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NUMERICAL AND VISUAL ANALYSES OF SOUND PROPAGATION AND SCATTERING IN CONCERT HALLS BY MEANS OF A FDTD METHOD

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Abstract

Among numerical methods recently adopted for evaluations of room acoustics, finite difference time domain (FDTD) method is prominent in ability to visualize sound waves as they propagate and to investigate numerically sound field in various materials of an arbitrary structure. The method has, however, restrictions due to insufficient capacity of main memory and CPU speed of the workstations. Our investigations are limited to two-dimensional wave propagations. Therefore, a precise representation of fine structures of the objects in the room should be adequately realized for the FDTD calculation. We have used an elastic FDTD program developed by us, and the number of grid points per wavelength was 40. The numerical dispersion error induced by this grid size was estimated as $2\pi \times 5.2 \times 10^{-4}$ rad per period. The temporal sampling interval was 10.10 μs.

We report here: (1) what can be analyzed by the FDTD method using continuous waves, burst waves, or an acoustic impulse in structured objects. The physical quantities are sound pressure of longitudinal waves, particle velocity and shear stress of transverse waves in solid materials. (2) Propagations of sound waves in concert halls, especially shoe-box type hall of 40.8 m $\times$ 18.7 m and small diamond-shaped hall of 27.6 m $\times$ 27.6 m. Both are reasonably considered by two-dimensional propagations of sound. (3) Echo-time patterns at representative positions in the halls up to 2.2 sec. The initial time delay gap was between 15 to 30 ms on average. It is shown that this method of analysis can give reasonable and useful results about fundamental acoustic properties of rooms and concert halls.

INTRODUCTION

Computer simulation of sound waves in concert halls, room acoustics in general, could not include their wave properties but geometrical ones. Namely, acoustical ray tracing and method of image sources were mainly used from ’80s up to now to analyze sound field in room[1]. Recently, as computers develop in CPU speed and in main memory size, various computer simulations which analyze sound waves have been performed even by personal computer systems. Among those computer simulations, finite difference time domain (FDTD) method is used for room acoustics and for visualization of sound waves[4, 5]. The FDTD method calculates spatial distribution of wave fields as time passes using finite difference equations derived by central or symmetrical difference approximation of the wave equations. Successive display of the calculated spatial distributions of sound waves with time makes directly movie-like visualization of propagation of sound waves. The calculation power of PCs is, however, still insufficient for detailed three-dimensional FDTD analyses of large halls or rooms. Even for two-dimensional one, an adequately simplified modeling of the surrounding structures are required with sufficiently fine sampling intervals for a small numerical (grid) dispersion to get a good numerical precision.

We have developed two types of acoustic FDTD programs. One is called sonic FDTD one, which solves longitudinal sound waves in the material of an arbitrary spatial distribution of elastic modulus, density and absorption. The other is called elastic FDTD one, which solves longitudinal and shear waves in the material together with shear modulus, in other terminology, Lame’s constants. For acoustics in the airspace inside the halls or rooms, sonic FDTD analysis is sufficient using normal acoustic impedance of the surface structures effectively realizing the reflection and absorption on the walls, seats, floor and ceiling. The characteristics of reflectors on the stage, diffusion walls and absorption walls are analyzed by elastic FDTD one to get the normal specific acoustic impedance and the effective absorption coefficient.

PARAMETERS FOR FDTD ANALYSES OF CONCERT HALLS

In this report, we have performed a two-dimensional FDTD analysis, because our original FDTD program, which was developed for precise analyses of sonic crystal waveguides, solves elastic equations for acoustic waves including propagations in solid materials and requires much memory space and fine sampling intervals for small numerical dispersion. Discarding these high qualities, we will do in the immediate future practical three-dimensional acoustic FDTD analyses.

