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# EMPIRICAL EQUATIONS FOR SOUND PROPAGATION FOR DIFFERENT ABSORBENTS

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## Abstract

The absorbent materials of the sound are already common in acoustic solutions in the scope of the architectural acoustics. These materials are part of multilayer configurations for sound insulation. They improve the insulating properties of the partition or they serve to control the excess of reverberation in enclosures in the acoustic conditioning to attenuate noise in conductions, etc. These types of materials are very diverse: recycled absorbent materials of plastic, organics, of fiber of polyester, conglomerates, etc., have appeared. At the moment, simple models exist with which characterize these materials, based on empirical formulas from different measurements. These models usually are valid only for the studied material. In this work, a study of the behaviour of these simple models is achieved and valid empirical equations are proposed for a set of absorbent materials.

## INTRODUCTION

There are many different types of sound absorbent materials. Nowadays, some simple models are used to characterize these materials. These models are based on empirical equations obtained from several measurements. The models are only valid for the studied materials. In the references there are different empirical models to predict the behaviour of some materials.

In this investigation, a study of the behaviour of these simple models has been carried out, and some empirical equations for several absorbent materials have been proposed.

## EMPIRICAL MODELS

Generally, the sound propagation through isotropic and homogeneous materials is determined by two complex values: the complex propagation constant (Γ) and the complex characteristic impedance (Z):

$$\Gamma = \alpha + j\beta \tag{1}$$

$$Z = R + jX \tag{2}$$

Nowadays, several models are based on the specific flow resistance (σ) to obtain those variables. Some of them are simple empirical models for absorbent materials that have been obtained by adjustment quadratic minimum procedures.

Like it has been shown in the Delany&Bazley or Miki models, the impedance and the propagation constant of the material are based on equations like the following ones:

$$R = \rho_o c_o (1 + A1 \cdot C^{-A2}) \tag{3}$$

$$X = -\rho_o c_o A3 \cdot C^{-A4} \tag{4}$$

$$\alpha = \frac{2\pi f}{c_o} A5 \cdot C^{-A6} \tag{5}$$

$$\beta = \frac{2\pi f}{c_o} (1 + A7 \cdot C^{-A8}) \tag{6}$$

$\rho_o$ is the air density and $c_0$ is the speed of sound in the air. The A1 to A8 coefficients are obtained by an adjustment procedure, and C is the normalized frequency:

$$C = \frac{\rho_0 f}{\sigma} \tag{7}$$

Probably the best-known empirical model is that of Delany and Bazley [1], who presented simple power-law relations obtained by best-fitting a large amount of experimental data. This model allows to calculate the values of Z and Γ from fibrous materials. Miki [2] reviews this model to simplify the coefficients and he concludes that A2 = A4 and A6 = A8.

Dunn and Davern [3] retained the same equation forms and calculated new regression constants for polyurethane foams, using few samples having low airflow resistivity values. Garai & Pompoli [4] develope a new empirical model to predict the acoustic impedance and sound absorption coefficient of polyester fibre materials.

There are manufacturers, like Rockwool ®, that have specific models. Table 1 shows the coefficients given by the mentioned authors.

*Table 1: Coefficients of the models.*

| Model | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 |
|---|---|---|---|---|---|---|---|---|
| Delany & Bazley | 0,057 | 0,754 | 0,087 | 0,732 | 0,189 | 0,595 | 0,098 | 0,7 |
| Miki | 0,079 | 0,632 | 0,120 | 0,632 | 0,179 | 0,618 | 0,122 | 0,618 |
| Dunn & Davern | 0,114 | 0,369 | 0,099 | 0,758 | 0,168 | 0,715 | 0,136 | 0,491 |
| Garai & Pompoli | 0,078 | 0,623 | 0,074 | 0,66 | 0,159 | 0,571 | 0,121 | 0,53 |
| Rockwool ® | 0,064 | 0,703 | 0,085 | 0,695 | 0,114 | 0,683 | 0,213 | 0,577 |

