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ANALYSIS AND ASSESSMENT OF THE SENSITIVITY OF TRIM PARAMETERS ON SEA SIMULATION FOR INTERIOR NOISE REDUCTION

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Abstract

To predict and analyse the acoustical behaviour of complex structures at higher frequencies the field of deterministic analysis methods like FEM or BEM has to be left. In this paper the energy based statistical approach called Statistical Energy Analysis (SEA) is used.

To optimize acoustic packages of vehicles a definition of interior equipment (trim) in SEA models is required. One possibility is to use absorption, damping and insertion loss as parameters for the modelling of each trim part. Using this definition all different trim parts can be described through the same type of parameters, which is a big advantage of this rather abstract approach.

In defining optimal acoustic packages for vehicles, engineers face many problems. Definition of acoustic packages is extremely time critical and changes of the geometry of the vehicle have to be avoided in the development process. Furthermore, acoustic packages are limited by weight and cost requirements.

Having knowledge about areas, where a change of trim behaviour has a large influence on the sound pressure level (SPL) in the passenger compartment, is extremely helpful during defining acoustical packages. To provide this information a method using a mathematical sensitivity analysis had been developed. This method computes influences of trim parameters (absorption, damping and insertion loss) of different trim parts on the SPL. A detailed description of this method and its mathematical background will be given in this paper.

Through principle examples the developed method will be illustrated and verification will be done using a full vehicle model. Application of a mathematical optimization, which is based on this method, will be a scope of future work.

INTRODUCTION

The application of the Statistical Energy Analysis (SEA) in the early stage of vehicle development has been increased during the last few years. SEA computations are used for prediction of sound pressure levels (SPLs) and surface vibration velocities at frequencies above 400 Hz.
During the vehicle development process, it is important to obtain the SPLs in the vehicle cabin with minimal effort of weight and costs. At higher frequencies the SPL fitting process is mainly trial and error based on the knowledge of acoustic engineers.
With the motivation to make the fitting process more effective and more deterministic, this paper focuses on a mathematically based sensitivity analysis. This method allows the user to identify the most important parameters of vehicle trim parts and to analyse their influence on the SPL of the interior cabin.

SEA Theory

For SEA applications, the object of interest has to be parted into several substructures. Substructures are either parts of the body or of the cavities. Each subsystem is defined by a characteristic length, area or volume, material and damping parameters.

Computation

Having two adjacent subsystems an energy balance equation can be drawn; [4]. There are input powers and power losses for each of the subsystems resulting in a power flow from one subsystem to the other. The power loss in one subsystem can be defined by the mean energy and the internal loss factor (ILF) of each of the subsystems. The power loss between subsystems is defined by the mean energy of the emitting subsystem and the coupling loss factor (CLF) between the emitting and the receiving subsystem.
The power balance equation (1) can be written in matrix form, whereas the vector of the two input powers P of both subsystems is equal to the angular frequency ω multiplied by the coupling matrix L (matrix of loss factors) between the two subsystems and multiplied by the vector of the relevant mean energies E; [4].
This system of linear equations completely describes the energy behaviour of the subsystems and can be expanded to any number of subsystems. Throughout this paper we assume that the number of subsystems is n. Then the power balance equation can be written as

$$\omega L E = P \tag{1}$$

where E and P are n-dimensional vectors, ω is the angular frequency and L is an n x n dimensional matrix.

Integration of TRIM

There are different possibilities to include trim into SEA models. One possible approach describes the influence of trim parts using BIOT theory; [1]. This proceeding requires a detailed description of each trim layer. For this reason results are strongly dependent on the quality of the parameters for each layer. On the one hand a lot of cost intensive measurements are needed to get adequate solutions. On the other hand obtaining sensitivities for all required parameters leads to very complex interpretations.
A second method introduced in [2] defines trim parts as their influence on the CLFs and ILFs. This influence is calculated from the absorption, insertion loss and additional damping of the trim part. In contrary to the BIOT theory approach sensitivities for this triple of parameters are straightforward and meaningful in practice. For this reason the sensitivity analysis, presented in this paper, is based on the second approach.

PROBLEM FORMULATION

SEA Basics

For calculations it is important to investigate mathematical properties of the loss matrix L. The off diagonal elements of L represent the negative CLF values η_{ij}. As stated in (2) ILFs (η_i) are placed on the diagonal of the L matrix. For the reason that ILFs correspond to the damping of their related subsystem they are always greater than zero.
Due to this restrictions for ILFs and that the CLF values are greater than or equal to zero L is strictly column diagonally dominant and therefore an invertible matrix [7].

$$L = \begin{bmatrix} \eta_1 + \sum\limits_{j \neq 1} \eta_{1j} & -\eta_{21} & -\eta_{n1} \\ -\eta_{12} & & \vdots \\ -\eta_{1n} & -\eta_{2n} & \eta_n + \sum\limits_{j \neq n} \eta_{jn} \end{bmatrix} \tag{2}$$