In room acoustics, sound waves are solved for the propagation in the airspace inside the hall, for reflections on the surfaces of the wall, ceiling, floor and seats, and for absorption in their inner structures. It is reasonable to include their external form concretely in the calculations. It is considered, however, not practical to solve simultaneously sound wave propagations both in the airspace and in the surrounding structures. Based on the experimental measurements or on the results obtained by the elastic FDTD analyses of a complicated faithful model of such structures, we construct a model of the hall with its surrounding materials whose properties are described by effective absorption coefficients. Namely, an effective acoustic characteristic impedance Z is given to each part of the inner surface of the hall for an absorption coefficient α by a well-known relation:

$$Z = \frac{1+\sqrt{1-\alpha}}{1-\sqrt{1-\alpha}} Z_0 \,, \tag{1}$$

where Z_0 is the characteristic impedance of air. The front side-wall of the hall is made of concrete, and has the lowest absorption coefficient of 0.03 almost independent of frequency. The rear side wall is structured by wood-ribs and rock-wool sheets or plaster-boards with a relatively high absorption coefficient of 0.345 on average at 125 Hz.

The grid size $\Delta x = \Delta y$ for the finite difference is defined to be a q-th wavelength of sound waves at a frequency f Hz. Here we adopt $q = 40$. This sampling size gives a phase error of $2\pi \times 5.2 \times 10^{-4}$ rad per wavelength due to numerical dispersion of FDTD method[3]. This is an erroneous phase difference between a sound propagation along one of the grid axes, namely x or y, and a sound propagation along the diagonal, namely $x = y$. The inner size of the halls under consideration is an order of 10 times wavelength at 125 Hz, and the phase error per one-pass sound propagation across the hall is 0.52 per cent of 2π rad. This error becomes only 5.2 per cent of 2π rad at 125 Hz after 10 times propagations in 0.8 s across the hall due to multi-reflections. It becomes, however, 41.6 per cent of 2π rad at 1 kHz. In order to get a practical estimation of the acoustic properties of concert halls, this amount of phase error is acceptable only with a dislocated interference pattern of sound waves by an order of the wavelength.

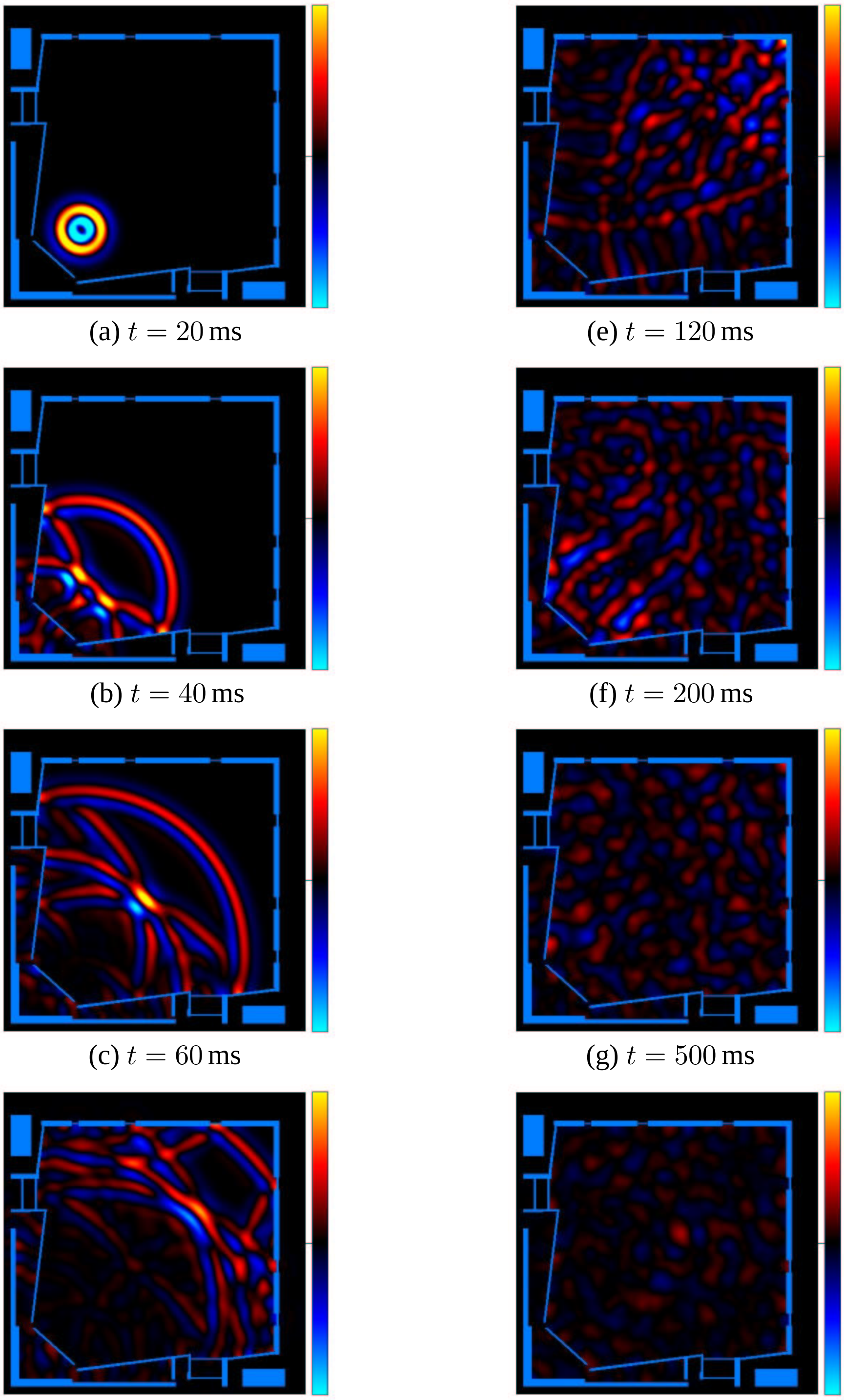
The temporal sampling interval Δt is set to $1/(\sqrt{2}\overline{c_l} q f)$, which is the maximum allowable value under the Courant condition and gives the minimum numerical dispersion as it is evaluated above. Here, $\overline{c_l}$ is the ratio of the highest longitudinal sound velocity in the hall to the velocity of sound in air. At a frequency of 125 Hz with a number of sampling points per wavelength $q = 40$, the temporal sampling interval is $\Delta t = 10.10\,\mu$s.

