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COMPARISON OF DETERMINISTIC AND STOCHASTIC METHODS FOR ROBUST IDENTIFICATION OF A LIGHTLY DAMPED FLEXIBLE BEAM

Allahyar Montazeri*, Hadi Esmaeilsabzali, and Javad Poshtan

ANVC/FDI Lab, Electrical Engineering Faculty, Iran University of Science & Technology, Narmak, 16846, Tehran, Iran
allahyar.montazeri@gmail.com

Abstract
Robust identification of lightly damped flexible structures, because of high amplitude modes in resonance frequencies, is a challenging task. In this paper, two different methods for robust identification of a lightly damped flexible beam are applied and compared. The first method is called NSSE using stochastic assumptions on the uncertainties and the second one is ellipsoid set-membership method by deterministic assumptions on the noise and unmodelled dynamics. Identification results show that NSSE with integrated random walk process gives better model for the in-bandwidth modes and unmodelled dynamics in comparison with other specified methods. In fact the identified model will exhibit a good compromise between the performance and robust stability of the controller designed base on it and its uncertainty band.

INTRODUCTION

Robust control theory plays an important role in the application of control theory in practical problems. The main concept is to consider a physical system as an uncertain model which may be represented as a family of mathematical models. Using robust control techniques, all models in this family will be stabilized in an appropriate manner. This family is described by a nominal model and a bounded uncertainty. Thus it is customary to identify not only a nominal model, but also an uncertainty bound associated to this nominal model. Identification methods producing a nominal model and its associated uncertainty are known as "*Robust Identification*" or *"(Robust) Control-Oriented Identification*" methods. Because of the outspread use of robust control techniques in practical problems, robust identification is an area which has received a growing interest of researchers since beginning of 1990's due to the weakness of classical identification methods to produce suitable models for robust control theory. Robust identification algorithms use *a priori information* on system and its input-output data (posteriori information) to produce a nominal model and its associated uncertainty.

Two main philosophies for description of model's uncertainties have been used. The first one is based on statistical assumptions and produces so-called "soft bound" on model's uncertainty. Second approach is based on deterministic hypothesizes and gives "hard bound" on uncertainty. Indeed in this approach, uncertainties are assumed to be "Unknown but Bounded" (UBB) [1]. Deterministic hypothesis on model's uncertainties, leads to set membership identification methodologies.

In all system identification problems, perturbation are potentially arise form two main sources: a variance error due to the measurement noises and a bias term due to effect of unmodeled dynamics (dynamics that have not been included by nominal estimated model- also known as model error). The nature of these two error types is quite different. Variance error generally uncorrelated with the input signal (in open loop data collection case), but bias error is strongly depends on nominal model's structure and identification experiment input signal [1].

Three main approaches for robust identification have been addressed in the literature, namely:

1. Stochastic Embedding (SE)
2. Model Error Modeling (MEM)
3. Set Membership (SM)

SE is a frequency domain method based on statistical hypothesizes about uncertainties. This method potentially has the ability of handling both variance and bias errors but is mostly used for the aim of non-parametric uncertainties modeling [2, 25, 26, 27]. This approach to robust identification was first introduced by Goodwin in [25]. Later in [26] this method was modified by using maximum likelihood technique for the estimating of parameters. To alleviate the problems associated with the identification procedure in [26], in [2] unmodeled dynamics relevant uncertainties are represented by a non stationary stochastic process whose variance increases with frequency. This method which is known as "*Non-Stationary Stochastic Embedding*", has a high ability of capturing typical cases of non-parametric uncertainties, including systems with unmodeled lightly damped modes [2].

Finally SM is a time/frequency domain method, based on deterministic assumptions on system's perturbations. In fact uncertainties deem to be unknown but bounded by a suitable norm. In the first works the idea is used for state estimation [5, 6]. Later, SM theory is employed for the aim of system identification [7, 8]. Because of its deterministic framework, this approach to robust identification is more popular than SE and other statistical based approaches. Both parametric and non-parametric uncertainties can be accounted in SM identification problem. In [7], [8], [9] and [10] just parametric uncertainties are considered while [1], [4], [11], [12], [13], [14] and [15] deal with parametric and non-parametric uncertainties.

