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CRACK DETECTION IN STRUCTURES, USING PROPER ORTHOGONAL DECOMPOSITION AND MORPHOLOGICAL PROCESSING

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Abstract

Vibration analysis based damage identification in structures has been a subject of intensive research. As a result, a plethora of analytical, numerical and experimental techniques have been developed, involving frequency or mode-shape based methods, wavelet analysis, or higher order statistical signal processing. Parallel, a number of pattern recognition and classification approaches have been used, such as neural networks, either as standalone procedures, or in combination to the above techniques.

In this paper, a method for crack detection in structures is presented. As a first step, using the Proper Orthogonal Decomposition (POD) method, a number of Proper Orthogonal Modes (POMs) of a crack cantilever beam are estimated. Proper Orthogonal Decomposition, closely related to Principal Component Analysis, is a powerful and elegant method of data analysis aimed at obtaining low-dimensional approximate descriptions of high-dimensional processes and eliminating noise effects.

Then, the POMs are further processed, using basic morphological operators (dilation, erosion, opening, closing). The basic concept of morphological processing is to modify the shape of an object, equivalently considered as a set, by transforming it through its interaction with another object, called the structuring element. Using appropriate morphological processing, the position of the crack can be estimated by isolating the local sudden change effect of the crack on the space variation of each POM.

Cantilever beams, containing a transverse surface opening-closing crack at different lengths and of different depths are considered and the influence of the type of the structuring element and of the noise level to the accuracy of the results is examined.

INTRODUCTION

In recent years, structural damage detection and health monitoring are subjects of intensive research due to their practical significance. For structures like bridges, offshore platforms, dams, transmission towers, and aircraft, early detection of damage is essential since propagation of defects might lead to a catastrophic failure. Detection of defects, as well as defining their position and size, might be obtained by vibration measurements, extracting from the data the information on natural frequencies, damping factors or mode shapes.

The main reason for the popularity of natural frequencies as damage indicators is that natural frequencies are rather easy to be determined with a high degree of accuracy. Problems exist however, when the size of the crack is small. To overcome the aforementioned difficulties related to natural frequencies, many research studies have been focused on utilizing changes in mode shapes. The idea of using mode shapes as crack identification tool is the fact that the presence of a crack causes changes in the modal characteristics. Recently, an interesting comparison between a frequency-based and a mode shape-based method for damage identification in beam like structures has been published [3]. The advantage of using mode shapes is the fact that their changes are much more sensitive, compared to changes in natural frequencies. Using mode shapes, however, has some drawbacks. The presence of damage may not significantly influence the mode shapes of the lower modes, which are usually measured [3]. In the last few years wavelet analysis has become a promising damage detection tool, due to the fact that it is very accurate to detect localized abnormalities in a mode shape caused by the presence of a crack. Different wavelets have been used by many authors trying to detect cracks [1, 5].

In the present work, a method for crack identification in beam structures based on Proper Orthogonal Decomposition and morphological analysis is presented. The proper orthogonal modes (POMs) of a cracked beam are processed using morphological analysis in order to overcome the difficulties related to modes and the location of the crack is estimated by the variation of the spatial morphological index at the site of the crack. The influence of noise has been also investigated.

PROPER ORTHOGONAL DECOMPOSITION

The method of Proper Orthogonal Decomposition (POD) or Karhunen-Loeve Decomposition (K-L-D) is a means of extracting spatial information from a set of time-series data available on a domain [2]. In acoustical and random signal decomposition, the Proper Orthogonal Decomposition has been widely used to ascertain the modes and the energy of the signals under consideration. This is very important in applications that involve compression and storage of stochastic signals. The use of Karhunen-Loeve (K-L) transform is of great value in non-linear settings, where traditional linear techniques, such as modal testing and power spectrum analyses cannot be applied. This is especially the case for non-linear engineering structures. The POD can be used to obtain low-dimensional dynamic models of distributed parameter systems, by computing orthogonal eigenfunctions, resulting form the post-processing of experimental or numerical data of the system response. These eigenfunctions are optimal in the sense that fewer K-L modes are needed to account for the same amount of vibrational energy, compared to modes resulting from application of standard Galerkin or Rayleigh-Ritz procedures. This technique can conveniently treat non-linear distributed parameter systems defined on irregular domains to yield discretized systems with only a few degrees of freedom, which can then be used to reconstruct the dynamic response.

