

ON THE IIR INVERSE FILTER MERITS FOR THE EQUALIZATION OF LOUDSPEAKER NON-MINIMUM PHASE SYSTEMS

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Abstract

In a previous paper [1] the authors have proposed a novel approach for the equalization of non-minimum phase loudspeaker systems based on the design of Infinite Impulse Response (IIR) inverse filters. This approach allow inverse filter based equalization solutions with lower computational requirements and with lower delay of the equalized loudspeaker system than the most used one, the FIR filter, nevertheless with some more effort in the design process due to nonlinear nature of the problem. This advantage has been demonstrated with some examples with off-line simulated data.

In this paper further developments and new results are presented concerning the proposed equalization method. An experimental set-up has been developed for loudspeaker response measurement and equalization that uses as excitation signals Maximum Length Sequences (MLS). With this set-up, a 2-way loudspeaker system was equalized using FIR inverse filters and IIR inverse filters. An objective comparison between the real results of these two solutions was done using as criterions the time and frequency domain equalization errors and the delay of the equalized loudspeaker; the results of this comparison are presented and discussed.

INTRODUCTION

Even with careful construction commercial loudspeaker systems are characterized by linear and nonlinear distortions that can degrade and introduce "colour" in the reproduced sound. The compensation of the linear distortion by pre-processing the audio signal with the inverse model (inverse filter) of the loudspeaker is known as equalization. However, as loudspeaker systems are in general non-minimum phase systems the equalization task is not so easily attainable because there are only approximated inverse filters.

The idea behind equalization is to compensate the linear distortion effects observed in a discrete-time linear time invariant system H(z), with a new system F(z), such that F(z) = 1/H(z). The system F(z) is the inverse system - inverse filter - of H(z). In the time domain this is equivalent to the convolution operation $h(n) * f(n) = \delta(n)$ where the result of convolving the system's impulse response h(n) with the inverse system's impulse response f(n) is the unit impulse sequence $\delta(n)$.

For non-minimum phase linear time-invariant systems the exact inversion is in general impossible, resulting two types of systems: an unstable causal inverse system or a stable noncausal inverse system; the former one can be made causal by adding a sufficient delay, reason why this solution is usually named "delayed approximated inverse filter". The necessary delay for this causal solution depends on the proximity of the non-minimum phase zeros of H(z) to the unit circle, as closer to the unit circle longer will be the necessary delay for a causal stable inverse solution.

In a previous paper [1] the authors have proposed a novel approach for the equalization of non-minimum phase loudspeaker systems based on the design of Infinite Impulse Response (IIR) inverse filters. This approach allow inverse filter based equalization solutions with lower computational requirements and with lower delay of the equalized loudspeaker system than the most used one, the Finite Impulse Response (FIR) filter, nevertheless with some more effort in the design process due to nonlinear nature of the problem. This advantage has been demonstrated with some examples with off-line simulated data.

In this paper further developments and new results are presented concerning the proposed equalization method. An experimental set-up has been developed for loudspeaker response measurement and equalization that uses as excitation signals Maximum Length Sequences (MLS). With this set-up, a 2-way loudspeaker system was equalized using FIR inverse filters and IIR inverse filters. An objective comparison between the real results of these two solutions was done using as criterions the time and frequency domain equalization errors and the delay of the equalized loudspeaker; the results of this comparison are presented and discussed.

LEAST SQUARES INFINITE IMPULSE RESPONSE (IIR) INVERSE FILTER DESIGN

This novel IIR inverse filter design technique for the equalization of non-minimum phase loudspeaker systems was detailed and presented in a previous paper [1]. For this reason only a brief review is outlined in this section mainly to explain the results that will be presented in the next sections.

This technique for IIR filter design is based on the deterministic inverse modeling "Output Error" configuration presented in figure 1, where F(z)=B(z)/A(z) is an IIR filter with M zeros and N poles and Δ is the modeling delay [2].



Figure 1 – IIR inverse filter design

The inverse filter coefficients in this deterministic output error configuration are obtained minimizing the error e(n). Using a least-squares criterion for the minimization of the error sequence e(n)=y(n)-d(n), it can be stated as follows

$$\mathbf{J}(\mathbf{b}, \mathbf{a}) = \frac{1}{2} \sum_{n=0}^{L-1} \mathbf{e}^2(n) = \frac{1}{2} \sum_{n=0}^{L-1} \{\mathbf{y}(n) - \mathbf{d}(n)\}^2 = \mathbf{e}^{\mathrm{T}} \cdot \mathbf{e}$$
(1)

where $\mathbf{b} = [\mathbf{b}_0 \cdots \mathbf{b}_q \cdots \mathbf{b}_M]^T$ and $\mathbf{a} = [\mathbf{a}_0 \cdots \mathbf{a}_p \cdots \mathbf{a}_N]^T$ are the inverse filter vector coefficients with q=0,1...M and p=1,2...N.

