



LOCAL PEAK FREQUENCIES AND THEIR DAMPING FACTORS FOR VIBRO-ACOUSTIC TRANSMISSION.

Hideo Shibayama and Tang Yong

Shibaura Institute of technology
3-7-5 Toyosu Koutou-ku, Tokyo 135-8458, Japan
sibayama@sic.shibaura-it.ac.jp

Abstract

Hitting a floor by an impact hammer, we generate vibration and acoustic waves. We estimate the transfer function between the vibro-acoustic transmission by the cross-spectrum method. Local peaks are found in the magnitude characteristic. The aim of this study is to estimate frequencies and damping factors of their local peak with high accuracy. From the impulse response that is calculated from the transfer function, we propose the estimation method by use of the forward linear prediction coefficients and show the estimated results.

INTRODUCTION

The vibration and the sound waves caused by rapping on the floor propagate to various positions through many transmission lines in a house. A lot of new sound sources are also generated at the various positions through the solid-borne and the air-borne paths. In addition, waveforms of their waves are impulsive. And, spectral band of waves is wide from infrasonic range to the audibility range. In generally, environmental sound under rapping on the floor is not comfortable. Noise propagation is likely to develop into noise problem that is bothering the residents who lives in multiple dwelling houses.

We protect the comfortable environmental sound, we should remove as much of environmental stress factors as possible. However, there are many cases that we unconsciously generate the vibration and the noise in a daily life. In advance, by knowing how unwanted noise sources are generated under any conditions, we can consider how to make the comfortable environmental sound. By this purpose, we made a vibration monitoring equipment, called a manner sensor, for detecting oscillatory reaction on the floor. The equipment is useful for the residents who live downstairs and upstairs. From some notices of the equipment, an evaluation of the influence on the environmental sound in the downstairs room caused by

the generated noise sources can be understood for the residents who lives upstairs. The vibro-acoustic transmission characteristics between the vibration waves in the upstairs room and the acoustic waves in the downstairs room are necessary to improve the accuracy evaluation of the equipment.

And, the measurements are carried out in a second floor house. Hitting the concrete slab of the upstairs floor by an impulse hammer, we generate the both vibration and acoustic waves. After, we calculate the vibro-acoustic transmission characteristics. In the magnitude characteristics, we can find many local peaks that make up features of the vibro-acoustic transmission. Hence, we desire to estimate highly accurate each frequency and damping factor of the local peaks. To modeling the impulse response that is calculated from the transfer function estimated by the non-parametric method, we propose the estimation method and show the results.

MEASUREMENT CONDITIONS

By hitting the concrete slab that is upstairs floor by the impulse hammer, we generate the vibration and acoustic waves. The generated waves are traveled to many positions though the solid-borne and the air-borne paths. We measure the vibration acceleration on the slab and sound pressure of the acoustic waves. The size of the slab is $3.64\text{m} \times 3.64\text{m} \times 0.2\text{m}$. We show a measuring setup diagram in Fig.1. Symbol by a black circle shows the hitting location, that is the center of the slab. The driven force is measured by the force sensor. And, symbol by a white circle shows the vibration acceleration sensor (Rion, PV-90B) set on the slab. The sound pressure meter (Rion, NL-21) is set in 120cm high in the measuring cubic downstairs room. The room's size is $3.64\text{m} \times 3.64\text{m} \times 1.82\text{m}$. A carpet is covered over the floor made by concrete. Four side walls without acoustic treatment are also made by concrete. There are a center table, a side table, an audiovisual apparatus and a bookcase. Four side walls that are also made by concrete are in no acoustic treatment .

We show the results of the vibration acceleration at the measuring point shown in Fig. 2. After the last sound waves that are enough attenuated, a new sound waves are generated. The reverberation time is 0.5s. The repetition time of hitting the floor by hand power is about 2s. The measured waves are quantized by an analog-to-digital converter at the rate of the sampling frequency 22.05kHz. The quantization resolution is 24 bit. The whole number of the data point is 16,384.

We show the waveform of the vibration acceleration that is standardized by the mean value = 0 and variance = 1 in Fig. 3(a). We show its magnitude frequency response that is normalized by the maximum spectrum in Fig. 3(b). From the shown spectrum characteristic, the magnitude of acceleration spectrum is relatively flat without high spectral components in the frequency band from 25Hz to 100Hz. In contrast, the spectrum characteristic of sound pressure is rapidly varying. In its same frequency band, frequencies with high spectral component are 50Hz, 82Hz and 100Hz. Their peaks have influence with elastodynamic characteristics of the floor.

