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A MARKOVIAN APPROACH TO THE PREDICTION OF NOISE PROPAGATION

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Abstract
The modelling of noise propagation in urban environments is a topic of considerable interest at the present time. Current methods include ray tracing, radiosity and diffusion models. In this paper a new method is presented based upon Markovian matrices. In this initial study a 2 dimensional model is employed to establish the principle of the approach in the context of noise propagation in a simple city street. The approach is then extended consider the effect of a distribution of scatterering surfaces in an urban street channel.

INTRODUCTION

There are a number of deterministic situations for which stochastic techniques have been employed to model because of their complexity. One such area is the modelling of noise propagation situations in which sound is scattered on encountering a surface rather than being reflected specularly. The stochastic model typically treats a sound ray as being scattered in a random direction whereas, in actual fact, the sound energy is re-distributed throughout the space surrounding the scattering surface in a deterministic manner.

Examples of the use of stochastic techniques to model sound propagation in the open air can be found in the paper by Oldham and Haron [1]. The propagation characteristics of city streets is a topic of considerable current interest and a number of models have been proposed based upon façade reflections which are specular, diffuse or mixed. However, these models generally assume that the street channel is empty and hence do not account for the effect of objects such as street furniture, vegetation or vehicles. In this paper we will examine the application of a particular stochastic method to the study of the propagation of sound in a street with distributed scattering elements.

MARKOV PROCESSES

A Markov process involves consideration of states which change with time [2]. The possible transitions between states can be described in terms of a probability or transition matrix where an element denoted by p_{ij} denotes the probability of a transition from state i to state j. The fundamental requirements for a Markov matrix are:

- All elements are non-negative
- The sum of each column is 1.

Gerlach [3] proposed that reverberation in a room could be modelled as a Markov process because the energy falling on a surface of an enclosure will be reflected and distributed to all the surfaces in a room according to their "visibility" with respect to the reflecting surface. This can be interpreted in terms of probability where the fraction of the reflected sound energy reaching a particular surface is equivalent to the probability of reflected energy reaching that surface. Transitions correspond to orders of reflection. This technique has been extended by Kruzin and Fricke to the study of sound propagation in an enclosure with obstructions [4]. More recently Alarcão and Bento Coelho have described the application of Markovian techniques to the study of auditorium acoustics. In this paper we will examine the application of a Markovian method to the study of the propagation of sound in a street containing scattering elements.

A 2D MARKOV MODEL FOR SOUND PROPAGATION IN A SIMPLE STREET

The basis of the Markov method is that all the boundaries of the street are divided up into patches. A series of transitions is assumed with each patch radiating sound energy to all of the other patches with a probability given by the ratio of the solid angle at the source patch subtended by the receiving patch to the total solid angle into which the source patch radiates. For this preliminary study the approach adopted was to use a 2 dimensional model in order to establish the potential of the method. Partial justification for this approach can be found from the results of a number of researchers who have observed that the noise level away from the immediate vicinity of the source tends to be uniform over the street cross section.

The 2D flanking walls of the street model were first divided into a number of patches as shown in Figure 1. Walls 1 and 2 were divided into linear patches denoted by i=1,2…n, and k=1,2..n, respectively. In developing a Markov model it is necessary to populate a matrix with the transition probabilities relating to the propagation of sound between patches. What makes the Markov method attractive is that there will be a pattern by which these transition probabilities will be repeated. For example $P_{i,\,k} = P_{i+1,\,k+1}$ and $P_{i,\,k+1} = P_{i+1,\,k+2}$ etc. Note also that the transition probabilities from patches on the bottom surface to the patches on the top surface will be identical. Thus with n patches per side there will be n probability values which will cover all n^2 possible transitions.