The Karhunen-Loeve analysis has additional distinctive advantages. The modes obtained from the K-L decomposition for a certain set of system parameters can, in most cases, be used to reconstruct the response of a system whose parameters are slightly different than the original system. Also, the Principal Orthogonal Modes (POMs) would at least serve as an orthogonal basis in the absence of a better basis. In the area of vibrations of structures, the Karhunen-Loeve modes can be used to solve the non-linear governing equations accurately. This method has been used successfully in the field of fluid dynamics. The key advantage of this method lies in the fact that it can be applied, not only to Hamiltonian systems, but also to dissipative ones, and provides information about the spatial structure of the dynamics as well as the energy contained in them. Hence, this method could be a valuable tool in the analysis and system identification of the dynamics of engineering structures.

MORPHOLOGICAL SIGNAL PROCESSING

Morphological signal processing [4] comprises a broad collection of theoretical concepts and mathematical tools for signal analysis, non-linear signal operators, design methodologies and application systems that are related to mathematical morphology (MM). Morphological signal processing was first used to analyse binary image data and was extended to grey-level images. Since its launching, it has grown rapidly, especially in the last 15 years. This growth is a consequence of the great variety of applications which can be treated by MM. The traditional tools of linear systems and Fourier analysis are of limited use for solving geometry-based problems because they do not directly address the issues of how to quantify the shape and the size of the signals. Contrarily, morphological signal processing is perfectly able to quantify all aspects, related to the geometrical structure of the signals. The basic concept of morphological signal processing is to modify the shape of a signal, equivalently considered as a set, by transforming it through its interaction with another object, called the structuring element.

A number of non-linear set processing transformations can be used in MM, all of which are based on two basic set operations: the Minkowski set addition and the Minkowski set subtraction. The two basic morphological operations, the dilation and the erosion can be defined as:

$$dil(A,B) = A \oplus B^r = \left\{ x, B_x \cap A \neq \varnothing \right\} = \bigcup_{b \in B} A_{-b} \tag{1}$$

$$er(A\,,B) = A \ominus B^r = \{x, B_x \subseteq A\} = \bigcap_{b \in B} A_{-b} \tag{2}$$

where $\subseteq$ denotes the set inclusion and B^r denotes the set obtained by reflecting the set of B with respect to the origin. Based on dilation and erosion, two other basic morphological operations, the closing and the opening can be further defined:

$$cl(A, B) = A \bullet B = (A \oplus B^r) \ominus B \tag{3}$$

$$op(f\,,g) = A \circ B = (A \ominus B^r) \oplus B \tag{4}$$

Since signals can be linked to sets, morphological filtering proceeds to the modification of their geometrical characteristics by morphologically processing the signal with another signal or function, called the structuring element, which in practice is compact and of a simple shape. Thus a morphological filter is a set mapping operation, which must satisfy four basic requirements: It must be: (1) translation invariant, (2) scale invariant, (3) dependent only on the local features of the signal and (4) Continuous with some possible positive jumps (upper semi-continuous). Using the definitions for Minkowski addition and subtraction between functions, the four basic morphological operations are defined as:

$$dil(f\,,g) = (f \oplus g^r)(x) = f(x) \oplus g(-x) = \sup_{y \in D}\{f(y) + g(y-x)\} \tag{5}$$

$$er(f\,,g) = (f \ominus g^r)(x) = f(x) \ominus g(-x) = \inf_{y \in D}\{f(y) - g(y-x)\} \tag{6}$$