## PROPOSED MODEL

As it has been shown in the previous section, equations from (3) to (6) are the base to obtain Z y Γ from different materials. The mentioned authors predict the behaviour of the materials by an adjustment procedure of the coefficients. With the purpose of regrouping models, the following change is proposed:

$$R / Z_o = 1 + A_{1i} C^{-A_{2i}} = 1 + A_1 K_{i1} C^{-A_2 K_{i2}} \tag{8}$$

$$X / Z_o = -A_{3i} C^{-A_{4i}} = -A_3 K_{i1} C^{-A_4 K_{i2}} \tag{9}$$

$$\alpha / k_0 = A_{5i} C^{-A_{6i}} = A_5 K_{i1} C^{-A_6 K_{i2}} \tag{10}$$

$$\beta / k_0 = 1 + A_{7i} C^{-A_{8i}} = 1 + A_7 K_{i1} C^{-A_8 K_{i2}} \tag{11}$$

Zo is the air impedance, ko is the wavenumber in the air, A1 to A8 represent the common coefficients for all the materials, A1i to A8i represent the coefficients of the i-model (e.g., Delany&Bazley coefficients) and Ki1 to Ki2 are the coefficients that will be calculated for each material.

Therefore, with this transformation, once that A1 to A8 coefficients are obtained for the group of models, it will only be necessary to obtain two values: Ki1 and Ki2. With this proposed change, the new formulation is independent of the parameter C. For a group of n models, two systems of nonlinear equations are obtained (one with Ki1 and another with Ki2). The system for Ki1 (the odd system):

$$A_{1i} = A_1 K_{i1} \quad i = 1,\ldots,n$$

$$A_{3i} = A_3 K_{i1} \quad i = 1,\ldots,n$$

$$A_{5i} = A_5 K_{i1} \quad i = 1,\ldots,n$$

$$A_{7i} = A_7 K_{i1} \quad i = 1,\ldots,n$$

In the same way, it can be done for Ki2 (the even system). Each obtained system has 2n equations and 4+n variables, for what an iterative nonlinear optimization algorithm must be used to solve this problem, where the initial solution for starting the search are the coefficients of one of the models.

## ERROR FUNCTION

Several error functions have been realized to carry out the adjustment to the new proposed coefficients. The first proposed error function is (odd system):

$$\frac{\sum_{j=1}^{4}\sum_{i=1}^{n}\sqrt{\left\|\left(A_j K_{i1}\right)^2 - A^2{}_{ji}\right\|}}{4n} \tag{12}$$

And the second error function:

$$\frac{\sum_{j=1}^{4}\sum_{i=1}^{n}\left\|\left(A_j K_{i1}\right) - A_{ji}\right\|}{4n} \tag{13}$$

For Ki2 (even system) similar error functions have been proposed.

A Matlab function, based in the fminsearch.m function, has been designed to carry out the adjustment procedure. The coefficients of the models showed in Table 1 have been introduced as initial iteration. The obtained results with these five models and the first error function are showed in Table 2. The results with the second error function are showed in Table 3. Table 4 summarizes the obtained errors. Figure 1 shows the results of the adjustment taking as initial iteration Delany&Bazley and the second error criterion. The expressions have been normalized.

All models have been evaluated for fibrous materials, except that of Dunn&Davern. For this reason, the procedure has been repeated with the models of Table 1, except Dunn&Davern. The obtained results with the first error function are shown in Table 5. The results with the second error function are shown in Table 6. Table 7 summarizes the obtained errors.

*Table 2. Obtained coefficients with the first error function.*

| **INITIAL ITERATION** | **A1** | **A3** | **A5** | **A7** | **K11** | **K21** | **K31** | **K41** | **K51** |
|---|---|---|---|---|---|---|---|---|---|
| Delany & Bazley | 0,064 | 0,087 | 0,189 | 0,098 | 1,000 | 1,113 | 1,011 | 0,841 | 0,996 |
| Miki | 0,086 | 0,110 | 0,172 | 0,126 | 1,100 | 0,931 | 1,086 | 0,925 | 0,769 |
| Dunn & Davern | 0,114 | 0,099 | 0,168 | 0,136 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 |
| Garai & Pompoli | 0,078 | 0,074 | 0,159 | 0,121 | 1,189 | 1,006 | 1,057 | 1,000 | 0,996 |
| Rockwool ® | 0,064 | 0,085 | 0,114 | 0,213 | 1,024 | 1,094 | 1,165 | 1,000 | 1,000 |
| | **A2** | **A4** | **A6** | **A8** | **K12** | **K22** | **K32** | **K42** | **K52** |
| Delany & Bazley | 0,754 | 0,732 | 0,606 | 0,700 | 1,000 | 1,019 | 1,035 | 0,963 | 0,949 |
| Miki | 0,632 | 0,632 | 0,618 | 0,618 | 1,158 | 1,000 | 1,157 | 0,986 | 1,105 |
| Dunn & Davern | 0,369 | 0,758 | 0,715 | 0,609 | 0,988 | 0,864 | 1,000 | 0,870 | 0,917 |
| Garai & Pompoli | 0,623 | 0,660 | 0,571 | 0,530 | 1,109 | 1,015 | 0,989 | 1,000 | 1,089 |
| Rockwool ® | 0,703 | 0,694 | 0,680 | 0,586 | 1,073 | 0,909 | 1,092 | 0,951 | 1,001 |