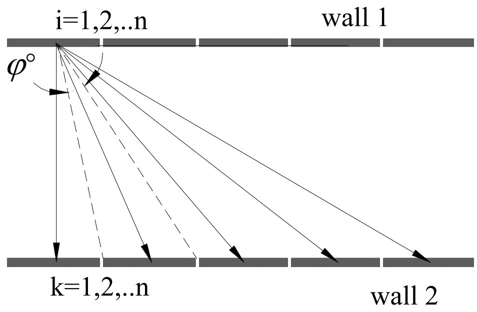

Figure 1. Radiation of sound from patch to patches on the opposite facade

To construct a true Markov matrix it is necessary to add transition probabilities relating to sound radiation from each patch out of the street ends. However, it is not necessary to divide the street ends into small patches as the transition probability will be given by the difference between the sum of all transition probabilities for patch to patch radiation and unity.

DISTRIBUTION OF SOUND ENERGY FROM SOURCE TO PATCHES AND PATCHES TO RECEIVER

Before the Markov model can be run it is necessary to distribute the sound energy of a source to the patches which can then be regarded as sound sources. The basic principle of the source energy distribution is that the fraction of energy at each patch is the same as the ratio of the angle subtended by the receiving patch divided by the total angle into which energy from the source radiates. The normal intensity at the centre of a patch can be determined using the inverse distance law as a 2D approach is being followed which means that the source is assumed to radiate cylindrically.

The source properties (sound power spectrum) can be separated out from the geometry to obtain a source function, S. For n patches per façade there will be n functions which can be written in the form of vector array.

$$S = \left[[S_1] \ [S_2] \right] \tag{1}$$

The subscripts refer to the two walls.

For two sides the array will have a dimension of 2n and, as can be seen from Figure 2, if the source is moved along the road by a distance equal to one patch length a new source vector can be constructed from the established distribution functions.

Figure 3 illustrates the radiation of sound reflected diffusely from a patch to a receiver. The intensity at the receiver can be calculated using the inverse distance law to yield a receiver function. For n patches there will again be n functions which can be written in the form of a vector array.

$$R = \left[\, [R_1] \;\; [R_2] \, \right] \tag{2}$$

For two sides the array will have a dimensions 2n and, as can be seen from Figure 3, if the receiver is moved along the road by a distance equal to one patch length then a new source vector can be constructed from the established distribution functions.

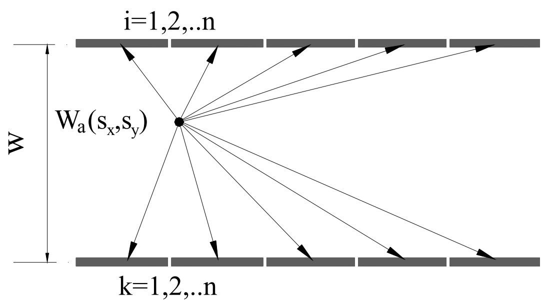

Figure 2 - Distribution of sound energy from source to patches

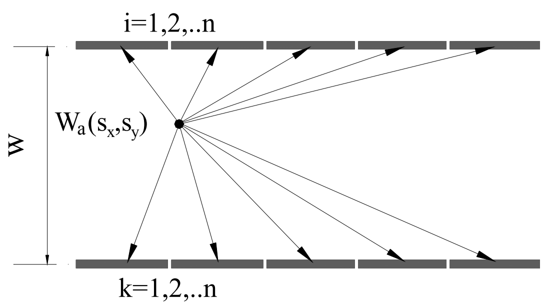

Figure 3 - Distribution of sound energy from patches to a receiver

The energy response at walls can be determined by taking all orders of transition into account. For the first transition, the energy incident on a wall, $E_{(1)}$, can be written as:

$$E_o = W_a S \tag{3}$$

Where W is the acoustic power of the noise source and S is the source function.

The energy distribution after the first transition can be obtained using the transition matrix as follows:

$$E_1 = [E_0] \quad [A] \quad [P]$$

$$(...) = (.....)\begin{pmatrix} (1-\alpha). & & \\ & (1-\alpha) & \\ & & (1-\alpha). \end{pmatrix}\begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \tag{4}$$

A is a diagonal matrix of the reflection coefficients of the wall surfaces (α is the absorption coefficient). The absorption coefficient of a building façade is commonly taken to be 0.1.

