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CRACK DETECTION IN BEAM-LIKE STRUCTURES USING A POWER SERIES TECHNIQUE AND ARTIFICIAL NEURAL NETWORKS

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Abstract

The analysis of experimentally measured frequencies as a criterion for crack detection has been extensively used in the last decades given its simplicity. However the inverse problem of the crack parameters (location and depth) determination is not straightforward. An efficient numerical technique is necessary to obtain significant results. Two approaches are herein presented: The solution of the inverse problem with a power series technique (PST) and the use of artificial neural networks (ANNs). The free vibration problem of a Bernoulli-Euler beam with an intermediate spring is stated and then solved with the PST. The first three flexural frequencies are measured and input in the algorithm. At this stage a numerical experiment on a 2D beam is employed. The crack depth is derived from a Mechanics of Fractures relationship. The ANNs technique is a different approach since it needs a training set of data. A single hidden layer back-propagation neural network is trained with data found with 2D finite element models with more than four hundred scenarios. These data are also analyzed and some curves are depicted to show the variables influence. The first methodology is very simple and straightforward though no optimization is included. It yields negligible errors in the location and very small ones in the depth values. When using one network for the detection of the two parameters the ANNs behave adequately. However better results are found when an ANN is used for each parameter. Finally a combination of the two techniques is proposed: The location found with the first technique and the depth with the second one.

INTRODUCTION

It is well known that a structural element shows changes in its behaviour due to the presence of a crack. The estimation of the crack parameters (location and depth) using the changes in the measured frequencies of a cracked member has been an extended criterion in the last years. One of the reasons is that frequencies are, among other dynamic parameters, easily obtained from measurements. So their experimental determination for a given cracked element is rather direct. However the inverse problem of crack parameters determination for a given set of frequencies, in a damaged element, is not as simple. So in order to obtain meaningful results an acceptable model and an efficient numerical technique have to be adopted.

Several researchers have tackled the problem with diverse techniques. Many works are available on crack detection in beams [2,5,6-8,12].

In the present work two approaches are explored. One is the solution of the inverse problem with a power series technique (PST) and the other is the use of artificial neural networks (ANNs).

The power series algorithm is a systematization of this well-known technique which results in an efficient numerical method appropriate for this inverse problem. The power series technique are a useful means to have an efficient numerical tool. The authors have solved several ordinary nonlinear problems using a similar approach [4]. Also boundary value problems were approached with power series [3]. Two structural elements were examined with PST, a BE beam and a spinning beam. In both cases the modeling of the crack is done by the introduction of springs of constant stiffness for each (crack depth)/(height of the section) ratio. The analytic model of the beam with springs is stated and the differential problem is then solved by an algorithm based on power series. Since the aim is the detection of the damage up to level 3 (its existance, depth and location), an inverse problem should be tackled. The power series algorithm is used and the natural frequencies measured in the damaged beam are input in it. The location and the value of the spring constant are obtained as a result. Some relationship from the Mechancis of Fracture theory [10] allows finding the related crack depth.

The second application refered to the crack detection of a damaged spinning beam ("rotor") was also performed though not presented here due to brevity. Similarly to the Bernoulli-Euler beam the inverse problem solution leads to the determination of the crack parameters and the results may be found in [11]. The methodology is shown with an illustrations: a cantilever beam with a vertical crack. A virtual experiment is performed with a finite element model of a damaged BE beam. A two-dimensional elasticity element is employed and the crack is simmulated as a notch. Finally the ANNs technique is explored. The ANNs technique is a different approach. It does not involve governing equations but it needs a training set of data. A single hidden layer back-propagation neural network is trained with data found with 2D finite element models with more than four hundred scenarios. These data are also analyzed and some curves are depicted to show the variables influence. Some preliminary results are commented. The obtained results are encouraging. At present other configurations of ANNs are being studied.

Bernoulli beam-spring vibrational problem

Governing equations

A cantilever cracked BE beam is considered. The crack influence is here simulated as a change in the flexibility at the crack location. A spring of stiffness constant k^* is then introduced. The beam has mass density ρ and Young's modulus E. For the sake of generality the beam is supposed to have two spans L_1 and L_2 of different cross sections F_1 and F_2 and second moments of inertia J_1 and J_2, respectively. The governing equations of the natural vibrations of the beam-spring systems, after non-dimensionalization, along with the boundary and continuity conditions are:

$$v_1'''' - \Omega_1^2 v_1 = 0; \;\; v_2'''' - \Omega_2^2 v_2 = 0 \tag{1}$$

$$\begin{gathered} v_1(0) = 0 \quad v_1'(0) = 0 \quad v_2''(1) = 0 \quad v_2'''(1) = 0 \quad v_1(1) = v_2(0) \\ \frac{EJ_1}{L_1} v_1''(1) + k\left[\frac{v_1'(1)}{L_1} - \frac{v_2'(0)}{L_2}\right] = 0 \end{gathered} \tag{2}$$