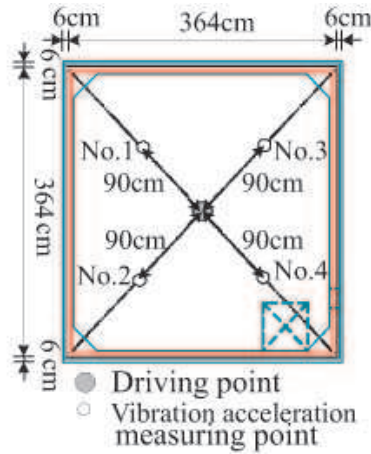


Figure 1: Plain view of the slab and sensors' location

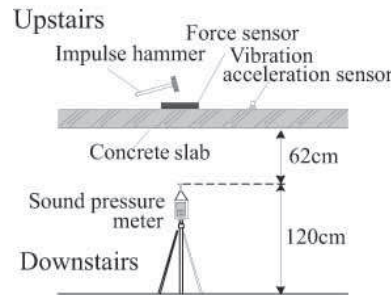


Figure 2: Setup of the microphone

VIBRO-ACOUSTIC TRANSFER FUNCTION

We measured acoustical effects in the downstairs room by the driving force. Duration of the acoustic waves shown in Fig. 4(a) is long in comparison to one of the vibration acceleration wave. And, spectrum characteristic shown in Fig. 4(b) has many local peaks, and their frequencies of peaks are in relation to the spectrum characteristic of the vibration acceleration wave shown in Fig. 3(b). Their local peaks cause by the natural frequencies of both the slab and the measuring room.

For detecting features of their local peaks of the transmission characteristic, we estimate the vibro-acoustic transfer function between the vibration acceleration and the sound pressure by the cross-spectral method. We show the vibro-acoustic transfer function normalized by the maximal value in Fig. 5(a). In the frequency range of 100Hz or less, local peaks at frequency of 50Hz, 83Hz and 100Hz are in magnitude response. In a range any more, we find many local peaks. The estimation method for each local peak frequency and damping factor will be shown in the following section. The phase characteristic is shown in Fig. 5(b). It can be considered a linear phase response though it is somewhat fluctuation. This is the reason by

the sound waves that reflected from many objects in the measuring room. Furthermore, this is because new acoustic waves are generated by the multiple vibrations of the derived slab.

LINEAR PREDICTION MODELING

We describe the estimation method for each local peak frequency and damping factor by use of the impulse response obtained from the transfer function. Suppose that the vibro-acoustic transfer function is measured under the condition of the time invariant system. Let $h(n)$ denote the *discrete impulse response* of a discrete linear system excited by a *unit impulse function* $\delta(n)$. It is defined as

$$\delta(n) = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

In this measuring system at discrete time n , the output response $y(n)$ of the acoustic waves to an arbitrary vibration acceleration sequence $x(n)$ is provided by the *discrete convolution summation*

$$y(n) = \sum_{k=0}^n x(k)h(n-k). \quad (1)$$

By a parametric method, we reconstruct a new impulse response from the impulse response estimated by the cross-spectral method that is the nonparametric method. We estimate the new impulse response by the linear prediction method. The magnitude response of the transfer function can be calculated by the discrete Fourier transform of the new impulse response. As the frequency characteristic between the magnitude response and the new magnitude response is same under the condition of an optimal modeling, each local peak frequency and damping factor shown in Fig. 5(b) can be calculated from the estimated impulse response $\hat{h}(n)$ by the linear prediction method that is the parametric analysis [1].

Consider the *forward* linear prediction estimate of the sample $h(n)$,

$$\hat{h}(n) = - \sum_{k=1}^M a_M(k)h(n-k) \quad (2)$$

where $\hat{a}_M(k)$ is the forward linear prediction coefficient at time index k . The symbol $\hat{\cdot}$ is used to denote an estimate. The prediction is forward in the sense that the estimate at time index n is based on M samples indexed earlier in time.

The forward linear prediction error $\epsilon(n)$ is given as follows:

$$\epsilon(n) = h(n) - \hat{h}(n). \quad (3)$$

And, from Eqs. (2) and (3), a variance ρ for the error is given as follows:

$$\rho = r_{hh}(0) + \mathbf{r}_M^H \mathbf{a} + \mathbf{a}^H \mathbf{r}_M + \mathbf{a}^H \mathbf{R}_{M-1} \mathbf{a} \quad (4)$$

in which, the superscript H denotes the Hermitian transpose operation,

$$\mathbf{a} = \begin{pmatrix} a(1) \\ a(2) \\ \vdots \\ a(M) \end{pmatrix}, \mathbf{r}_M = \begin{pmatrix} r_{hh}(1) \\ r_{hh}(2) \\ \vdots \\ r_{hh}(M) \end{pmatrix},$$

$$\mathbf{R}_{M-1} = \begin{pmatrix} r_{hh}(0) & r_{hh}(1) & \dots & r_{hh}(M-1) \\ r_{hh}(1) & r_{hh}(0) & \dots & r_{hh}(M-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_{hh}(M-1) & r_{hh}(M-2) & \dots & r_{hh}(0) \end{pmatrix}.$$

