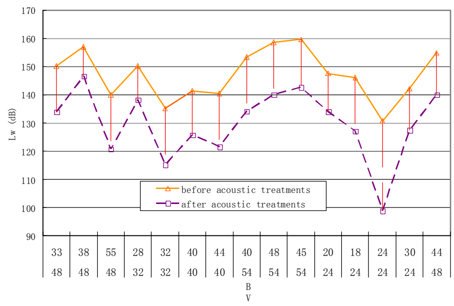

Figure.4 Effects of the combination of B and V on sound power level before and after acoustic treatments

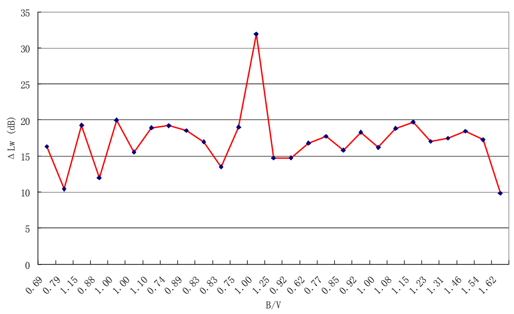

Figure.5 Effects of B/V on the in-duct sound attenuation

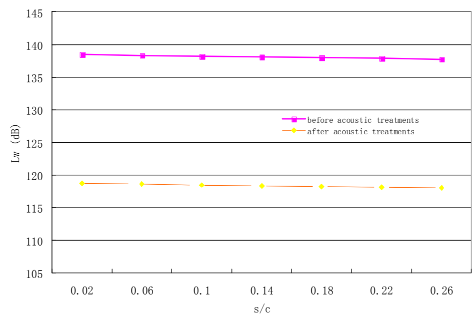

Figure.6 Effects of rotor-stator axial spacing s/c on sound power level before and after acoustic treatments

The effects of blade passing frequency BPF on the sound attenuation ΔLw due to acoustic liners shown in Figure1 and on the total sound power levels Lw before and after three-segmented ducts are estimated in Figure 2. It can be shown from Figure1 that the fluctuation of the sound attenuation appears with the changes of BPF, and the local peak values of sound attenuation do exist. For the given treatments, there is an optimum operating condition in which the in-duct sound suppression approaches to the maximum. The nominal design condition must not be an optimum operating condition. Figure 2 shows that BPF is one of the important factors, which affects the in-duct sound attenuation and noise levels. On the other hand, by changing fan speed, the suppressing performance of the in-duct liners with given structure in variable operating conditions can be determined. The effects of the different combination of blade/vane number B and V, and blade/vane number ratio B/V on the sound attenuation ΔLw due to in-duct acoustic treatments and on the total sound power level Lw before and after three-segmented ducts, as shown in Figure 3, Figure4 and Figure 5 respectively, are calculated numerically. It can be shown from Figure 3-Figure 5 that it is an important for reducing noise in a engine duct and increasing the in-duct sound attenuation, to avoid the mismatch between the blade number and vane number, to choose the pertinent blade number and vane number which will be helpful for producing the cut-off phenomena for acoustic modes. Moreover, Figure 6 shows that the effect of rotor-stator spacing s/c on the in-duct sound power level before and after three-segmented ducts. It has been shown that the noise levels before and after three-segment ducts decrease a little as the rotor-stator spacing increases within a range of (0.02 - 0.5)c. Here, it should be seen that although the increase of the axial spacing between rotor and stator and pertinent choice of rotor blade / stator vane number will result in the changes of the in-duct noise, but the increase of the axial spacing is restricted by aerodynamic performance such as the efficiency of fan/compressor stage, and the weight of the engine and etc. Perhaps in contrast with the increase of the rotor-stator axial spacing, choosing the pertinent blade/vane number in order to produce the cut-off phenomena for acoustic modes seems to be more effectively.

CONCLUSIONS

By utilizing the acoustic mode matching between the source modes and the propagation modes, the theoretical prediction method for rotor-stator interaction noise generation and its propagation and attenuation in an annular duct with multi-treatments is developed. The establishment of this method means that the prediction of sound propagation and attenuation in the segmented ducts may no longer completely depend on the in-duct mode measurements, and the investigation on the sound propagation and attenuation in ducts can be accomplished not only by acoustic mode measurement, but also by using the source prediction to determine the source modes excited by rotor-stator interaction, and in light of mode analysis method, to investigate the sound field in an annular duct with linings. The effects of blade passing frequency, the combination of blade number and vane number, axial spacing between rotor and stator on the in-duct sound attenuation and sound power level before and after ducts have been calculated by using this prediction method. By analyzing the results of the calculating cases, some theoretical basis and reference criteria are presented for the engineering design of the suppressing engine nacelle in aircraft.

ACKNOWLEDGEMENTS

The supports of the Natural Science Foundation of China (NSFC) and the Beijing Municipal Natural Science Foundation under grant number 10274092 and 3032003 respectively are gratefully acknowledged.

REFERENCES

[1] Kaji, S. and Okazaki, T., "Generation of sound by rotor-stator interaction", Journal of Sound and Vibration, **13,** 281～307 (1970).
[2] Ventres C.S., Theobald, M.A., Mark, M.O. "Turbofan noise generation", NASA CR-167952, (1983).
[3] Envia E. and Kerschen E.J. "Noise produced by the interaction of rotor wake with a swept stator blade", AIAA-84-2326 (1984).
[4] Kemp N. H. and Sears W. R., "The unsteady forces due to viscous waves in turbomachines", Journal of the Aeronautical Sciences, **22**, 478-483 (1955).
[5] Goldstein, M.E., *Aeroacoustics*, (McGraw-Hill International Book Company, 1976).
[6] Zorumski W.E., "Acoustic theory of axisymmetric multisection ducts", NASA TR-R-419, (1974).
[7] Syed A. A., Motsinger R.E., Fiske G.H. and Joshi M.C. Kraft R.E., "Turbofan aft duct suppressor study", NASA CR-175067 (1983)
[8] Kantola R.A., "Effects of vane/blade ratio and spacing on fan noise", AIAA-81-2033, (1981).
[9] Sun Xiaofeng, Hu Zongan, Zhou Sheng, "Noise prediction of rotor-stator interaction for fan/compressor", Acta Aeronautica et Astronautica Sinica, **10**(1), A41-A47 (1989).
[10] Kovich G., "Performance of transonic fan stage with weight flow per unit annular aera of 19.8 kilograms square meter", NASA X-2905 (1973).
[11] Lu Yadong, Hu Zongan, Sun Xiaofeng, Zhou Sheng, "Numerical simulation of sound propagation in an annular duct in the presence of uniform flow", Acta Aeronautica et Astronautica Sinica, **14**(8), 337-343 (1993).
[12] Lu Yadong, *Research on Aeroacoustics in Engine Duct*, Ph. D. Dissertation of Bejing University of Aeronautics and Astronautics (1993).