$$cl(f\,,g) = (f \bullet g)(x) = [(f \oplus g^r) \ominus g](x) \tag{7}$$

$$op(f\,,g) = (f \circ g)(x) = [(f \ominus g^r) \oplus g](x) \tag{8}$$

where f(x) is the function under consideration, g^r(x) denotes the reflected (symmetric) function of g(x) with respect to the origin of the x-axis. All the above four operations are translation invariant. Moreover, opening and dilation are extensive, while erosion and dilation are non-extensive operations. Opening and closing are also idempotent operations.

SIMULATIONS ON A CANTILEVER BEAM

Vibration model of a breathing cracked cantilever beam

A cantilever beam of length *l*, of uniform rectangular cross-section *wxw* with a breathing crack located at *lc* is considered. The crack has uniform depth *a*. The beam is excited by a lateral impact causing the crack to open and close. It has been shown that a crack in a structure such a beam may cause the structure to exhibit non-linear behaviour if the crack opens and closes ("breathes") during the vibration. However, in the most commonly used models up to now, it has been assumed that the crack is fully open during vibration. When a crack opens and closes during vibration, contact phenomena must be taken into account. Thus in the present paper, a Plexiglas cantilever beam of a total length *l*=300mm and of a rectangular cross-section 20 x 20 mm^2 is considered. The beams were modelled using the ANSYS finite-element method and Gaussian white noise was added externally to simulate field conditions. A crack of relative depths *a*=10%, 20%, 30%, 40% and 60% is introduced each time at distances of *lc*=60 mm, *lc*=150 mm, *lc*=240 mm from the clamped end. The displacement time records were obtained with a space resolution of 1 mm, resulting in a number of 301 points available.

Determination of crack location and size

As it was noted before, changes in mode shapes are much more sensitive compared to changes in natural frequencies. However, the presence of damage may not significantly influence mode shapes of the lower modes usually measured. In order to overcome these difficulties, the first 3 Proper Orthogonal Modes were extracted from the displacement data, using Proper Orthogonal Decomposition. It must be emphasized here that using POD it is possible to extract a number of POMs equal to the number of the measurement points.

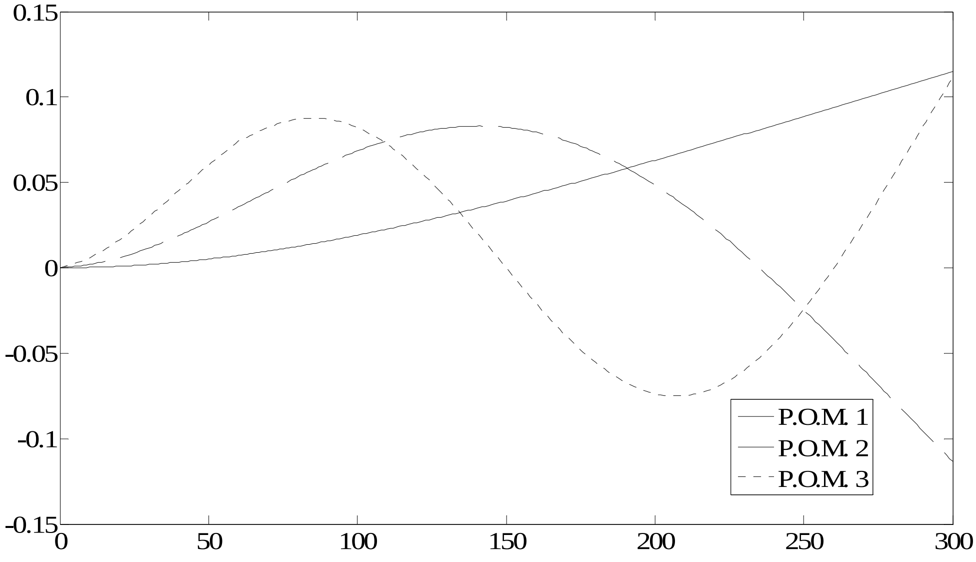

Figure 1 – Principal Orthogonal Modes with crack location at 60 mm, crack depth 20% and SNR=0%