The forward linear prediction coefficient vector \mathbf{a} that minimizes the variance ρ is found as the solution to the normal equation given by

$$\begin{pmatrix} r_{hh}(0) & \mathbf{r}_M^H \\ \mathbf{r}_M & \mathbf{R}_{M-1} \end{pmatrix} \begin{pmatrix} 1 \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} \rho \\ \mathbf{0}_M \end{pmatrix}. \quad (5)$$

This matrix expression is structurally identical to the Yule-Walker equation of an autoregressive process. The solution of the Hermitian Toeplitz equation (5) can be computed with Levinson's algorithm. The solution for a linear prediction filter of M coefficients is given by the recursion for $m=1$ to $m=M$

$$\Delta_m = \begin{cases} r_{hh}(1) & \text{if } m=1 \\ \sum_{k=1}^{m-1} a_{m-1}(k)r_{hh}(m-k) + r_{hh}(m) & \text{if } m \succ 1 \end{cases}$$

$$a_m(k) = \begin{cases} -\Delta_m / \rho_{m-1} & \text{for } k=m \\ a_{m-1}(k) + a_m(m)(a_{m-1}(m-k)) & \text{for } 1 \preceq k \preceq m-1 \text{ and } m \succ 1 \end{cases}$$

$$\rho_m = \rho_{m-1}(1 - |a_m(m)|^2). \quad (6)$$

The single initial condition required is

$$\rho_0 = r_{hh}(0). \quad (7)$$

LOCAL PEAK FREQUENCY AND ITS DAMPING FACTOR

The impulse response of the forward linear prediction filter is equal to M coefficients, and the impulse response $h_M(n)$ of the estimating model is followed

$$h_M(n) = \sum_{k=0}^M a_M(k)\delta(n-k), \quad (8)$$

here, $a_M(0) = 1$. If the Hermitian Toeplitz matrix is positive definite, then the polynomial $H_M(z)$, the z transformation of the sequence $h_M(n)$, may be written as follow:

$$H_M(z) = \sum_{k=0}^M a_M(k) z^{-k} \quad (9)$$

The function $H_M(z)$ forms a polynomial in which the powers of z^{-1} correspond to the sequence time index. And, the polynomial has roots at $z_M(k) = \exp([- \alpha_k + j2\pi f_k]T_s)$ for $k=1$ to $k=M$. Here, T_s is the sampling period. For a decaying damping factor ($\alpha < 0$), the roots of the forward linear prediction characteristic polynomial fall inside the unit z -plane circle.

ESTIMATED RESULTS

In this section, we show the estimated results of the local frequencies and damping factors calculated from the roots of the polynomial. Figure 6 shows the impulse response estimated by the vibration force and the sound pressure measured under the condition of the measuring setup shown in Fig. 2. The impulse response, $h(n)$ that is depicted in Eq. (1), is calculated by the inverse discrete Fourier transform of the transfer function shown in Fig.5.

As shown previously, the number of the data is 16,384. We can freely select the number M which is the order of the forward linear prediction filter. Akaike suggested the selection criterion using a maximum likelihood approach to derive a criterion termed the *Akaike information criterion* (AIC). The AIC determines the model order by minimized an information theoretic function. Assuming the process has Gaussian statistics, the AIC for the forward linear prediction model has the form

$$AIC_M = N \lg(\rho) + 2M \quad (10)$$

,where M is the order, N is the number of data samples, and ρ is the linear prediction error variance. We apply to determination of the optimum order and get the optimum order 17. By use of its optimum order, we can estimate the only local frequency with the maximum peak value in magnitude response.

For aids to find the local peak frequencies and their damping factors, we must select the greater order of the model. We show the results calculated under the condition of $M=1024$. We show both of the measured magnitude characteristic by the thin line and the estimated magnitude characteristic by the heavy line by use of the calculated filter's coefficients in Fig. 7. The calculated magnitude characteristic is close agreement with the measured one over 80Hz frequency ranges.

In the Table 1, we show these results are greater or equal than 20 dB in magnitude of the magnitude characteristic shown in Fig. 7. The peak-frequency of magnitude is 884.1Hz and the damping factor at its frequency is -15.6. Damping factors at 82.8Hz and 100.4Hz are -3.5 and -4.5, respectively. Damping of the response over their frequency ranges is small, and damping time becomes longer than the other frequency ranges.

