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MODEL-BASED OPTIMIZATION OF COMPRESSOR MOUNTING USING GENETIC ALGORITHMS

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Abstract

The reciprocating motion of a piston in a reciprocating compressor excites the vibration of the compressor, which further exerts forces acting via the compressor mounting on its surroundings. In this article, genetic algorithms are employed to optimize the axial and radial stiffnesses of the springs supporting the compressor unit within the housing, the stiffnesses of the so-called pressure tube and the locations of their attachment points such that the amplitudes of the forces exerted on the surroundings are minimized. The results show that non-symmetric supporting springs, having different stiffnesses in the two radial directions, could significantly reduce the force amplitudes.

INTRODUCTION

The typical reciprocating compressor built into common domestic appliances consists of an operating unit mounted in a housing (Figure 1). The reciprocating motion of the piston excites the vibration of the unit, which via its mountings further exerts forces on the surrounding of the compressor. In other words, the compressor unit vibrations in turn excite the vibrations of the domestic appliance, which is an unfavourable effect that should be minimized.

The investigations of compressor vibrations reported in the literature have mainly focused on industrial, large-piston compressors, sometimes with more than one piston (C.G. Ong, 2000), or they have dealt with different, non-reciprocating compressors. In these investigations, analytical models of compressor dynamics were usually derived and used for prediction of various characteristics of compressor operation. However, these models appear to be rarely used for the optimization of any aspect of compressor design.

This paper reports on results of optimization of the compressor unit mounting in order to minimize the amplitudes of forces exerted on the domestic appliance. The parameters optimized are the location of the supporting springs and pressure tube (Figure 1), as well as their axial and radial stiffnesses. The optimization procedure was carried out using genetic algorithms and employing an analytical model of compressor unit dynamics (Dufour, 1995). The numerical results show that combining non-symmetric supporting springs, having different stiffnesses in the two radial directions, and relocation of the pressure tube could significantly reduce the force amplitudes transmitted to the compressor on the domestic appliance.

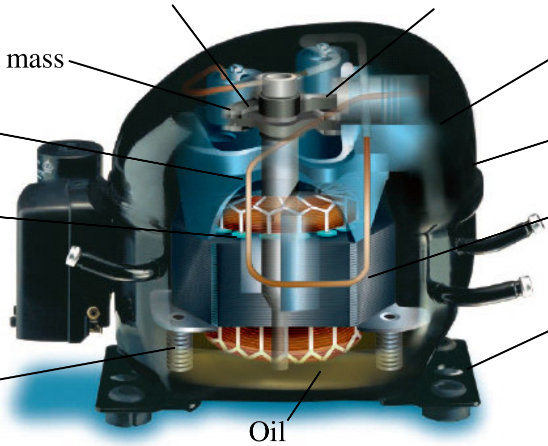

Figure 1 – Internal components of the compressor (Technical documentation)

ANALYTICAL MODEL OF THE COMPRESSOR UNIT

For the optimization procedure, an analytical model of compressor unit dynamics was used (Dufour, 1995). Since the dominant source of the force exerted by the compressor on the domestic appliance is the motion of compressor unit within the housing, the analytical model included only the compressor unit dynamics on its supporting springs, while neglecting the influences of the compressor housing and the external rubber mounting pads (Figure 1), the properties of which are of importance for force transmission.

The sketch of the compressor unit used to derive the analytical model is shown in Figure 2. The compressor unit is modeled as a rigid square block, mounted on four elastic-damping elements and connected to the housing by a fifth elastic-damping element, the pressure tube, at the top. The point T denotes the centre of inertia of the block, while the point O denotes the intersection of the motor shaft and piston motion axes. T and O are located at the origins of the Galilean reference frames xyz and $x_o y_o z_o$, respectively. It is also assumed that the displacement of T is small and that the slider crank mechanism remains in the xyz plane – thus the gyroscopic effect is neglected (Dufour, 1995).