The error function $J(\mathbf{b},\mathbf{a})$ is highly nonlinear in the inverse filter parameters (vectors **b** and **a**) that an analytical solution is generally not possible. As stated the design of an inverse filter for a non-minimum phase system requires the inclusion of a modeling delay, Δ , for which the error function $J(\mathbf{b},\mathbf{a},\Delta)$ appears also highly nonlinear.

The minimization of this error function, $J(\mathbf{b},\mathbf{a},\Delta)$, is done using an iterative search procedure like the Gauss-Newton method or the Levemberg-Marquardt method. The nonlinear least squares optimization routine *lsqnonlin* of Matlab [3,4] was chosen for this task and requires the Jacobian matrix of the error function as stated and defined in [1].

The algorithm proposed for a given number of zeros of the filter does an iterative search for the minimum of the error function for a defined interval of the number of poles and for a defined interval of the modeling delay; the mean value of loudspeaker's group delay is used as reference to establish the lower limit of the modeling delay interval used in the search of minimum error solutions. During the iterative optimization procedure when an unstable solution is reached, for a certain value of the modeling delay, it is made stable by pole reflection towards the inside of the unit circle.

RESULTS OF THE EQUALIZATION OF LOUDSPEAKER SYSTEMS

The validity of the proposed methodology for the equalization of loudspeaker systems using the proposed IIR filter based solution was checked using:

- loudspeaker's responses from a database ([5]), as presented in [1], extended with new equalization results;
- measured loudspeaker's responses obtained with a measuring system

developed by the authors.

This measuring system, developed with a 16 bit data acquisition card from National Instruments, uses as loudspeaker excitation signals Maximum Length Sequences (MLS) [6].

The criterions chosen for comparison purposes of the equalization results are: - the root mean square (rms) value of the error sequence (equation (1)) in time domain, defined as

$$e_{\rm rms} = \left(\frac{1}{L}\sum_{n=0}^{L-1} e^2(n)\right)^{1/2}$$
(2)

being L the length of the loudspeaker impulse response;

- the standard deviation of the modulus of Fourier transform of the equalized response from a constant level, E_{freg} , as a measure of the spectral distortion, defined as

$$E_{\text{freq}} = \left[\frac{1}{N_{\text{freq}}} \sum_{k=N\min}^{N\max} \left[20\log_{10} \left| H_{\text{equal}}(k) \right| - H_{\text{mean}}(k)\right]^2\right]^{1/2}$$
(3)

where $H_{equal}(k)$ is the Fourier transform with N points of the equalized loudspeaker's response, N_{min} and N_{max} are the frequency limits and N_{freq} the number of frequencies used in this frequency error evaluation; $H_{mean}(k)$ is the mean value of the modulus of the Fourier transform of the equalized response, given by

$$H_{mean}(k) = \frac{1}{N_{freq}} \sum_{k=N\min}^{N\max} 20 \log_{10} |H(k)|$$
(4)

Equalization results using a loudspeaker's response from a database

One of the loudspeaker's impulse response used in the objective evaluation of the loudspeaker equalization results using IIR inverse filters is from a small-sized two way vented-box with a 127 mm low-frequency element, a 14 mm dome tweeter and a passive crossover. This response was used in [1] and it is from a database ([5]) without access to the real loudspeaker system; however extensions to the reported equalization results and new ones were achieved. The application of the criterion expressed in equation (3) to this un-equalized measured frequency response points to a deviation of 2.8 dB.

The proposed technique was applied for the design of IIR inverse filters with 64 zeros, a number of poles between 12 to 64 and with delays, Δ , between 12 to 64 samples. The rms (in dB) of the time equalization error (equation (2)) is presented in figure 2, on the left, as function of the number of poles and the delay.



Figure 2 – Left: rms (in dB) of the time equalization error for IIR filters with 64 zeros, as function of the number of poles and the delay. Right: partial view with 61 to 63 poles and with delays from 18 to 64 samples

The error surface in figure 2 on the left is highly nonlinear; this is also suggested in the figure on the right where it is presented a partial view of this error surface for N from 61 to 63 poles with delays from 18 to 64 samples, where some minimums of the equalization error are attainable.

Some of these minimums of the equalization error allow the design of efficient IIR inverse filters, as suggested in figure 3 where are presented the equalization results of the application of an IIR inverse filter of order 64/62 with a delay of 64 samples.



