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DIRECT SIMULATION OF DUCT AEROACOUSTICS USING CE/SE METHOD

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Abstract
Subsonic flow through duct has been a topic of interest among aeroacoustics researchers. Seldom can the fluid inside the ductworks flow through without encountering any obstructions or restrictions. In reality because of design requirements and space limitations, the flow needs to change direction, thus leading to branching and the necessity to introduce internal guide vanes to smooth transition, bends, and different types of junctions. As a result, devices/elements are introduced into the flow and they will invariably affect the flow structure and behavior inside the air-conditioning and ventilation ductworks. Turbulence is also generated at these duct devices. It is a common goal of the engineering designers that the occurrence of high-pressure amplitude oscillations should be minimized since they are the major source of destructive unsteady loads and noise. Because of high complexity of the sound-flow interaction at duct devices/elements, a direct solution of the duct aero-acoustics is desired for understanding of the relevant flow physics.

Owing to the scale disparity between acoustic and unsteady flow disturbances, high-order Pade scheme is needed for duct aero-acoustics simulation using DNS. In addition non-reflecting inlet/outlet buffer regions are required to suppress spurious numerical waves. The buffer region size is usually comparable to the computational domain to ensure satisfactory performance and the computational resources incurred are significant. In the present paper direct numerical simulation scheme for subsonic duct aeroacoustics based on conservation-element/solution-element (CE/SE) method is developed. The CE/SE method is built with strict conservation within a ‘conservation element’ spanning in spatial and time dimensions; therefore, the scheme can ensure local and global flux conservation for both in time and space. The generation of the numerical error waves is expected to be less severe and inlet/outlet buffer regions are not required for an accurate solution. Benchmark comparison with DNS solutions will be reported in the present paper to reveal the effectiveness and efficiency of the CE/SE based simulation.

INTRODUCTION

Noise control of flow through a duct is a very important issue in many engineering applications, such as automobile industry, ventilation duct and exhaust pipe, etc. There are two major types of noise generation mechanisms. The first type is the interaction between vortices in shear layers emanating from structural discontinuities or in the wake at the downstream of immersed structures in the duct flow. The second type is the interaction between vortices and solid boundaries of structural discontinuities, which appear as a result of flow management devices in the ducts. Sound waves are generated when the unsteady flow passes over gaps or obstacles inside the duct. The interaction becomes more complicated if aeroacoustic resonance occurs in the ductworks. Therefore, it is a common objective for the engineers to minimize the noise level generated by the unsteady flow past flow devices. This is particularly important for high-pressure gas transport because the resonant high-pressure oscillations would be a possible excitation source of flow-induced vibration and fatigue failure of the duct works. In order to obtain a better understanding of the noise generation mechanisms, a great deal of experimental and numerical works have been carried out [1] [2] [3] [4].

Direct simulation is always the preferred simulation methodology as this scheme is able to solve accurately the far-field aeroacoustics disturbances, near-field unsteady flows, and consequently their interaction, simultaneously. Direct simulation usually calls for a low dispersive and low dissipative finite-difference DNS scheme in order to capture accurately the acoustic fluctuations, the magnitudes of which are at least three orders of magnitude smaller than the associated flow fluctuations. The high-order schemes are usually required so as to suppress the magnitude of truncation errors well below the acoustics. In addition, the treatment of inflow and outflow boundary conditions does have significant effect in the accuracy of the aeroacoustic simulations. Buffer regions are usually required at the inlet and outlet boundaries so as to allow the unsteady flow fields to pass through them with minimal reflection of outgoing acoustic waves. The buffer regions are usually quite large and in most cases comparable to the actual physical domain so much computational resources are spent for the non-productive error suppression processes. Therefore, the aeroacoustics research community has been looking for a more efficient direct aeroacoustic simulation scheme that has the same accuracy as the finite-difference approach yet involves smaller buffer regions.