Fundamentally lightly damped flexible structures are distributed parameter systems and thus have infinite dimensional analytic models. In order to design a controller one has to have a finite dimensional model. Using truncated or reduced order model, "*spill over effect*" is a possible phenomenon. Spill over effect is called to the degradation of controller's performance due to excitation of unmodeled dynamics [16]. To fulfill this problem robust controller is a beneficial tool. So, robust identification of lightly damped flexible structures is an evident necessity. The following section describes briefly the robust identification algorithms used in simulations. In section three, the characteristics of the simply-supported beam and the results of simulation bases on three robust identification techniques are explained and finally conclusions come.

ROBUST IDENTIFICATION PROBLEM FORMULATION

Non-Stationary Stochastic Embedding Technique

Our approach in this section is similar to [2], which can be present as follow:
Suppose that the true system's frequency response is given as:

$$G(j\omega) = G_0(j\omega) + \Delta G(j\omega) \tag{1}$$

where $G_0(j\omega)$ is the nominal model that we want to estimate and $\Delta G(j\omega)$ is a stochastic process independent of nominal model whose variance increases with frequency and is stand for model errors. Let $\hat{G}_k$ to be the noisy observations of the true system at certain frequency:

$$\hat{G}_k = g(j\omega_k) + v_k \; ; \quad k = 1,2,..,m \tag{2}$$

where v_k is the measurement noise. One way to estimating the nominal model is to parameterize it using some orthonormal basis as [18,19]:

$$G_0(q) = \sum_{i=1}^{n} \theta_i b_i(q) = \mathrm{B}^T \theta \tag{3}$$

where $\mathrm{B} = [b_1 \, b_2 \, ... b_n]^T$ is the vector of basis functions and θ is the vector of parameters. It is also possible to use this structure to determine $\Delta G(j\omega)$. So (1) can be represented as follow:

$$G = \mathrm{B}^T \theta + \mathrm{B}^T \overline{\theta} \Lambda \tag{4}$$

where Λ is the (integrated) random walk process over frequency. Now by (2) and (3):

$$\hat{\mathrm{G}} = \mathrm{B}^T \theta + \mathrm{B}^T \overline{\theta} \Lambda + v_k \tag{5}$$

So the non-stationary stochastic embedding robust identification process cab be stated as follow:
1. Point-wise least square estimation of the transfer function at certain frequencies. The input u for this purpose must be sum of sinusoids. This step delivers a raw estimation of the real transfer function at certain frequencies which is considered

as $\hat{G}_k$. Additionally, statistical properties of the noise are calculated assuming Gaussian white noise.

2. Choice of basis functions **B**.

3. Estimation of the parameter θ and the (integrated) random walk process Λ in (5) based on the frequency function point estimation $\hat{G}_k$ according to following procedure:

a) Least square estimation of $\boldsymbol{\theta}$ based on frequency point estimation $\hat{G}_k$.

b) Using this estimate for model error parameterization as shown in (4).

c) Computation of an unbiased estimate of the variance of the (integrated) random walk process.

d) Quantification of the model error for any frequency (calculation of its statistical properties).

Set Membership Technique

Suppose that N samples of input-output data that have been generated by real system *G(q)* are available:

$$y_m(k) = G(q)u_m(k) + v(k) \tag{6}$$

where *v(k)* is the measurement noise and is bounded by a suitable norm:

$$\|v(k)\|_\beta \le \delta(k) \tag{7}$$

It is possible to represent the real system as follow:

$$G(q) = G(q,\theta) + \Delta G(q) \tag{8}$$

where $G(q,\theta)$ is the parameterized nominal model and $\Delta G(q)$ stands for possible unmodeled dynamics and is also bounded by suitable norm in the space of transfer functions. More details on driving bound of $\Delta G(q)$ can be found in [13]. For our identification problem we choose ∞-norm. Using this, the effect of the frequency response amplitude of unmodeled dynamics can be considered effectively.