The governing equation of motion of the compressor unit reads:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ denote the mass, damping and stiffness matrices. $\{X\}^{\mathrm{T}} = [x, y, z, \phi_x, \phi_y, \phi_z]$ is the displacement vector, combining translation and rotation of the compressor unit with respect to T, while $\{F\}^{\mathrm{T}}=[F_x, F_y, F_z, M_x, M_y, M_z]$ is the force vector. The equation of motion was solved with a modal analysis framework. The values of the mass, damping and stiffness matrix elements were determined experimentally (Pušavec, 2005). A proportional damping model was assumed (Dasgupta, 1991; Ewins, 2000), since it is the most effective way to treat damping within the modal analysis framework (Hatch 2001). For this purpose, the eigen frequencies and viscous damping ratios were determined from modal tests.

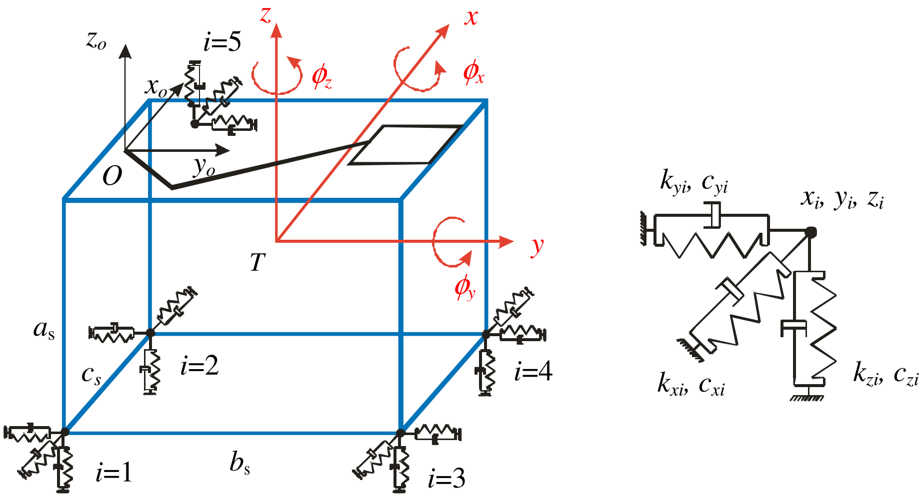

Figure 2 – The compressor reference frame

The force exerted by the compressor unit is caused by inertial and centrifugal forces and by the torque from the slider-crank mechanism (Hartog, 1972). The excitation force vector $\{F\}$ therefore depends on the motion of the slider-crank mechanism. Its equation of motion can be derived from the second-order Lagrange equation:

$$\sum_{j=1}^{n}\left[\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_j}\right)-\frac{\partial E_k}{\partial q_j}\right]\delta q_j=\sum_{j=1}^{n}Q_j\delta q_j \tag{2}$$

resulting in an expression for the angular position of the crankshaft:

$$\ddot{\theta}=f\left(\theta(t),\dot{\theta}(t),M_t\left(\dot{\theta}\right),p(\theta)\right) \tag{3}$$

where M_t and p denote the driving motor torque and the pressure in the compression chamber, respectively. The motor torque characteristic was supplied by the compressor manufacturer, while the pressure dependence on θ was calculated assuming the cooling medium to be an ideal gas. Further details about the derivation of Eq. (3) can be found in (Pušavec, 2005).