The following parameters have been introduced in order to obtain the nondimensionalized governing equations:$x_1 = \frac{X_1}{L_1},\ x_2 = \frac{X_2}{L_2},\ 0 \leq X_1 \leq L_1\ 0 \leq X_2 \leq L_2,\ 0 \leq x_1 \leq 1\ 0 \leq x_2 \leq 1\ \Omega_1^2 = \rho F_1 \omega^2 L_1^4 / EJ_1\ \Omega_2^2 = \rho F_2 \omega^2 L_2^4 / EJ_2\ k = k^* L_1 / EJ_1$ and where $v_1(x_1)$ and $v_2(x_2)$ are the *elastica* of each beam segment. The prime(s) in Eq. (1-2) denote derivative(s) with respect to x_1 or x_2 correspondingly.

Direct and inverse problem solution via a power series algorithm.

In the direct problem the spring constant and its location are input data and the natural frequencies the output. The power series is a very well-known technique and also straightforward. Its systematization yields an efficient method which is useful to solve the derived inverse problem. The authors have made use of this approach to solve strongly nonlinear problems [4]. Since the direct problem is governed by linear equations, the algebra is even simpler. The unknowns are the functions $v_1(x_1)$ and $v_2(x_2)$ which are expanded as follows:

$$v_1(x_1) = \sum_i^N A_i x_1^i; \qquad v_2(x_2) = \sum_i^N B_i x_2^i; \quad N \to \infty\text{(theoretically)} \tag{3}$$

After introducing this expansions in Eqs. (1-2) the next relationships derive

$$\begin{aligned} B_0 &= \sum A_i; \;\; \frac{k}{\alpha} B_1 = \sum \varphi_{2i} A_{i+2} + k \sum \varphi_{1i} A_{i+1}; \;\; 2\frac{\gamma}{\alpha^2} B_2 = \sum \varphi_{2i} A_{i+2} \\ 6\frac{\gamma}{\alpha^3} B_3 &= \sum \varphi_{3i} A_{i+3}; \qquad A_{i+4} = \Omega_1^2 \frac{A_i}{\varphi_{4i}}; \qquad B_{i+4} = \Omega_2^2 \frac{B_i}{\varphi_{4i}} \end{aligned} \tag{4}$$

where $\varphi_{lk} = (k+l)!/k!$ with $k,\ l$ integer number. Also $\alpha = L_1/L_2,\ \beta = L_1/(L_1+L_2),\ \gamma = EJ_2/(EJ_1)$. These are necessary equations to construct the solution algorithm. The possible input data are the spring constant k, the spring location β and the natural frequency parameters Ω's. Given two of them, the third may be obtained as an eigenvalue.

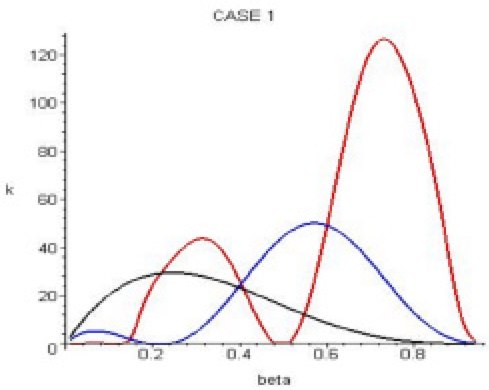

Figure 1: Curves k vs. β corresponding to the first three frequencies. $a = 1cm$.

In order to solve the inverse problem a measurement of the first three natural frequencies of the damaged beam is carried out. Each of this values is introduced as input in the power series algorithm. For each frequency a curve $\beta\ vs.\ k$ is obtained. The detected spring location and constant are given by the intersection of the three curves. The obtained value of β is proportional to L_1 and the value of k is related to the crack depth by some relationship from Fracture Mechanics [10].

Numerical results with PST: cracked BE.