Regarding (8), the input-output relationship (6) can be presented as:

$$y_m(k) = [G(q,\theta) + \Delta G(q)]u_m(k) + v(k) \tag{9}$$

$$y_m(k) - G(q,\theta)u_m(k) = \Delta G(q)u_m(k) + v(k) \tag{10}$$

As it has been addressed earlier, we choose *L*∞ and *H*∞ norms for noise and unmodeled dynamics respectively, so:

$$\|y_m(k) - G(q,\theta)u_m(k)\|_\infty = \|\Delta G(q)u_m(k) + v(k)\|_\infty \tag{11}$$

$$\left\| y_m(k) - G(q,\theta)u_m(k) \right\|_\infty \le \left\| \Delta G(q)u_m(k) \right\|_\infty + \left\| v(k) \right\|_\infty \tag{12}$$

$$\left\| y_m(k) - G(q,\theta)u_m(k) \right\|_\infty \le \left\| \Delta G(q) \right\|_\infty \left\| u(k) \right\|_\infty + \left\| v(k) \right\|_\infty \tag{13}$$

where $\left\| \Delta G(q) \right\|_\infty$ and $\left\| v(k) \right\|_\infty$ are nonparametric and parametric perturbation bounds respectively and come from a priori information on system to be identified. Let:

$$\left\| \Delta G(q) \right\|_\infty \le \gamma \,; \quad \left\| v(k) \right\|_\infty \le v_k \,; \quad \left\| u(k) \right\|_\infty = u_k$$
$$w_k = \gamma u_k + v_k$$

Thus (13) can be expressed as:

$$\left\| y_m(k) - G(q,\theta)u(k) \right\|_\infty \le w_k \tag{14}$$

Another way in determination of perturbation bound for set membership problem is to use a constant upper bound instead of variable bound. In order to do this, we can choose the maximum value of the variable perturbation bound over all N samples and consider it in (14) for all data samples. Although considering constant upper bound on system's perturbations increases the conservativeness of identification algorithm but it reduces the computational complexity of the algorithm.

Know we have to determine structure of $G(q,\theta)$ in order to complete the set membership inequality in (14).different model structures are available for nominal model [17]. Among them, output error (OE) structure is a popular model structure. To avoid high computational complexity due to nonlinear optimization in the process of parameter estimation and to obtain linear in model structure, we use the linear combination of orthonormal basis functions for OE model structure. This choice has an another advantage in the way that much more a priori information can be imported to the identification algorithm by proper choice of basis functions. In other words by selecting basis functions whose dynamics are close to the dynamics of the real system, it will be conceivable to estimate the nominal model by minimum number of parameters [18, 19]. Because of resonant nature of our system, we use so-called *"Kautz"* or two-parameter basis functions [20]:

$$G(q,\theta) = \sum_{i=1}^{n} \theta_i \psi_i(q) \tag{15}$$

$$\begin{aligned} \psi_{2k-1}(q,b,c) &= \frac{\sqrt{1-c^2}\,(q-b)}{q^2 + b(c-1)q - c}\left[\frac{-cq^2 + b(c-1)q + 1}{q^2 + b(c-1)q - c}\right]^{k-1} \\ \psi_{2k}(q,b,c) &= \frac{\sqrt{(1-c^2)(1-b^2)}}{q^2 + b(c-1)q - c}\left[\frac{-cq^2 + b(c-1)q + 1}{q^2 + b(c-1)q - c}\right]^{k-1} \\ -1 < b < 1 \quad & -1 < c < 1 \end{aligned} \tag{16}$$

where n is the order of nominal model and $\psi_i(q)$ is Kautz basis function. Now by (14) and (15):

$$\left\| y_m(k) - \sum_{i=1}^{n} \theta_i \psi_i(q) u_m(k) \right\|_\infty \le w_k \tag{17}$$

or equivalently:

$$\left\| y_m(k) - \theta^T \mathbf{x}_m(q,k) \right\|_\infty \le w_k \tag{18}$$

where $\mathbf{x}_m(q,k)$ is the regression(information) vector and computed as:

$$\mathbf{x}_m(q,k) = [\psi_1(q)u_m(k) \quad \psi_2(q)u_m(k) \ldots \psi_n(q)u_m(k)] \tag{19}$$

And $\theta = [\theta_1 \quad \theta_2 \ldots \theta_n]^T$ is the vector of parameters. For each time stamp (k=1, 2,…, N), (18) produces a so-called strip in the space of parameters. By intersecting these strips, "*Feasible Parameter Set*" (FPS) will be obtained as follow:

$$\Theta = \{\theta : \bigcap_{k=1}^{N} \left\| y_m(k) - \theta^T \mathbf{x}_m(q,k) \right\|_\infty \le w_k \} \tag{20}$$