Comparison of the model and experiment

The analytical dynamic model of compressor unit motion was solved numerically (Matab, 2004). Figure 3 compares the predicted and experimentally recorded forces exerted on the compressor mounting. In both cases, the largest force amplitudes appear in the *xy* plane, in which the slider-crank mechanism moves, while the force amplitudes in the vertical, *z*, direction are much smaller. The amplitudes of predicted and recorded force traces agree quantitatively quite well, whereas the qualitative agreement is rather poor. In the experiment (right panel in Figure 3), there is a significant difference between the start-up (the first 0.8 sec) and the subsequent steady-operation mode. During start-up, the predominant large-amplitude oscillations are due to the low-frequency (approx. 7 Hz) oscillation of the compressor unit on the supporting springs. These vibrations die out abruptly, and small amplitude vibrations due to the compression rhythm (approx. 50 Hz) dominate in the steady state. In the model (left panel in Figure 3), the start-up appears to last much longer than 0.8 sec and seems to pass over into steady state operation rather smoothly, which is in stark contrast to the abrupt transition observed in the experiments. However, the dominant frequencies during start-up and steady operation in the model compare well to those observed in the experiments. The main reasons for these qualitative discrepancies between the predicted and recorded force traces can be attributed to the assumptions regarding damping, the pressure dependence in the compression chamber and other simplifications in the analytical model.

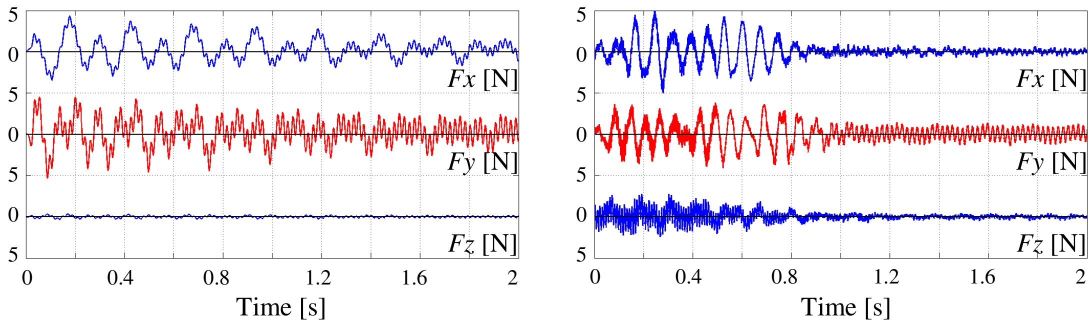

Figure 3 – Predicted (left) and measured (right) forces exerted on the domestic appliance in all directions xyz

Since the quantitative agreement between the predicted and recorded force amplitudes was quite good, the derived model was then employed in the optimization of the compressor unit mounting using genetic algorithms.

BASICS OF OPTIMIZATION USING GENETIC ALGHORITMS

A genetic or evolutionary algorithm applies the principles of evolution found in nature to find an optimal solution to a problem (Goldberg, 1998). An algorithm begins with a set of solutions called the "population" that are characterized by a set of parameters (chromosomes). Solutions from one population are then taken and modified to form a population of the next generation. The modifications are motivated by the expectation that the new population will be better than the old one. The solutions which are chosen from the old population to form the new one (offspring) are selected according to their fitness with respect to the given criteria; the more fit they are, the more chance they have to reproduce (Goldberg, 1998).

Genetic algorithms use three main operations to create the next-generation population: (1) Selection is inspired by the role of natural selection in evolution – an evolutionary algorithm performs a selection process in which the "most fit" members of the population survive; (2) Crossover is inspired by the role of reproduction in the evolution of living organisms – an evolutionary algorithm attempts to combine elements of existing solutions in order to create a new solution, with some of the futures of each parent; (3) Mutation is inspired by the role of mutation of an organism's DNA in the evolution – an evolutionary algorithm periodically makes random changes or mutations to one or more members of the current population, yielding a new candidate solution, which can be better or worse than the preceding population members.

The outline of the genetic algorithm employed in this paper is: (1) Generate a random population of *n* suitable solutions for the problem; (2) Evaluate the fitness f_s for each solution *x* in the population; (3) Create a new population by repeating the steps of selection, crossover, mutation and accepting until the new population is complete; (4) Use the newly generated population for the subsequent run of the algorithm; (5) If the end condition is satisfied, stop and return the best solution in the current population with respect to the fitness function f_s.