In the present paper the conservation element/solution element (CE/SE) [5] [6] [7] scheme is proposed for direct simulation of duct aeroacoustics. An advantage of CE/SE scheme is that its formulation guarantees strong conservation of local and global flux in both space and time dimensions and consequently the truncation errors of the solutions would be minimal. Owing to its strong flux conservation, a non-reflecting boundary condition can be easily built by introducing an additional line of ghost points just outside the computational boundaries [8]. In the present paper, benchmark aeroacoustic problems of compressible flow over a cavity is calculated using CE/SE scheme and compared with finite-difference DNS solutions. The credibility and efficiency of the CE/SE scheme will be discussed.

THE NUMERICAL SCHEME

Two-dimensional space-time conservation element and solution element scheme is adopted. A detailed description of the CE/SE scheme for Euler and Navier-Stokes equations is referred to references [5] [6] [7]. Only a brief description of the scheme is given as follows. Two dimensional Navier-Stokes equations for compressible flow can be written in strong conservation form as

$$\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial(\boldsymbol{E}-\boldsymbol{E}_v)}{\partial x}+\frac{\partial(\boldsymbol{F}-\boldsymbol{F}_v)}{\partial y}=0 \tag{1}$$

With

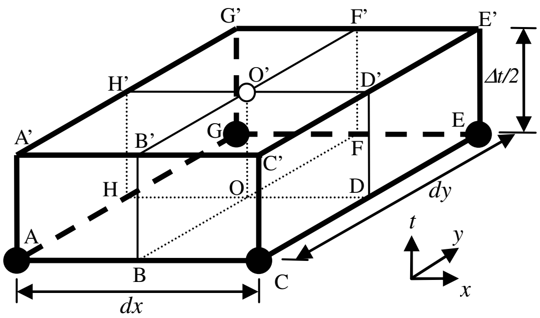
$$\begin{gathered}\boldsymbol{U}=[\rho,\rho u,\rho v,e]^T,\\ \boldsymbol{E}=\left[\rho u,\rho u^2+p,\rho uv,(e+p)u\right]^T, \qquad \boldsymbol{F}=\left[\rho v,\rho uv,\rho v^2+p,(e+p)v\right]^T\\ \boldsymbol{E}_v=\left[0,\tau_{xx},\tau_{xy},u\tau_{xx}+v\tau_{xy}+q_x\right], \qquad \boldsymbol{F}_v=\left[0,\tau_{xy},\tau_{yy},u\tau_{xy}+v\tau_{yy}+q_x\right]\ ,\\ \tau_{xx}=\frac{2}{3}\mu\left(2\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right),\quad \tau_{xy}=\mu\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right),\quad \tau_{xy}=\frac{2}{3}\mu\left(2\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\end{gathered} \tag{2}$$

where ρ, p, u, v and e are the density, thermodynamic pressure, velocities in x- and y-directions, and the total energy respectively. The quantities μ, q_x, and q_y are the fluid viscosity and heat flux respectively. These equations can be made dimensionless by normalizing the variables using reference flow velocity, density, temperature and length, i.e. U_∞, ρ_∞, T_∞, and H. Except otherwise stated, all quantities in the forthcoming discussions are dimensionless.

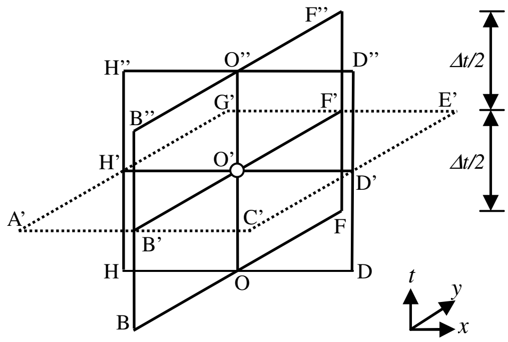
Let (x,y,t) be the coordinates of a computational domain consisting of space in two dimensions and time dimension. Apply Gauss divergence theorem, the integral form of equations (1) becomes