Table 1: Local peak frequency and damping factor

Freq.(Hz)	Damping	Freq.(Hz)	Damping
82.8	-3.5	288.8	-20.3
100.4	-4.5	299.4	-11.6
109.6	-10.9	403.6	-9.8
130.9	-18.9	487.9	-18.5
143.9	-12.6	496.8	-37.3
208.7	-15.0	515.8	-18.1
225.3	-13.3	529.2	-14.3
251.6	-20.5	535.8	-17.1
260.7	-16.9	752.3	-13.8
269.9	-17.5	884.1	-15.6

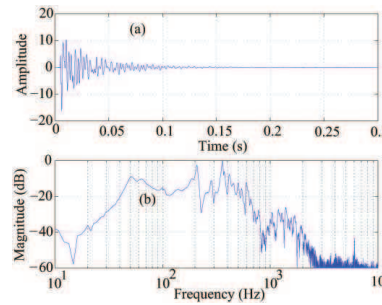


Figure 3: The measured vibration acceleration signal and its spectral characteristics. (a) Waveform of the measured vibration acceleration signal. (b) The spectral characteristics of the measured vibration acceleration signal

CONCLUSIONS

We protect the comfortable environmental sound. To know how the noise sources are generated under any conditions, we can consider how to make the comfortable environmental sound. The evaluation of the influence on the environmental sound in the downstairs room caused by the generated noise sources is understood for the resident who lives upstairs. The vibro-acoustic transmission characteristics between the vibration waves of the upstairs and the acoustic waves downstairs room are necessary to improve the accuracy of the equipment. We calculate the vibro-acoustic transmission characteristic. In the magnitude characteristics, we find many local peaks and dips that influence features of the acoustic transmission and the reverberation time in the room. Hence, we desire to estimate highly accurate each frequency and damping factor of the local peaks. To modeling the impulse response that is calculated from the transfer function estimated by the non-parametric method, we describe the estimation method and the results.

Reference [1] S. L. Marple *Digital Spectral Analysis With Applications*. (Prentice Hall, Englewood Cliffs, New Jersey, 1987)

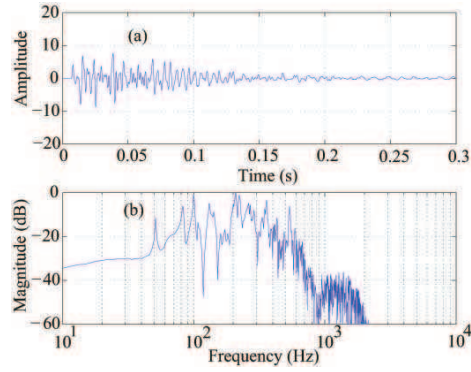


Figure 4: The measured acoustic waveform and its spectral characteristics. (a) Waveform of the measured sound pressure. (b) The spectral characteristics of the measured sound pressure waveform.

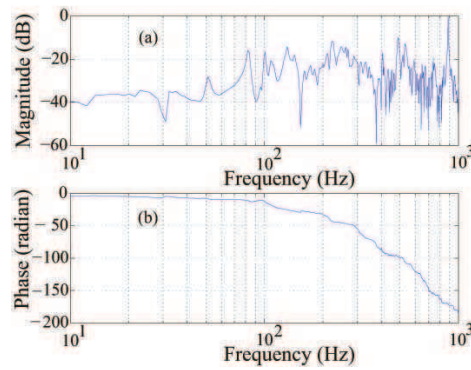


Figure 5: Transfer function between the vibration acceleration signal and acoustic signal. (a) Magnitude frequency response. (b) Phase frequency response.

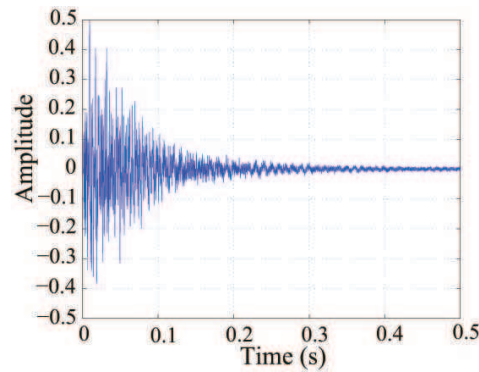


Figure 6: The impulse response between the vibration acceleration signal and acoustic signal.

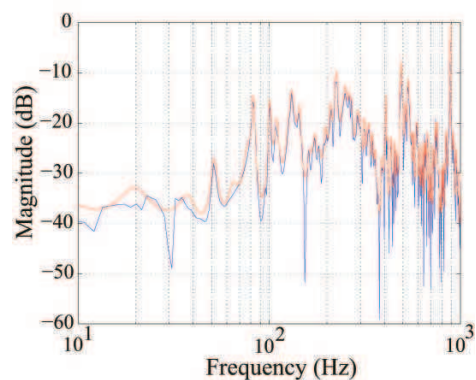


Figure 7: Comparison between the transfer functions. Thin line: The magnitude response of the signal estimated by the modeling. Heavy line: The magnitude response of the transfer function.