Afterwards, Morphological Erosion was applied on the 3 first POMs in order to determine the location and the size of the crack. A spline is used as the structuring element. This spline is created using the corresponding POM of an uncracked beam. The length of the structuring element was selected equal to 1/60 times the length of the beam. The resulting morphological indexes are shown in Figure 2. A peak corresponding to the location of the crack is clearly observed, even when the crack is at the first stages (relative depth 10%) The method gives very good results even when the crack is located away from the clamped end at *lc*=240mm.

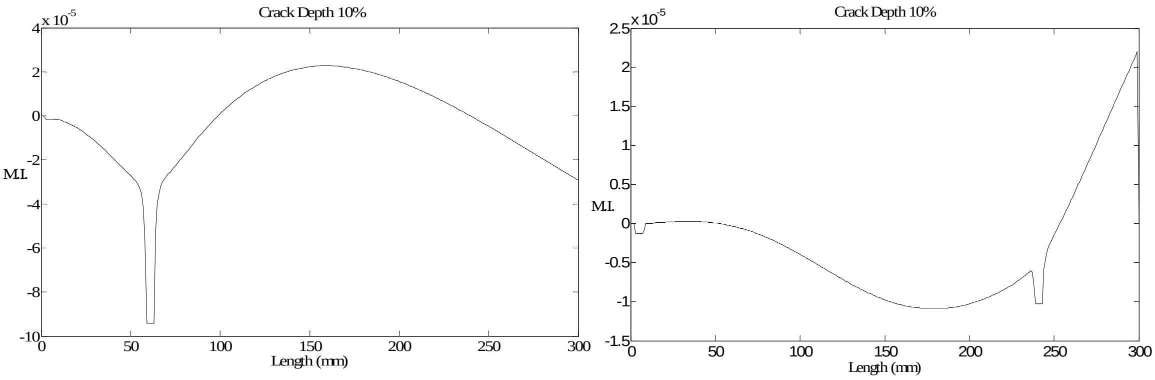

Figure 2 – Morphological index using P.O.M.1 with crack location at 60 mm (left) & 240 mm (right), 10% depth and SNR=0%

The evolution of the morphological index with the increase of the crack depth is shown in Fig. 3. It can be noted that the index follows the depth of the crack