$$\oint_{S(V)} \boldsymbol{H}\cdot d\boldsymbol{S}=0 \quad , \tag{3}$$

where $\boldsymbol{H}=(\boldsymbol{E}-\boldsymbol{E}_v,\boldsymbol{F}-\boldsymbol{F}_v,\boldsymbol{U})$ and $S(V)$ denotes the surface around an arbitrary region V. In the present paper the computational domain is described using Cartesian grid as illustrated in Figure 1. In the CE/SE scheme, the computational domain is divided by the conservation elements as shown in Figure 1(a). The flux between two conservation elements is approximated by the solution element as shown in Figure 1(b). Solid circles A, C, E and G denote known variables $\boldsymbol{U}$ at current time level n. Open circles O' denote unknown variables $\boldsymbol{U}$ at time level $n+1/2$. Grid points A, C, E and G support the marching of $\boldsymbol{U}$ at O to the next half time step level O'. The cube ACEGA'C'E'G' includes of cubical conservation elements OHABO'H'A'B', OBCDO'B'C'D', ODEFO'D'E'F' and OFGHO'F'G'H' as shown in Figure 1 (a). Each solution element includes two vertical planes BFF'' F'' and DHD'' H'', and a horizontal plane A'C'E'G' (Figure 1(b)). The total flux leaving each conservation element has to be zero according to equation (3). This requirement is implemented as follows. Conservation element OHABO'H'A'B' is formed where the variables at A are known but those at O' are not. The flux through the surfaces ABOH, ABA'B' and AHA'H' is approximated by the solution element formed with point A and the flux on the surfaces OBO'B', OHO'H', and O'H'A'B' is approximated by the solution element formed with point O' by Taylor's expansion with first order accuracy. Then equation (3) is applied to all four conservation elements and $\boldsymbol{U}$ at grid point O' for new time level can be obtained explicitly in term of the marching variables at grid point A, C, E, and G for old time level. Once $\boldsymbol{U}$ at new time level has been computed, their derivatives can also be computed by weighted averaged central-difference approximation [5] with second order accuracy. Non-reflecting boundary condition [8] is applied for inlet and outlet boundaries. No-slip boundary condition is applied for all solid boundaries.



(a) Conservation elements



(b) Solution element

Figure 1 – Schematic diagram of conservation elements and solution element.

RESULT

A test case of subsonic flow over a single cavity has been carried out to evaluate the accuracy of the proposed CE/SE scheme. It aims to replicate the experiment of unsteady flow dynamics and acoustics of the flow over a cavity reported by Kirshnamurty [9]. Therefore, the geometry and configuration of the simulation follows the experiment (Figure 2). As the boundary layer approaching the cavity is laminar in the experiment, we adopt a Blasius flat-plate boundary layer along the wall and spanning the cavity for the initial condition. The aspect ratio cavity length over cavity depth is $L/H = 2$. The Reynolds number $\mathrm{Re} = \rho_\infty U_\infty H / \mu$ and Mach number $M = U_\infty / c$ are 1499.5 and 0.6 respectively, where c is the ambient speed of sound. The size of the computational domain is 12×9 . A non-uniform rectangular mesh with grid spacings $\Delta x = 0.016$ and $\Delta y = 0.011$ is adopted in the calculation. For a stable explicit time marching, the time step is chosen as $\Delta t = 0.002$ such that the Courant Friedrichs and Lewy number based on the minimum grid spacing is equal to 0.2.

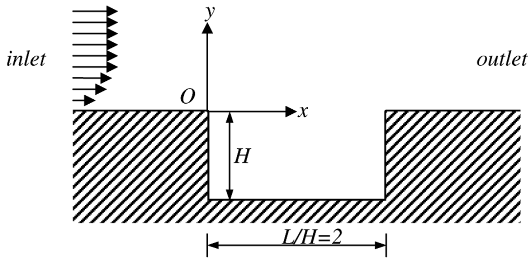


Figure 2 – Schematic diagram of a flow past a cavity

Figure 3 shows the four snapshots of instantaneous vorticity contours over one cycle of oscillation. The flow structures are characterized by the large-scale Kelvin-Helmholtz vortex formed as a result of shear layer separation from the cavity leading edge. The shed vortex grows up during its passage and hit on the trailing edge the cavity. Part of the vortex is ingested into the cavity interior, which then induces flow separation from cavity bottom and produces vorticity of opposite sign.