In order to validate the proposed algorithm, a computational simulation of the cracked BE beam was carried out using a 2D finite element model. The crack was introduced as a notch. The standard package ALGOR [1] was employed for the analysis. The data for the damaged beam is the following: length $L = 100$ cm, rectangular cross-section of height $h = 5$ cm and width $b = 1$ cm, Young's Modulus $E = 2.1 \times 10^11$ N/m^2 and Poisson's ratio $\nu = 0.3$. The frequency parameter is $\Omega = \omega L^2\sqrt{\rho A/EJ}$. In order for the computational modeling of the structure and the parameters to be homogenized the *zero setting* correction is applied (see for instance Nandwana y Maiti [8]). Such correction is done on each value of frequency.

A cantilever beam is analyzed with three different values of the crack depth a, CASE 1 with $a = 1$ cm, CASE 2 with $a = 2$ cm, CASE 3 with $a = 3$ cm. In all the cases the 2D FEM was built for a cantilever beam with a notch 0.2 cm wide, located at $\beta = L_1/L = 0.4$. The first three natural frequencies are input in the power series algorithm, after the zero setting correction. Three curves k vs. β for each natural frequency are obtained. Figure 1 shows the three curves obtained for CASE 1. Curves for CASES 2 and 3 are similar with different vertical scales. The intersection point gives the detected values of the parameters β and k. The value of the crack depth a may be estimated from the relationship $k = Ebh^2/[72\pi f(r)]$(Ostachowics y Krawczuk [10]), $f(r) = 0.6384r^2 - 1.035r^3 + 3.7201r^4 - 5.1773r^5 + 7.553r^6 - 7.3324r^7 + 2.4909r^8$ $(r = a/h)$). Results and estimations for CASES 1, 2 and 3 are depicted in Table 1. In all

Table 1: Crack parameter estimates $\hat{\beta}$ and $\hat{a}$. Relative percent error is shown between parentheses.

CASE	Ω_1	Ω_2	Ω_3	k	Estimated values	
					$\hat{\beta}$	$\hat{a}$
1:0.2 x 1	3.4905	21.7876	61.2885	23.63	0.4 (0%)	0.906(−9.4%)
2:0.2 x 2	3.3980	20.9411	59.9480	4.87	0.402(0.5%)	1.901(−4.9%)
3:0.2 x 3	3.1526	19.1435	57.2844	1.41	0.4(0%)	3.111(3.7%)

cases the computational experiment of the cracked beam was performed with $\beta = 0.4$ and a notch 0.2 cm wide. The natural frequencies were obtained from this computational model (2D FEM beam).

ARTIFICIAL NEURAL NETWORKS APPROACH

The Artificial Neural network technique is a different approach [9]. It does not involve neither governing equations not an inverse problem but it needs a training set of data. A single hidden layer back-propagation neural network was trained with data found with 2D finite element models with more than four hundred scenarios. As an illustration of the data, Figure 3 shows the first four natural frequencies of a cantilever beam with a crack. The surfaces represent the variation of the value of the respective frequencies with the crack location and depth. A single hidden layer back-propagation neural network was trained with data found with 2D finite element models with more than four hundred scenarios. From the complete set a percentage of the data was separated randomly for the validation. Several variables were modified such as the number of neurons in the hidden layer, the learning rate, the number of samples for the validation. Figure **??** shows an example of a typical output of the algorithm of training and validation after 50000 epochs, with the first three natural frequencies as input and with three neurons in the hidden layer. It may be seen that the training and evaluation error are similar with no overfitting. It was also observed that input of the four frequency in the training improved the performance. However the handling of two output variables forces the ANN to accomplish an higher average error level. To overcome this situation the authors propose the finding of the crack location with the first technique PST and the crack depth with the ANNs. The authors have already performed physical experiments on a cracked cantilever beam, with length $L = 40, 50, 60$ cm, crack location at $20, 30$ cm far from the free end and crack depth of $1, 1.2, 1.48$. The corresponding crack width were $1.3, 1.4, 1.5$ mm. As an example Table 2 shows one of the experiments results. Case 1: nondamaged beam. $L = 40$cm, square section of 2.5 cm side, steel. Case 2: (crack location, crack depth, crack width) (20, 1, 1.3), Case 3: (30,1.48,1.5). Case 4: (20,1.2,1.4). The comparison of these results and the ANN output will be presented in the Congress.