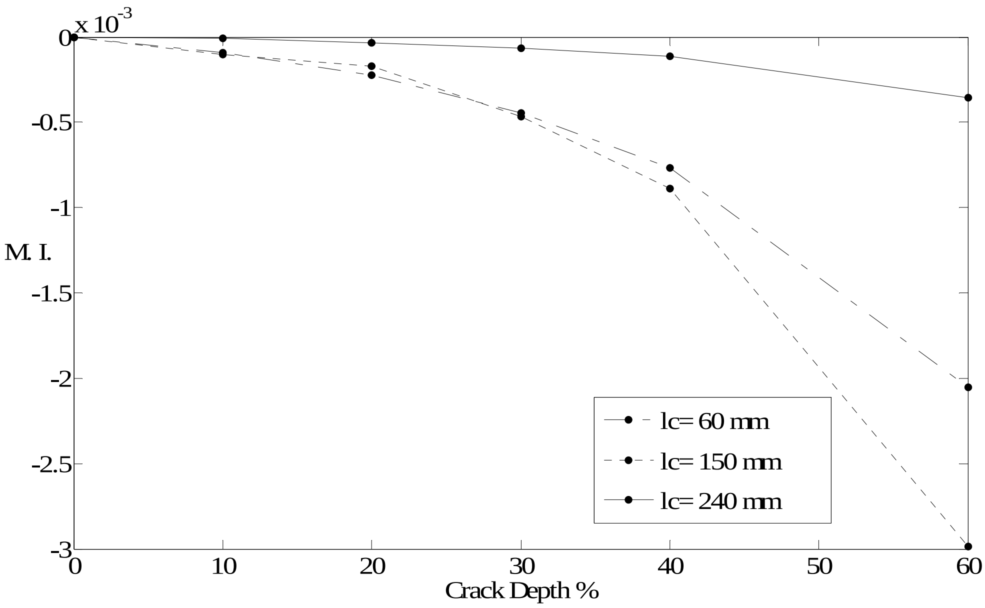

Figure 3 - Morphological index at 3 different locations & 5 different depths (SNR=0%)

Noise effects

In order to examine the effect of noise or measurements errors on the proposed detection process, a noise ratio with a *SNR* of 30% was introduced to the displacement records of the previous case.

Keeping the crack location fixed at *lc*=60 mm, several cases with varying crack depth have been investigated. From the results obtained, it follows that the crack localization is clearer in the case of large cracks, but even in case of small cracks (relative depth 10%) the results are clear enough. In all cases, however, the location of a crack can be accurately determined.

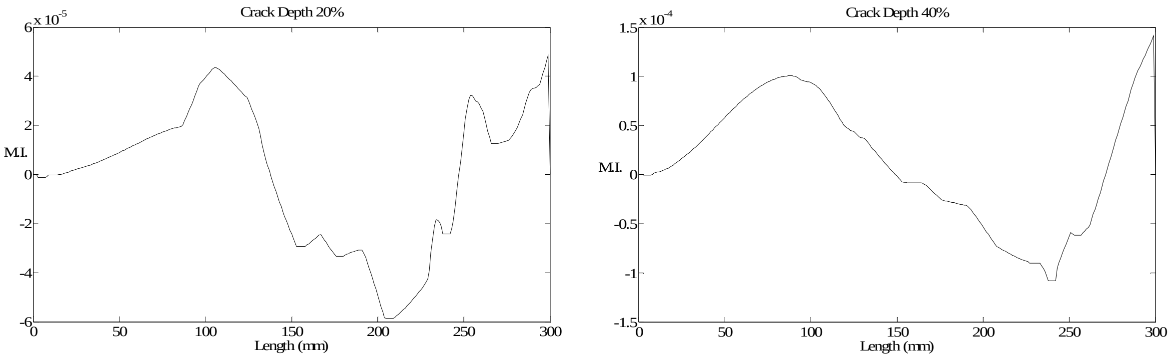

Figure 4 – Morphological Index using P.O.M 1 with crack location at 240 mm, SNR=30% and depth 20% (left) and 40% (right)

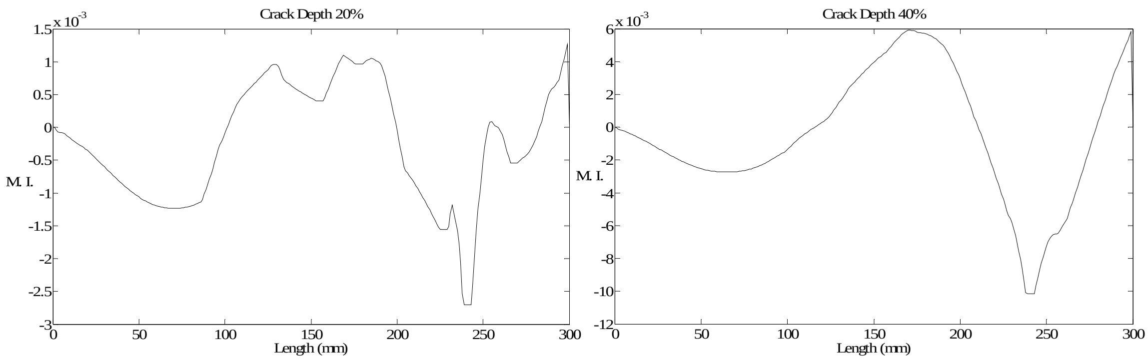

Figure 5 – Morphological index using P.O.M.3 with crack location at 240 mm, SNR=30% and depth 20% (left) and 40% (right)