The energy distribution after the q'th order transition is given by:

$$E_q = E_{q-1}[A][P] = E_0(AP)^q \tag{5}$$

For q orders of transition the total energy at the receiver can be written as:

$$R_q = \frac{W_a}{2\pi r_{sR}} + \sum_{j=1}^{q} E_q AR \tag{6}$$

Figure 4a shows the build up of sound pressure level in a street channel as a function of transition order calculated using the 2D Markov model for a street 100m long for a street width of 10m. Figure 4b shows the calculated propagation characteristics.

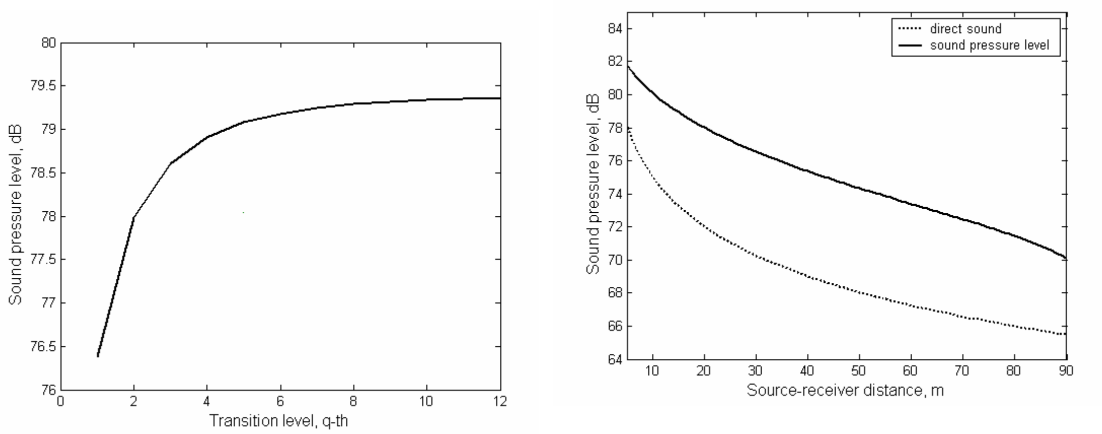

Figure 4 – (a) Sound pressure level build up as a function of transition level; (b) Sound pressure level along the street length for source-receiver distance 5 to 90m

A 2D MARKOV MODEL FOR SOUND PROPAGATION IN A STREET CONTAINING SCATTERING OBJECTS

Extension of the Markov model for the introduction of scattering objects into the street channel as illustrated in Figure 5 is, in principle, relatively simple and involves the introduction of new sound re-distributing patches and thus an enlarged transition probability matrix. It is necessary to take into account any screening effect for situations in which the scattering objects impede the line of sight between façade patches. In this case the transition probability of sound radiation between patches will be zero

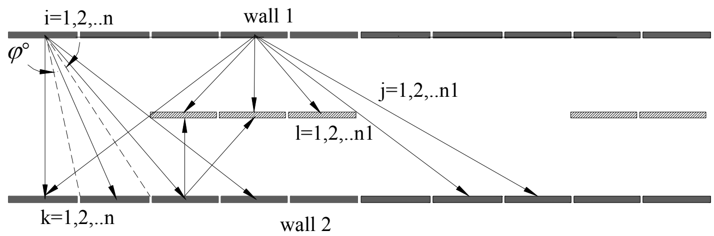

Figure 5 - Model of street containing simple obstructions

Transition probabilities required are:

i. from wall 1 to wall 2
ii. from wall 1 to screen side s1
iii. from wall 2 to wall 1
iv. from wall 2 to screen side s2