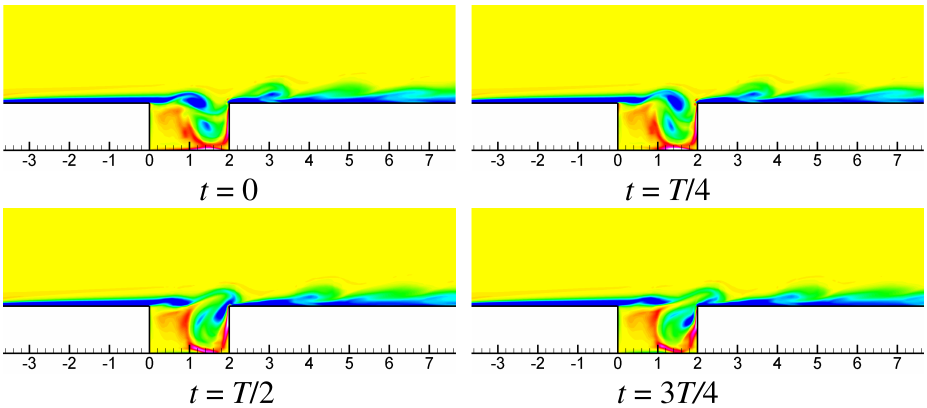
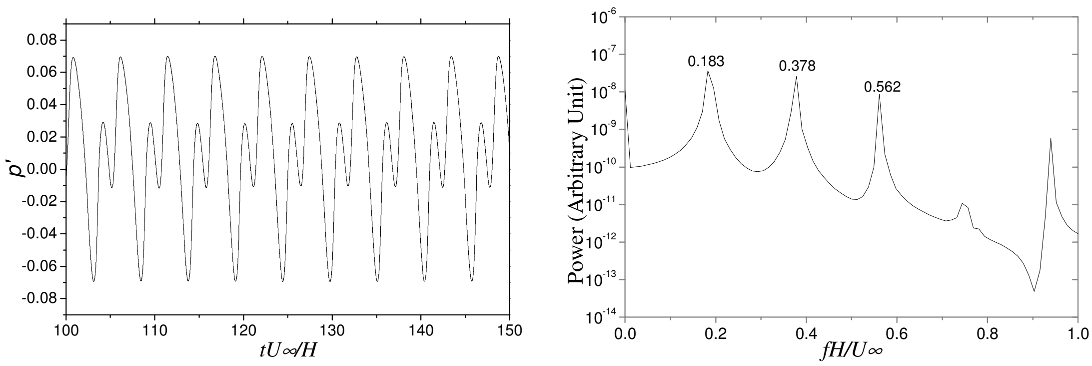


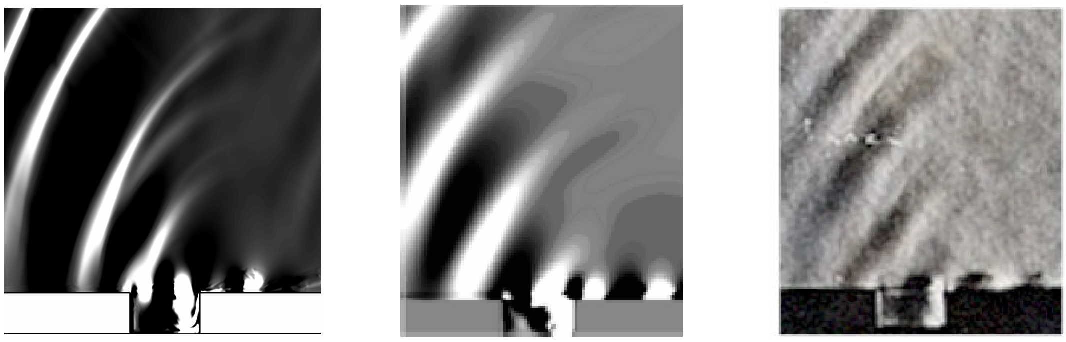
Figure 3 – Evolution of vorticity in one oscillation cycle with period *T*.