In fact, Θ is the set of all parameters compatible with input-output data, a priori information on system and the uncertainty bounds. For the case that inequalities are linear in parameters, as (20), the FPS is a convex polytope in the space of nominal model's parameters. The aim of set membership robust identification problem is to compute the FPS and determine an optimal point in FPS (in some sense) as the nominal model's parameters. Exact computation of FPS and nominal model's parameters is a laborious task and requires high amount of numerical computations and is not conceivable in practical situations [21, 22]. An alternative is to outbound the FPS by simple geometrical shapes like "*Ellipsoid*" and "*Parallelotope*" and consider their center as the parameters of nominal model [7, 8, 9, 13].

SIMULATION RESULTS

This section presents the identification results for a lightly damped simply-supported flexible beam (fig. 2). The simply-support flexible beam which is considered in this work is assumed to be out of steal whose exact specifications are given in table1.
The identification experiment has been simulated using a "*Finite Element*" model of the beam. The input signal applied to this model is force and the output collected for identification purpose is displacement corrupted by a normally distributed Gaussian noise with SNR of 1% (fig. 3). The input signal is the combination of 180 sinusoidal

Table 1. The properties of the beam

Length	500 *mm*
Width	20 *mm*
Thickness	1 *mm*
Modulus of Young	2.07e+11
Density	7800 *Kg/m3*
Damping	5e-3

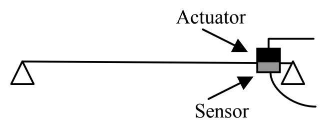

Figure 2 – The beam with location of sensor and actuator

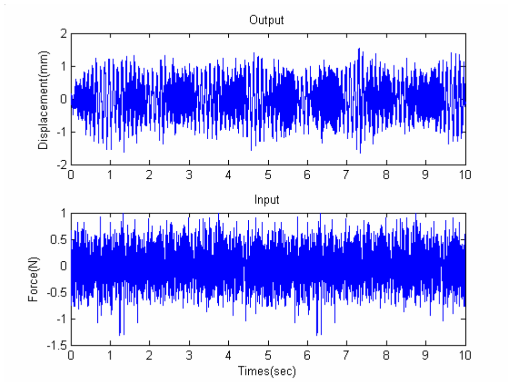

Figure 3 – Input and output data

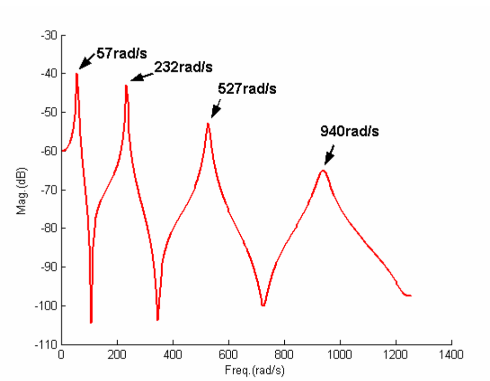

Figure 4 – FRF of the beam under study

with proper frequencies that have been picked according to the priori information of system which is in this case the FRF of the beam (fig. 4).

Distribution of these frequencies is a key point in the experiment design. Here, the first two modes are considered as the in-bandwidth modes which are aimed to be modelled and controlled and the last two modes are assumed as the uncertainty of the identified model. Because of lightly damped nature of the model and its considered uncertainties, having a good identification of in-bandwidth modes as well as including high amplitude modes uncertainties in the identified model is not a straight forward task.

For the identification of the first two modes, two continuous-time kautz basis functions are selected. The parameters of these bases are tuned based on the FRF of the beam. Fig. 5 shows the point estimation of the FRF of the beam used in NSSE algorithm. The estimated model and its uncertainty cloud using random walk process and with 99.99% confidence level are plotted in fig. 6. The same results for integrated random walk are shown in fig. 7. The second method used for robust identification of this lightly damped model is SM approach using non-stochastic but bounded assumptions on the amplitude of the noise and uncertainty of the model. The estimated model and its corresponding estimated uncertainty band are computed using ellipsoid set-membership method. In this case, two discreet-time kautz models are used to identify the first two modes of the beam. As it can be seen from fig. 6 and fig. 8, NSSE method with random walk process and SM methods deliver high amplitude uncertainty bands at the in-bandwidth modes while the band for uncertainty modes is tight. This identified model will reduce the performance of the controller for the first two modes. Otherwise, the identified model using NSSE with integrated random walk process shows tight fitting at the first two modes, while the amplitude of the third mode is not covered totally by the uncertainty band. In comparison with the previous identified model this model will result in better performance for the damping of the first two modes, while the stability of the controller is somewhat questionable because of uncertainties. However, since there is always a trade-off between stability and robustness of the controllers, the identified model in fig. 7 shows good compromise between these two goals.