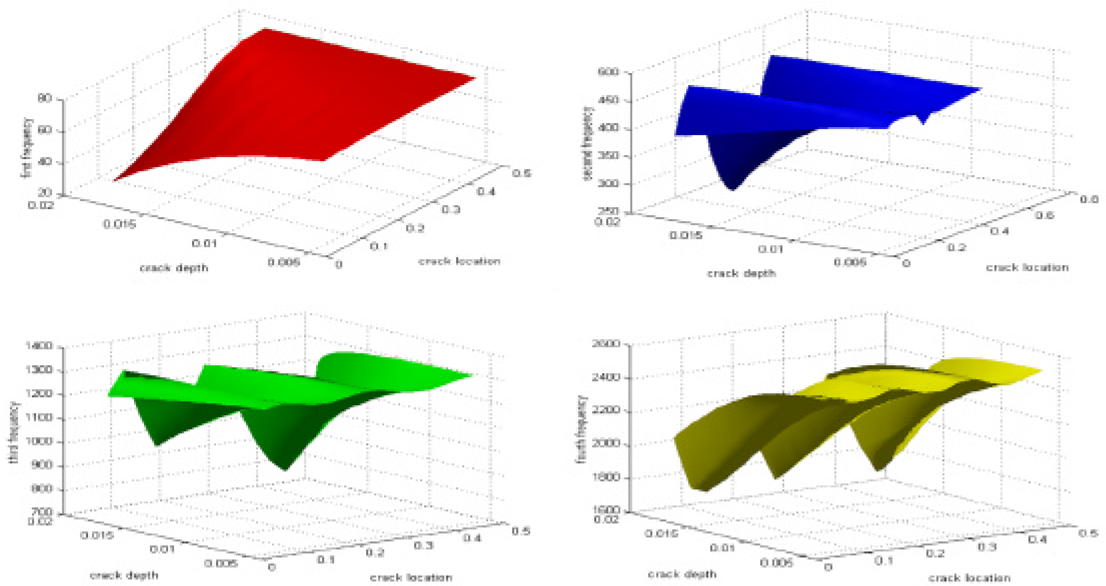

Figure 2: Influence of crack parameters on the first four natural frequencies of a cantilever cracked beam

Table 2: Physical experiment on cracked beam. $L = 40$ cm. Frequencies in Hz.

$Case$	$1st.freq.$ (m)	2nd. freq.	3rd. freq.	4th. freq.
1 Exp	110	645	1845	3437
1 2D	113	689	1866	3515
2 Exp	105	565	1830	3225
2 2D	110	609	1855	3390
3 Exp	85.5	610	1696	3325
3 2D	80	664	1498	3461
4 Exp	109	580	1825	3275
4 2D	107	566	1848	3388

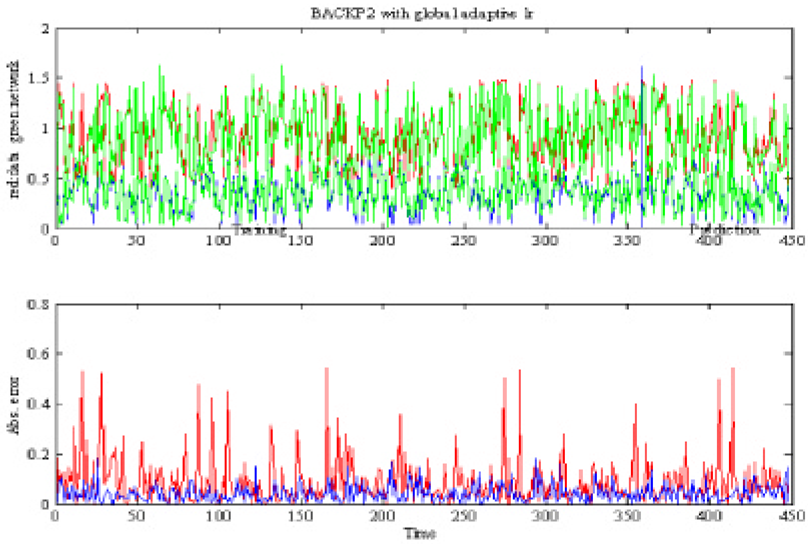

Figure 3: Typical ANN output. Training and validation. 50 neurons in one single hidden layer. Backpropagation algorithm in Matlab environment.

FINAL COMMENTS

A crack detection method for beam-type elements was presented. The detection criterion employed is that of the analysis of changes in the frequencies. The use of a power series algorithm provides of a straightforward and efficient numerical technique to solve the inverse problem. The crack is modeled by introducing springs to represent the stiffness diminution.

The results are excellent in the location value an with acceptable errors in the depth. It was observed that the width of the crack (in the axis direction) affects the accuracy of the depth resulting value. Also, as was expected, the angular velocity value does not affect the crack detection.

The ANN technique is in development and preliminary conclusions are herein presented. Also frequency values have been found through physical experiments and the comparison between these and the ANN output will be presented during the Congress.

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