The time history of acoustic pressure fluctuation, $p' = p - p_m$ where p_m is the time-averaged pressure, and its spectrum measure at point $(x, y) = (0,2)$ is shown in Figure 4. It is evident the acoustic pressure fluctuation is dominated by three frequencies at $fU_\infty / H = 0.183$, 0.378 and 0.562. These calculated frequencies are in good agreement with the Rossiter's formula [10], which gives Strouhal numbers equal to 0.159, 0.372 and 0.584 respectively. The first two calculated frequencies are also consistent with finite-difference DNS results of 0.2 and 0.376 by Leung et al. [11]. The calculated directivity of the acoustic field is shown in Figure 5. As shown in Figure 3, a weak and relatively vortex occupies the downstream half of the cavity. The vortex oscillates whenever a boundary layer passes over the cavity and the oscillation of the shear layer is responsible for the acoustic radiation. Such shear-layer mode oscillation is the dominant mechanism for the generation of acoustic waves from the cavity flow. Figure 5(a) shows the instantaneous distribution of density gradient $(\partial\rho/\partial x)$ calculated by the present CE/SE scheme which agrees with the schlieren photograph taken by Krishnamurty [9] (Figure 5(c)). The calculated directivity is also to the DNS result of Leung et al. [11] as shown in (Figure 5(b)). Therefore the comparison shows clearly the present CE/SE scheme is capable of capturing the acoustic fluctuation emitted by the unsteady cavity flow.



(a) (b)

Figure 4 – Time history and spectrum of acoustic pressure fluctuation at $(x, y) = (0,2)$.



(a) (b) (c)

Figure 5 – Instantaneous density gradient distribution. (a) CE/SE result; (b) DNS result; (c) Krishnamurty's schlieren photograph.

In practical ductwork of gas transport system, the cavity does not exist alone. It may experience the influence from opposite duct wall or exist as a component of an in-duct structural discontinuity, for example, an expansion chamber which is common in many ductworks. Its shape can be considered as two cavities installed face-to-face in the duct. Although a properly designed expansion chamber is able to absorb tonal acoustic disturbance in the duct, the unsteady flow around the cavity may generate additional acoustic wave which may reduce the silencing performance of the chamber. Therefore *a priori* knowledge of self-generated noise of expansion chamber is important in noise control design.

A calculation of self-noise generation of expansion chamber is attempted using CE/SE scheme. The chamber is composed of the cavity defined in Figure 1 and its mirror image on the top. The duct width is equal to $2H$. The Reynolds number, Mach number and the initial conditions are the same as those in previous calculation. Figure 6 shows the snapshots of instantaneous vorticity and density gradient distributions within one cycle of oscillation. It is evident that acoustic waves are generated at both cavities, as a result of unsteady vortex motion at the both leading edges, and travel both upstream and downstream. The acoustic waves hit the unsteady vortices and give rise to more complicated sound-flow structures as compared to single cavity case. Figure 7 shows the time history and spectrum of acoustic pressure fluctuation at point $(x, y) = (0,1)$. The three dominant frequencies are 0.191, 0.381 and 0.565. They are in general higher than in single cavity case which shows the change of the acoustic behaviours due to the presence of an additional cavity and duct walls.