*Table 3. Obtained coefficients with the second error function.*

| INITIAL ITERATION | A1 | A3 | A5 | A7 | K11 | K21 | K31 | K41 | K51 |
|---|---|---|---|---|---|---|---|---|---|
| Delany & Bazley | 0,062 | 0,082 | 0,139 | 0,105 | 1,065 | 1,154 | 1,212 | 1,147 | 1,041 |
| Miki | 0,064 | 0,085 | 0,142 | 0,108 | 1,021 | 1,126 | 1,183 | 1,119 | 0,998 |
| Dunn & Davern | 0,076 | 0,100 | 0,170 | 0,129 | 0,868 | 0,941 | 0,988 | 0,935 | 0,847 |
| Garai & Pompoli | 0,070 | 0,093 | 0,157 | 0,120 | 0,937 | 1,016 | 1,067 | 1,010 | 0,915 |
| Rockwool ® | 0,063 | 0,083 | 0,141 | 0,107 | 1,048 | 1,135 | 1,192 | 1,128 | 1,024 |
| | **A2** | **A4** | **A6** | **A8** | **K12** | **K22** | **K32** | **K42** | **K52** |
| Delany & Bazley | 0,676 | 0,672 | 0,657 | 0,575 | 1,089 | 0,940 | 1,088 | 0,922 | 1,039 |
| Miki | 0,668 | 0,665 | 0,650 | 0,568 | 1,101 | 0,951 | 1,099 | 0,933 | 1,051 |
| Dunn & Davern | 0,642 | 0,642 | 0,628 | 0,546 | 1,140 | 0,985 | 1,138 | 0,971 | 1,088 |
| Garai & Pompoli | 0,659 | 0,653 | 0,638 | 0,561 | 1,121 | 0,968 | 1,120 | 0,945 | 1,066 |
| Rockwool ® | 0,696 | 0,691 | 0,676 | 0,592 | 1,059 | 0,914 | 1,058 | 0,895 | 1,010 |

*Table 4. Obtained errors.*

| INITIAL ITERATION | Criterion 1 – Ki1 | Criterion 2 – Ki1 |
|---|---:|---:|
| Delany & Bazley | 0,0533 | 0,0149 |
| Miki | 0,0494 | 0,015 |
| Duna & Davern | 0,0645 | 0,0149 |
| Garai & Pompoli | 0,046 | 0,0149 |
| Rockwool ® | 0,0753 | 0,0149 |
| | **Criterion 1 – Ki2** | **Criterion 2 – Ki2** |
| Delany & Bazley | 0,244 | 0,046 |
| Miki | 0,1664 | 0,046 |
| Duna & Davern | 0,2362 | 0,046 |
| Garai & Pompoli | 0,1899 | 0,0461 |
| Rockwool ® | 0,1758 | 0,046 |

*Table 5. Obtained coefficients with the first error function.*

| INITIAL ITERATION | A1 | A3 | A5 | A7 | K11 | K21 | K31 | K41 |
|---|---|---|---|---|---|---|---|---|
| Delany & Bazley | 0,057 | 0,087 | 0,189 | 0,098 | 1,000 | 1,112 | 1,000 | 1,000 |
| Miki | 0,080 | 0,090 | 0,179 | 0,124 | 1,057 | 0,895 | 1,098 | 0,977 |
| Garai & Pompoli | 0,078 | 0,074 | 0,159 | 0,121 | 1,189 | 1,006 | 1,057 | 1,000 |
| Rockwool ® | 0,072 | 0,086 | 0,147 | 0,112 | 1,008 | 0,977 | 1,147 | 1,085 |
| | **A2** | **A4** | **A6** | **A8** | **K12** | **K22** | **K32** | **K42** |
| Delany & Bazley | 0,754 | 0,732 | 0,595 | 0,700 | 1,000 | 1,000 | 1,036 | 1,000 |
| Miki | 0,632 | 0,646 | 0,618 | 0,618 | 1,133 | 1,000 | 1,157 | 0,986 |
| Garai & Pompoli | 0,623 | 0,660 | 0,571 | 0,530 | 1,109 | 1,014 | 1,148 | 1,000 |
| Rockwool ® | 0,694 | 0,715 | 0,669 | 0,645 | 1,086 | 0,924 | 1,060 | 0,897 |