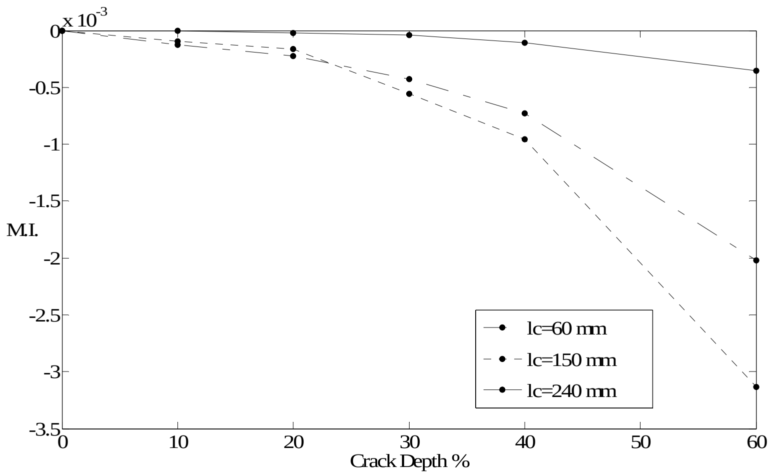

Figure 6 - Morphological index at 3 different locations & 5 different depths (SNR=30%)

Crack locations fixed at *lc*=150mm and *lc*=240 mm are also investigated. In the first case, crack determination is clear, even when the relative depth is only 10%. In the second case, which is close to the free end of the beam, crack location is clearer in the case of big cracks (greater than 40%) and somewhat obscured for smaller ones (less than 30%). The results are more clear when the 3rd PODs are used (Figure 5).

The evolution of the morphological index with the increase of the crack depth is shown in Fig. 6. It can be noted that the index follows the depth of the crack.

CONCLUSION

A method for crack identification in beam structures based on morphological analysis has been presented. For that purpose, a cracked cantilever beam having a transverse surface breathing crack has been investigated using Finite Element Analysis.

The location of the crack is determined by the variation of the morphological index of the spatial POM at the site of the crack. Using POMs instead of modes, noise effects are eliminated. Moreover, a certain number of POMs are easily extracted from the measured data. Morphological index follows the evolution of crack depth and gives good results even when SNR reaches 30%. The results become clearer when more than one POMs are examined and compared.

The results of the present work refer to a cantilever beam but can be easily extended to include more complex structures and boundary conditions. Work is already under way to explore the application of the proposed prediction technique to more complicated structures, including multicracked beams and cracked plates.

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REFERENCES

[1] Douka E., Loutridis S., Trochidis A., “Crack identification in beams using wavelet analysis”, International Journal of Solids and Structures, **40**, 3557-3569 (2003).

[2] Kerschen G., Golinval J. C., Vakakis A. F., Bergman L. A., “The method of Proper Orthogonal Decomposition for Dunamical Characterization and Order Reduction of Mechanical Systems: An Overview”, Nonlinear Dynamics, **41**, 147-169 (2005).

[3] Kim J. T., Ryu Y. S., Cho H. M., Stubbs N., “Damage identification in beam-type structures: frequency-based method vs mode-shape-based method”, Engineering Structures, **25**, 57-67 (2003).

[4] Nikolaou N. G., Antoniadis I. A., “Application of morphological operators as envelope extractors for impulsive-type periodic signals”, Mechanical Systems and Signal Processing, **17(6)**, 1147-1162 (2003).

[5] Rucka M., Wilde K., “Crack identification using wavelets on experimental static deflection profiles”, Engineering Structures, **28(2)**, 279-288 (2006).