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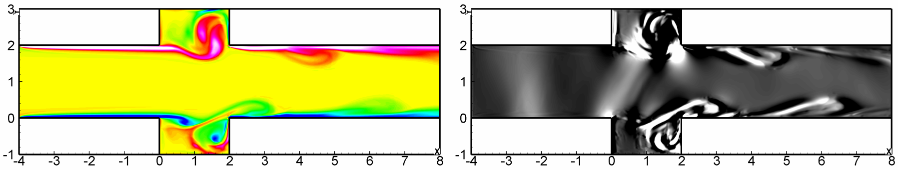
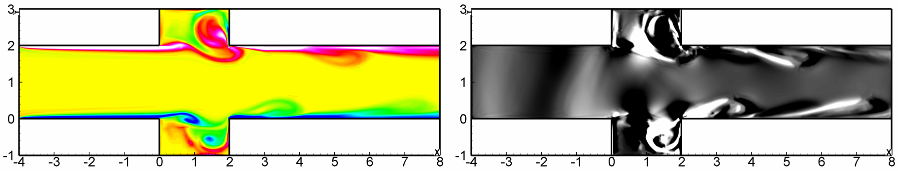
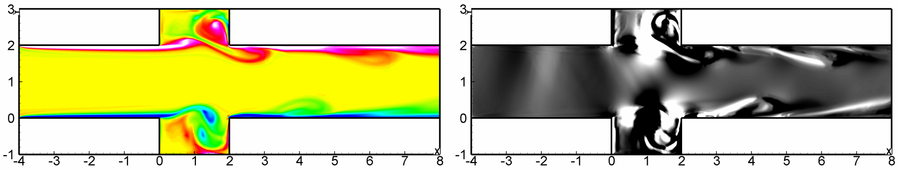


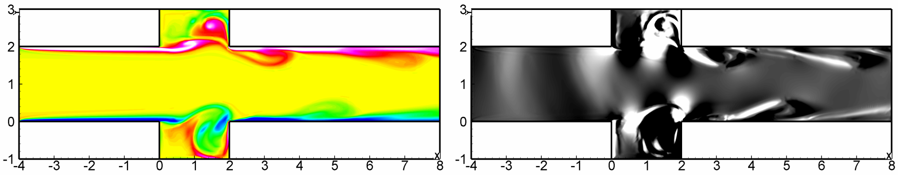
$t = 0$



$t = T/4$



$t = T/2$



$t = 3T/4$

Figure 6 –Evolution of vorticity (left column) and density gradient (right column) in one oscillation cycle with period *T*.

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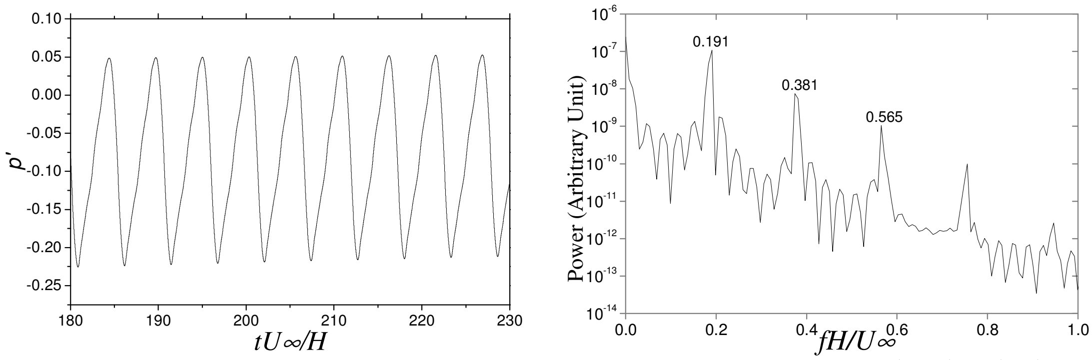

Figure 7 – Time history and spectrum of acoustic pressure fluctuation at $(x, y) = (0,1)$.

CONCLUSIONS

The CE/SE scheme for direct simulation of duct aeroacoustics has been established and compared with experimentsand finite-difference DNS results. The benchmark case of flow past a single cavity has been calculated for scheme validation. It has been found that the present CE/SE scheme is not only able to replicate the essential unsteady flow dynamics but also calculate accurately the generation of acoustic wave. It correctly predicts the dominant acoustic modes due to oscillation of shear layer covering the cavity and the calculated frequencies agree well with Rossiter's formula and finite-difference DNS results. The scheme is then used to calculate the self-generated noise of an expansion chamber. It is found that acoustic waves are generated and travel both upstream and downstream. The acoustic mode frequencies are in general higher than the single cavity case due to the complicated sound-flow interaction in the chamber geometry.

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