*Table 6. Obtained coefficients with the second error function.*

| **INITIAL ITERATION** | **A1** | **A3** | **A5** | **A7** | **K11** | **K21** | **K31** | **K41** |
|---|---|---|---|---|---|---|---|---|
| Delany & Bazley | 0,080 | 0,090 | 0,163 | 0,124 | 0,963 | 0,980 | 1,096 | 0,974 |
| Miki | 0,086 | 0,097 | 0,174 | 0,133 | 0,900 | 0,917 | 1,025 | 0,912 |
| Garai & Pompoli | 0,078 | 0,086 | 0,159 | 0,118 | 1,017 | 1,009 | 1,157 | 1,003 |
| Rockwool ® | 0,083 | 0,094 | 0,169 | 0,129 | 0,929 | 0,947 | 1,058 | 0,941 |
| | **A2** | **A4** | **A6** | **A8** | **K12** | **K22** | **K32** | **K42** |
| Delany & Bazley | 0,676 | 0,709 | 0,661 | 0,575 | 1,032 | 0,935 | 1,069 | 0,922 |
| Miki | 0,677 | 0,704 | 0,662 | 0,576 | 1,040 | 0,933 | 1,077 | 0,920 |
| Garai & Pompoli | 0,662 | 0,662 | 0,607 | 0,633 | 1,106 | 0,955 | 0,776 | 0,941 |
| Rockwool ® | 0,670 | 0,694 | 0,655 | 0,641 | 1,054 | 0,944 | 1,092 | 0,930 |

*Table 7. Obtained errors.*

| **INITIAL ITERATION** | **Criterion 1 – Ki1** | **Criterion 2 – Ki1** |
|---|---|---|
| Delany & Bazley | 0,0507 | 0,0102 |
| Miki | 0,0371 | 0,0103 |
| Garai & Pompoli | 0,0342 | 0,0103 |
| Rockwool ® | 0,0343 | 0,0103 |
| | **Criterion 1 – Ki2** | **Criterion 2 – Ki2** |
| Delany & Bazley | 0,2643 | 0,0557 |
| Miki | 0,1701 | 0,0557 |
| Garai & Pompoli | 0,1813 | 0,0554 |
| Rockwool ® | 0,1827 | 0,0558 |

Figures 1, 2, 3, 4 and 5 show the models of the mentioned authors and the adjustment.