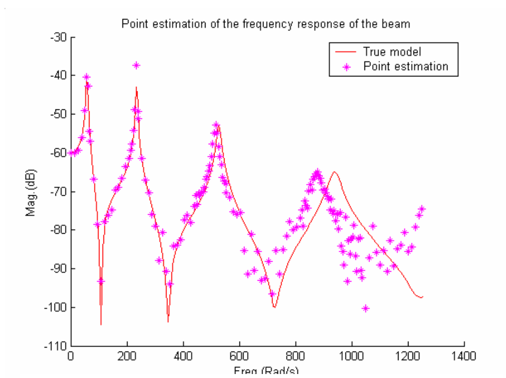

Figure 5 –Point estimation of the FRF of the beam

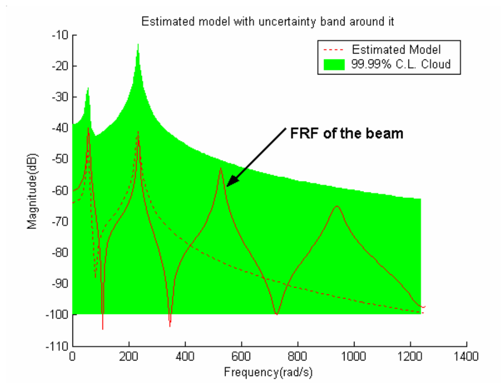

Figure 6 –Estimated model (dashed line) and its uncertainty band for random walk process (green cloud)

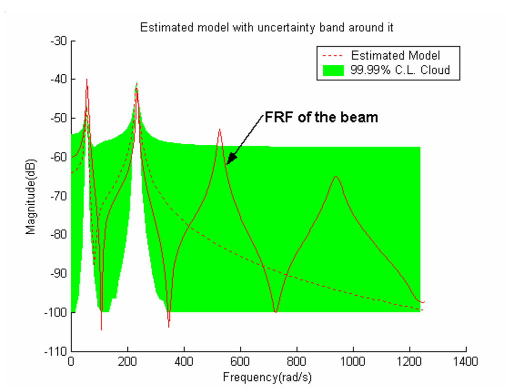

Figure 7 –Estimated model (dashed line) and its uncertainty band for integrated random walk process (green cloud)

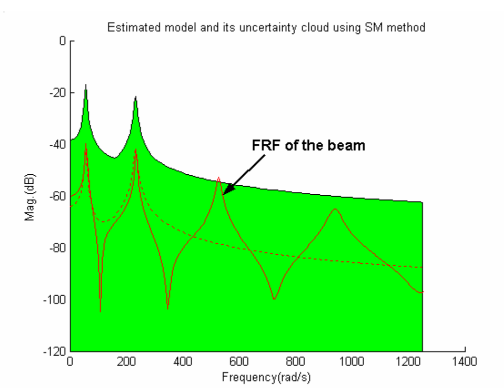

Figure 8 –Estimated model (dashed line) and its uncertainty band using ellipsoid SM method (green cloud)

CONCLUSIONS

Because of reducing the spill-over effect and increase the stability and performance of the robust controller used in active vibration control applications, robust identification of lightly damped flexible structures is an evident necessity. So here, the model of a lightly damped flexible beam with high amplitude modes as uncertainty was identified using two different robust identification approaches. The first one was NSSE with both random walk and integrated random walk processes and stochastic assumptions on uncertainties. The second approach was set-membership with ellipsoid out-bounding of uncertainties and deterministic assumptions. Identification results show that in comparison with other proposed methods, the model obtained by NSSE with integrated random walk process exhibits a better compromise between performance and robust stability of the identified model and so it is a good candidate for use in robust active vibration control algorithms.

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