Note: it is necessary to define patches on both sides of the simple screens employed here. The complete transition probability matrix can thus be constructed a follows:

$$P = \begin{bmatrix} [P_{11}] & [P_{12}] & [P_{1s1}] & [P_{1s2}] \\ [P_{21}] & [P_{22}] & [P_{2s1}] & [P_{2s2}] \\ [P_{s11}] & [P_{s12}] & [P_{s1s1}] & [P_{s1s2}] \\ [P_{s21}] & [P_{s22}] & [P_{s2s1}] & [P_{s2s2}] \end{bmatrix} \quad (7)$$

It should be noted that as all surfaces are planes it is not possible for a patch to radiate sound to another patch on the same surface thus resulting in half of the above sub-matrices being null matrices. In addition, due to reciprocity the remaining sub-matrices consist of pairs of identical matrices.

Using a similar procedure to that described above, the distribution of sound energy between patches on the walls and the scattering screens can be determined. The following four cases have to be considered:

i. source to wall 1
ii. source to wall 2,
iii. source to screen side s1
iv. source to screen side s2,

The complete array vector for the source distribution is thus given by:

$$S = [[S_1] \quad [S_2] \quad [S_{s1}] \quad [S_{s2}]] \tag{8}$$

Using a similar procedure to that described above, the distribution of sound energy from patches on the walls to a receiver can be determined. The following four cases have to be considered:

i. wall 1 to receiver
ii. wall 2 to receiver
iii. screen side s1 to receiver
iv. screen side s2 to receiver

The complete array vector for the receiver distribution is thus:

$$R = [[R_1] \quad [R_2] \quad [R_{s1}] \quad [R_{s2}]] \tag{9}$$

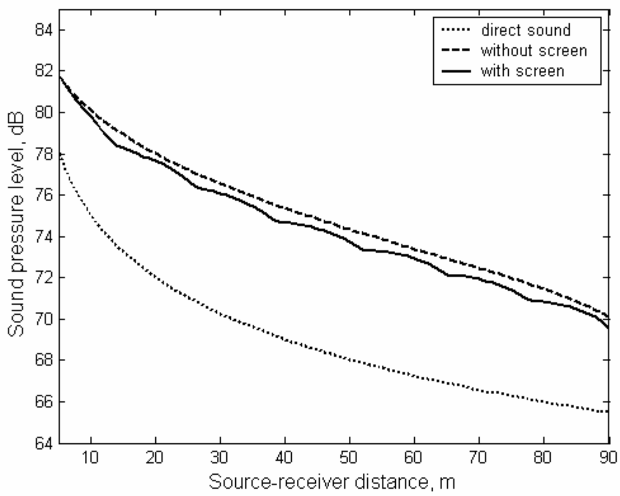

Figure 6 - Comparison between with and without screen.(solid-without screen, dashed-with screen, dashed dot- direct sound)

Figure 6 shows the propagation characteristics obtained using the 2D Markov model for a street 100m long for a street width of 10m containing a distribution of scattering screens with absorption characteristics similar to those of the facades. It can be seen that the effect of the obstacles on the street propagation characteristics is to reduce the sound pressure level along the street by approximately 2dB. As a 2D model has been assumed no sound will be scattered out of the street channel and this reduction will be due to the extra absorption introduced by the scatterers and the reduced mean free path between reflections. In the case of a 3D street there will also be the possibility of sound being scattered out of the street channel.

SUMMARY

The use of stochastic modelling techniques for investigating the propagation of sound has been discussed briefly and a particular approach, the use of the Markov chain, in the study of the propagation of sound in streets has been proposed. In this preliminary study a 2 dimensional model has been employed. It has been shown that this approach has the potential for enabling a systematic investigation of the effect of the distribution of scattering objects on the sound propagation characteristics of a street. Further work is now required to extend the method to 3 dimensions in order to enable a more detailed study of the effect of scattering objects on street propagation characteristics. In this context, it should be noted that the method has the potential to deal with scattering objects which have some degree of acoustical transparency. This aspect would be particularly useful is studying the effect of objects such as trees.

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