RESULTS OF OPTIMIZATION

The subject of the optimization is the mounting of the compressor unit, consisting of five independent springs: four supporting springs and one pressure tube (Figure 1). Each of the springs is characterized by its axial and radial stiffness, and by its location or point of attachment to the compressor unit. Therefore, there are three plus two independent parameters per individual spring respectively describing its stiffness and position in the *xy* plane. Altogether there are 25 independent parameters (chromosomes) to optimize.

The fitness function, which characterizes each solution, should be determined based on the aim of the optimization. In the present example, the aim is to reduce the amplitudes of forces exerted on the compressor mounting. Thus, one of the possible fitness functions is:

$$f_s = p2p(F_x) + p2p(F_y) + p2p(F_z) \tag{4}$$

where *p2p*(*x*) represents the peak-to-peak range of *x*, i.e. a difference between the maximum and minimum of *x*. Since the smaller the value of the function in Eq. (4), the more favorable the solution, the function given in Eq. (4) is not exactly a fitness function – it is more of an “unfitness” function.

Optimization was run for 200 generations, each having a population size of 20, of which 2 individuals are guaranteed to survive. The mutation probability decreased linearly with subsequent generations, $p_{mut}=1-i_{gen}/200$, and the crossover probability was 0.8. Beside the optimization parameters, some restrictions on compressor unit translation and rotation had to be defined in order to avoid collision of the unit with the housing. The unit was allowed to move at most 20mm in x, y, and z directions, and rotate at most 1.2° around the x and y axes and 3.5° around the z axis.

The optimization was run three times with different sets of parameters being optimized: (1) spring position only (10 parameters), (2) spring stiffness only (15 parameters), and (3) both spring location and stiffness (25 parameters). The value of the fitness function before the optimization was 18.9, while its values after the three optimizations were respectively 14.5, 7.0, and 10.0. The best results were therefore obtained in optimization (2), in which the force amplitude ranges were decreased by more than 50%. It is interesting that optimization (3), in which both location and stiffness of the springs were optimized, yielded poorer results than the optimization of stiffness alone (2). A possible explanation for this difference in the optimization results is the fact that the same number of generations was used in all three optimizations despite different numbers of parameters optimized. Using more generations in optimization (3) would probably further decrease the final value of the fitness function.

Results of optimizations (1) and (2) are shown in the left and right panels of Figure 4, respectively. It is interesting that the positions of the four supporting springs remained almost unchanged after the optimization, while the position of the bent pressure tube was moved along the y axis, closer to the compression chamber. This indicates that the position of the supporting springs was already close to optimal.

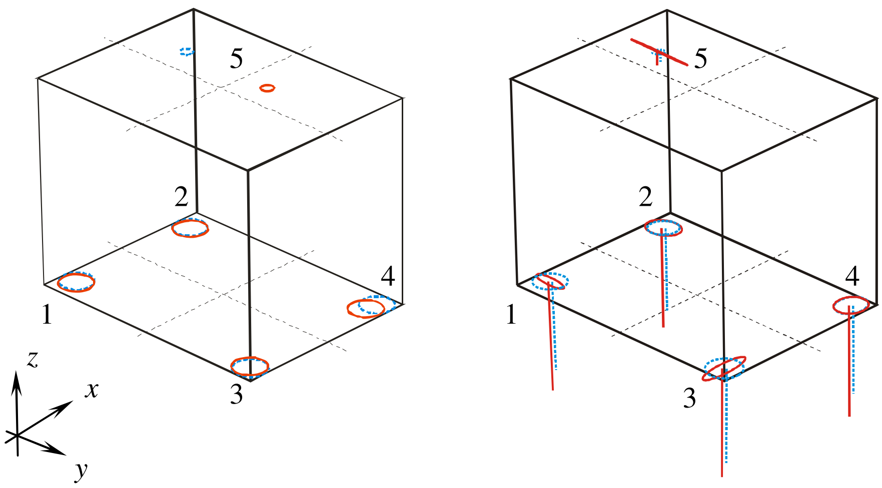

Figure 4 – Spring position (left) and stiffness (right) before (--) and after (–) the respective optimization of positions and stiffness