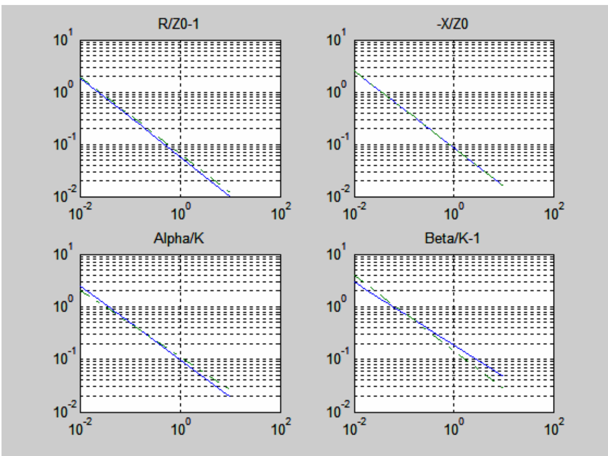


*Figure 1 - Delany & Bazley*

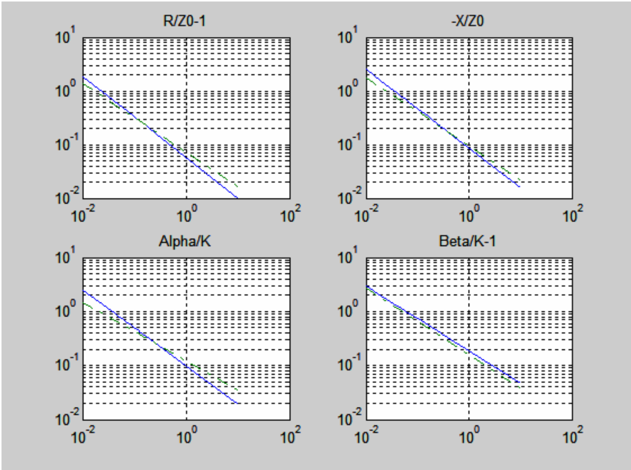


*Figure 2 - Miki*

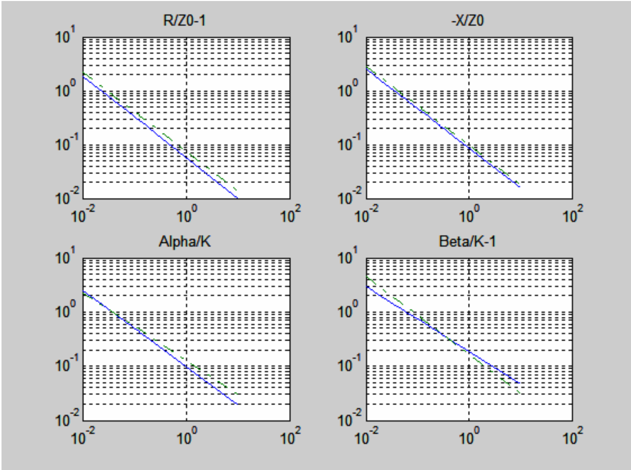


*Figure 3 - Dunn & Davern*

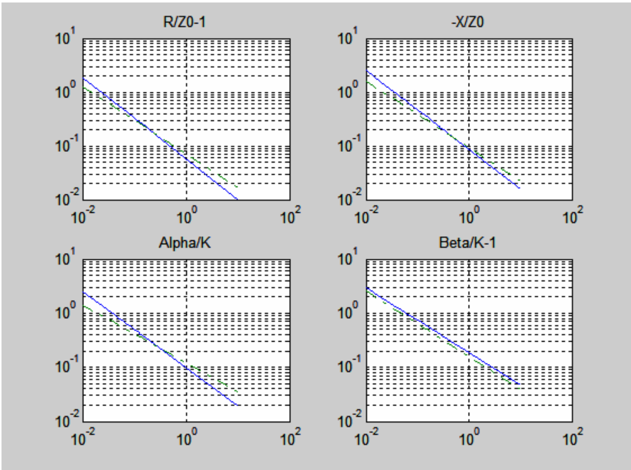


*Figure 4 - Garai & Pompoli*

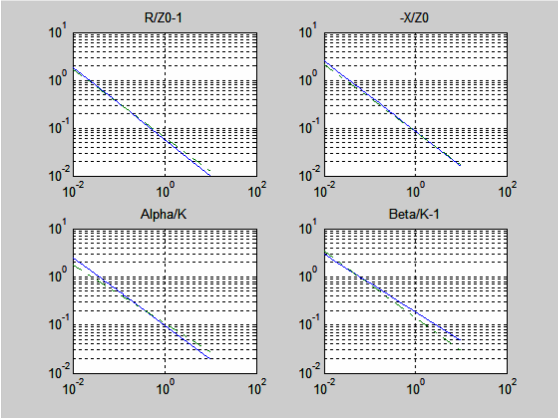


*Figure5 - Rockwool ®.*

## CONCLUSIONS

The first empirical models have been adjusted again with the new equations. As it can be observed, there are several solutions for these systems of nonlinear equations. This allows to have a bigger adjustment possibility. Another evidence of this is the fact that the coefficients, obtained with independent systems, are coherent results. Some solutions do not adjust to the original equations. However, in the original work, the dispersion in the measurements makes that the obtained solution is in the range of these original measurements (e.g., Dunn&Davern).

As future investigations, it can be appropriate to readjust the coefficients only in the range of the normalized frequency where the model is valid (the global adjustment has been carried out independently of the parameter C). Another possibility is to readjust the original measured values in each mentioned work.

## ACKNOWLEDGMENTS

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