SOUND WAVES IN A SMALL DIAMOND-SHAPED CONCERT HALL AND IN A SHOE-BOX TYPE CONCERT HALL

We have simulated propagations of sound waves in two types of concert halls, and evaluated the obtained spatial impulse responses and echo time patterns. One is called Hall D and has a shape of diamond, or a square rotated by 45°, with an outside measurement of 27.6 m $\times$ 27.6 m including backsides of the walls which construct the hall. The other Hall S is a shoe-box type one with an outside measurement of 40.8 m $\times$ 18.7 m.

Sound waves in concert halls with an impulse source

The calculated spatial impulse responses are shown in Fig. 1 for Hall D and in Fig. 2 for Hall S. In these figures, instantaneous sound pressure is shown by pseudo-color, namely positive one is by warm colors and negative one by cool colors according to the color bar attached



(a) $t = 20\,\mathrm{ms}$ (e) $t = 120\,\mathrm{ms}$

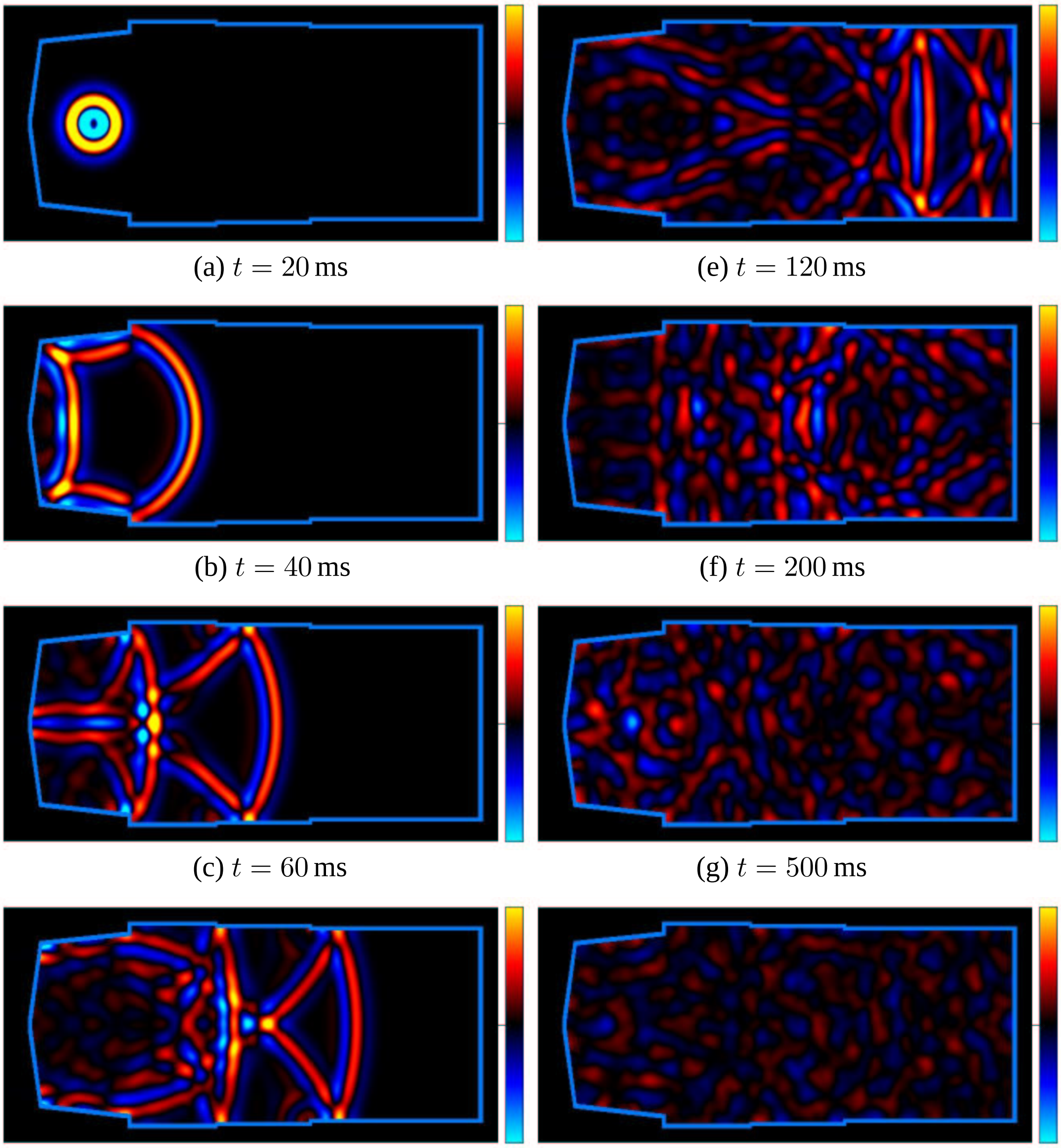
(b) $t = 40\,\mathrm{ms}$ (f) $t = 200\,\mathrm{ms}$

(c) $t = 60\,\mathrm{ms}$ (g) $t = 500\,\mathrm{ms}$

(d) $t = 80\,\mathrm{ms}$ (h) $t = 1000\,\mathrm{ms}$

Figure 1: Sonic impulse propagating in a small diamond-shaped concert hall: Hall D.

on the right side of each picture. Temporal shape of the sound source is made of the second



(a) $t = 20\,\mathrm{ms}$ (e) $t = 120\,\mathrm{ms}$

(b) $t = 40\,\mathrm{ms}$ (f) $t = 200\,\mathrm{ms}$

(c) $t = 60\,\mathrm{ms}$ (g) $t = 500\,\mathrm{ms}$

(d) $t = 80\,\mathrm{ms}$ (h) $t = 1000\,\mathrm{ms}$

Figure 2: Sonic impulse propagating in a shoe-box type concert hall: Hall S.