The optimization of stiffness (right panel in Figure 4) yielded higher axial stiffnesses k_z for all supporting springs and markedly different radial stiffnesses k_x and k_y for supporting springs 1 and 3 and the pressure tube (spring 5). The spring stiffnesses before and after the optimization (2) are also listed in Table 1. Closer examination of Table 1 also reveals that the optimal four supporting springs should all be radially non-symmetric and of different stiffnesses.

Finally, the predicted force traces before and after optimization (2) are compared in Figure 5. The amplitudes of forces in x and y directions after optimization are indeed considerably smaller than before optimization, while the force amplitude in the z direction remains almost unchanged. Moreover, the optimized spring stiffnesses have also changed the force oscillations in x and y directions in a qualitative manner: the low-frequency supporting spring vibrations appear to be much less pronounced than before optimization.

Table 1 – Spring stiffness before (BO) and after (AO) optimization

	Axis\Position *i*		1	2	3	4	5
Radial stiffness	k_x [N/m]	*BO*	1700	1700	1700	1700	600
		AO	583	1672	2627	1781	107
	k_y [N/m]	*BO*	1700	1700	1700	1700	675
		AO	1977	2181	706	1593	3993
Axial stiffness	k_z [N/m]	*BO*	4900	4900	4900	4900	400
		AO	6110	5963	5667	6249	658

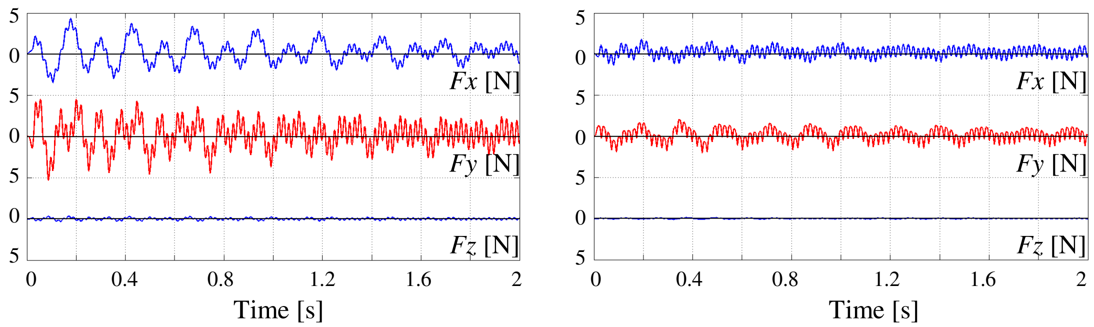

Figure 5 – Predicted forces exerted on the domestic appliance before (left) and after (right) optimization of spring stiffness

CONCLUSIONS

The paper presented the optimization of the mounting of a reciprocating compressor unit within a housing with the purpose of minimizing the amplitude of the forces exerted by the unit on the compressor's surroundings. The optimization procedure was based on genetic algorithms. The procedure relied on an analytical model of compressor unit dynamics, the predictions of which agreed quantitatively well with the experiment. The parameters optimized were the stiffness and location of the supporting springs and the pressure tube. The results of the optimization showed that the locations of the supporting springs were already close to optimal, while the pressure tube should be moved closer to the compression chamber. Each of the four supporting springs should have a different stiffness. In general, their axial stiffnesses should be increased, while their radial stiffnesses should be non-symmetric. According to the model predictions, such an optimal mounting would reduce the amplitudes of force exerted by the compressor unit on its surroundings by approximately 50%.

In summary, it was shown that a judiciously designed mounting of the reciprocating compressor unit within the housing, employing supporting springs with radially non-symmetrical stiffnesses can markedly decrease the amplitudes of forces exerted by the compressor on its surroundings.

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