Figure 3 – Left: a) and b) measured impulse response; c) and d) equalized impulse response ((b) and d) in log scale). Right: magnitude and unwrapped phase of the frequency response a) loudspeaker b) inverse filter and c) equalized loudspeaker

The IIR filters designed based on minimums of the error surface presented in figure 2 are summarized in table 1 regarding time and equalization errors and are compared with the equivalent FIR filters in terms of the number of coefficients of the filters; for example, an IIR of order 64/62 is equivalent to an FIR filter of length 127 (64+62+1); also in this table are presented the FIR filters requiring much more coefficients to achieve the same equalization error as the better IIR filters. The IIR

Table 1									
Filter type	Order/Length	Delay	Error (dB)	Frequency deviation (dB)					
		37	-42,51	0,88					
IIR	64/40	46	-44,73	0,78					
		78	-44,81	0,65					
	64/62	59	-45,27	0,64					
		64	-45,62	0,62					
	105	78	-42,98	0,83					
	127	82	-43,42	0,79					
FIR	164	80	-44,78	0,58					
	200	88	-45,60	0,48					

filter based equalization solutions allow lower equalization error than the FIR filter based ones with lower delay of the equalized system.

Real equalized loudspeaker's responses using a developed measuring system

A two-way loudspeaker system from Acoustic Research designed with a 1^{a} order crossover in the tweeter path, with a closed box of 27x44x20 cm, was used in this experimental evaluation.

The developed experimental set-up with a current power amplifier, working at a sampling rate of 72 kHz, was used in the measurement of this loudspeaker response at a distance of 1 meter from the microphone; in figure 4 on the left it is presented the time response and on the right it is presented the frequency domain response, where the crossing of the two drivers at approximately 6 kHz is notorious. The application of the criterion expressed in equation (3) to this un-equalized measured frequency response points to a deviation of 3.7 dB.



Figure 4 – Left: time domain response. Right: frequency domain response

The IIR inverse filter design technique was applied in the design of filters with 64 zeros, with 24 to 64 poles and with delays between 14 to 88 samples. The rms (in dB) of the time equalization error (equation (2)) is presented in figure 5, on the left, as function of the number of poles and the delay; a partial view of this error surface for N from 59 to 61 and Δ from 14 to 88 samples is also presented in figure 5, on the right.



Figure 5 – Left: rms (in dB) of the time equalization error for IIR filters with 64 zeros, as function of the number of poles and the delay. Right: partial view with 59 to 61 poles and with delays from 14 to 88 samples

In a similar way to the previous evaluation presented, the minimums of the error surface presented in figure 5 (on the left) are summarized in table 2 regarding time and equalization errors and are compared with the equivalent FIR filters in terms of the number of coefficients of the filters. As in the previous comparison presented, IIR filter based equalization solutions allow lower equalization error than the FIR filter based ones.

Filter type	Order/Length	Atraso	Error (dB)	Frequency deviation (dB)			
IIR	64/60	75	-46,34	0,46			
	64/62	48	-44,20	0,88			
		70	-46,17	0,42			
		77	-46,21	0,46			
	64/64	69	-46,27	0,42			
FIR	125	71	-44,68	0,48			
	127	73	-44,81	0,49			
	129	75	-44,94	0,48			

Table 2

Using table 2, for example a minimum error based equalization solution is possible using an IIR inverse filter with 64 zeros and 60 poles with a delay of 75 samples, as it is also suggested in the partial view presented in figure 5. This inverse filter was applied in the pre-processing of the MLS signal to be used in the measurement of the loudspeaker equalized response with the measuring system.

In figure 6 are presented the results of the application of this filter for real loudspeaker equalization; on the left the impulse response reveals an equalized loudspeaker with a delay of 78 samples (after the 204 samples related the distance to the microphone), 3 more samples than in the IIR off-line filter design process due to delays on the measurement chain; also note that the equalized impulse is not a perfect impulse due to the frequency response of the antialiasing filters of the measurement chain; in this figure on the right, the equalized frequency response – almost a perfect flat one - is easily seen when compared with the un-equalized one.



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CONCLUSIONS

In this paper further developments and new results are presented concerning a novel proposed method for the equalization of non-minimum phase loudspeaker systems, based on IIR inverse filters design in the time domain using a least squares criterion.

For real equalization of loudspeaker systems an experimental set-up has been developed for electro-acoustic response measurement that uses as excitation signals Maximum Length Sequences (MLS). With this set-up, a 2-way loudspeaker system was equalized using IIR inverse filters and FIR inverse filters.

An objective comparison between the real results of these two solutions was done using as criterions the time domain and frequency domain equalization errors and the delay of the equalized loudspeaker; the results of this comparison reveal the merits – lower equalization error and lower delay of the equalized system – of the IIR filter based solution.

The results of preliminary listening tests with real equalized loudspeaker systems also reinforce the good behave of the proposed equalization methodology.

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