derivative of Gaussian function, namely

$$g^{(2)}(t) = \left\{1 - 2\left(\frac{t-t_0}{\sigma}\right)^2\right\} \mathrm{e}^{-\left(\frac{t-t_0}{\sigma}\right)^2}, \tag{2}$$

whose central frequency $f_\mathrm{c} = 1/\pi\sigma$ is adjusted to the desired frequency, e.g., 125 Hz. Spatial distribution of the sound source has a shape of a Gaussian function truncated with a width of a half wavelength. Temporal evolution of the sound waves are shown from $t = 20\,\mathrm{ms}$ to $t = 1\,\mathrm{s}$. In Hall D, three arcs of early reflections follow the direct sound in almost the same direction, as seen in Fig. 1(c). On the other hand in Hall S, they are composed of two early lateral reflections and a reflection at the rear wall of the stage, as shown in Fig. 2(c). It is well known that these lateral reflections are necessary for envelopment[2]. Reflections on the side wall of the stage are clear in Hall S, as shown in Figs. 2(b) and 2(c), and the performers will have the ability to hear other clearly. This effect is relatively small in Hall D, as shown in Figs. 1(b) and 1(c). In both halls, the performers on the stage have the ability to receive from the space a reverberant return. A diffuse field is established at 500 ms $\sim$ 1000 ms as shown in Figs. 1(g) and 1(h) and also in Figs. 2(g) and 2(h), sound wave coming from any direction with equal probability.

Echo time patterns

With a small sound pressure source of a spatial width of a half wavelength on the stage, echo time patters are calculated at several positions in each hall. Typical examples are shown in Fig. 3. The initial time delay gap t_{I}, namely temporal difference between the direct sound

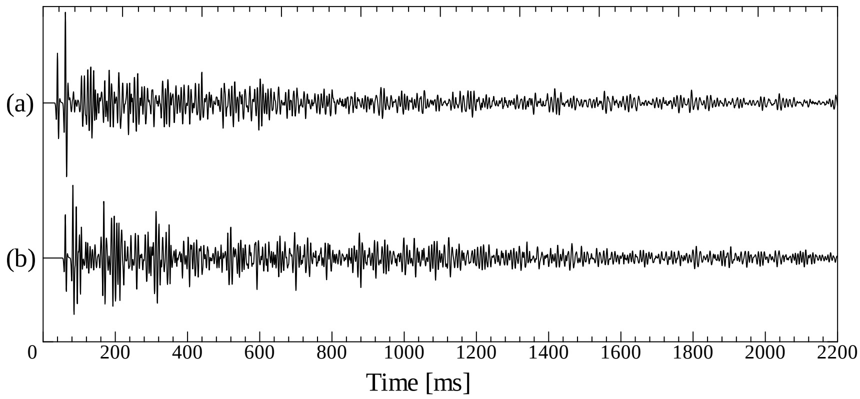


Figure 3: Echo time patterns at representative positions in two concert halls. (a) Small diamond-shaped concert hall D with $t_{\mathrm{I}} = 22.1$ ms and (b) Shoe-box type concert hall S with $t_{\mathrm{I}} = 24.3$ ms.

and the first early reflection, does not change practically along the diagonal line in Hall D except positions just before the rear wall, and estimated as 20.6 ms to 23.2 ms. In contrast, t_{I} decreases monotonously along the central line from 30.5 ms just in front of the stage to 12.4 ms just before the rear wall in Hall S. Although the concrete values of t_{I} may change depending on the location of the sound sources on the stage, it can be considered that the above mentioned characteristics are consistent.

SUMMARY

It has been shown that FDTD analyses can give reasonable and useful results about fundamental acoustic properties of rooms and concert halls, e.g., movie-like presentation of sound propagations and echo time patterns at arbitrary positions in the hall. The structures or components of the propagating sonic wavefronts and the arriving sound at arbitrary positions have been clearly observed. Reverberation time of the halls can be also determined from the echo time patterns. Discarding unnecessary high qualities included in our FDTD program, we will do in the immediate future practical three-dimensional acoustic FDTD analyses of concert halls.

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