For a single SEA computation and a fixed frequency L is a constant matrix. To determine the sensitivity of trim parameters, the influence of single parameter changes on L has to be taken into account.
Concerning computational reasons L is written as a sum of two matrices (3). The constant part of L, as used for a single SEA calculation, is represented in matrix $L_{default}$ with elements d_{ij} The changes of L with respect to the change of m trim parameters which are collected in the vector x are defined by matrix L_{update}, where the

scalar-valued functions u_{ij} represent the influence of the parameters on the ILFs and CLFs.

$$L(x) = \begin{bmatrix} l_{11}(x) & \dots & l_{1n}(x) \\ \vdots & & \vdots \\ l_{n1}(x) & \cdots & l_{nn}(x) \end{bmatrix} := \begin{bmatrix} d_{11} + u_{11}(x) & \dots & d_{1n} + u_{1n}(x) \\ \vdots & & \vdots \\ d_{n1} + u_{n1}(x) & \cdots & d_{nn} + u_{jn}(x) \end{bmatrix} = L_{default} + L_{update}(x) \quad (3)$$

As stated in (2) $L_{default}$ is a strictly column diagonally dominant matrix. Due the restrictions of physical laws the addition of L_{update} to $L_{default}$ maintains the diagonally dominance property. Therefore *L(x)* is still an invertible matrix for any admissible values of the parameter vector *x*.

Mathematical Problem Formulation

In future optimization problems of the following type will be considered:

$$\begin{aligned} &\boldsymbol{min}\ \hat{f}(x) = f(E(x)), \\ &\text{where } E(x) = \frac{1}{\omega} L(x)^{-1} P \end{aligned} \qquad (4)$$

The function $\hat{f}$ defines a cost value for each vector in the parameter space. Hence, the sensitivity of costs with respect to changes in the parameters can be analyzed.

Cost Function

To apply the sensitivity analysis, it is assumed that $\hat{f}$ is twice continuously differentiable. To formulate an adequate cost function energies of the defined subsystems within the SEA model have to be taken into account. This energies open the possibility to consider SPLs and vibration velocities in the cost function.
An example for a cost function formulation is

$$\hat{f}(x) = f(E(x)) = (E_i - E_{desired})^2 \qquad (5)$$

where E_i represents the energy of the subsystem of interest and $E_{desired}$ is the energy that should be reached in this subsystem.

Application of Taylor's Theorem

Let $\hat{f}$ be the cost function and $\bar{x} \in \Re^m$ be an arbitrary, but fixed vector of the defined parameters. Due to smoothness of $\hat{f}$ this function can be approximated in a ball around $\bar{x}$ through a polynomial function using Taylor's theorem; equation (6).

Let B be a ball in $\Re^m$ centered at $\bar{x}$, and $\hat{f}$ be the introduced cost function defined on the closure $\bar{B}$ being twice continuously differentiable. First order Taylor's theorem asserts that for any $h \in \Re^m$ with $(\bar{x}+h) \in \bar{B}$

$$\begin{aligned} &\hat{f}(\bar{x}+h) = \hat{f}(\bar{x}) + \nabla\hat{f}(\bar{x})h + \frac{1}{2}h^T\nabla^2\hat{f}(\bar{x}+\vartheta h)h \qquad (0<\vartheta<1) \\ &\left|\frac{1}{2}h^T\nabla^2\hat{f}(\bar{x}+\vartheta h)h\right| \le c\|h\|^2 \qquad c\in\Re \end{aligned} \tag{6}$$

The quality of the approximation is given by the second order term and is small for small values of *h*. Using this approximation the change of the function value can be described through the changes of parameter values.
Because of the power balance equation stated in equation (1) and the definition of the loss matrix L as subject to x in equation (3), energies are subject to the changes of trim parameters x. Therefore the used cost function, which is subject to these energies is dependent on x too.
Hence, to compute the gradient of f with respect to x the chain rule is applied, as stated in equation (7). For calculation of the gradient of E with respect to x applicable methods are given in [5].

$$\begin{aligned} &\nabla_x\left(\hat{f}(x)\right) = \nabla_x\left(f(E(x))\right) = \nabla_E f(E(x))\nabla_x E(x) \\ &\text{where } E(x) = \frac{1}{\omega}L(x)^{-1}P \end{aligned} \tag{7}$$

Formulation of Assessment Criterion

Considering the first order Taylor's theorem the difference between the function value at x and $x+h$ can be determined through the distance vector h, the function value at x and the gradient of $\hat{f}$ with respect to x. For the reason that the function value at x is constant the change of the function value can be described through the multiplication of h and the gradient of $\hat{f}$ in respect to x.
Application of Taylor's theorem gives the sensitivity of the cost function with respect to changes in the parameter vector. To formulate an assessment criterion for the sensitivity of a specific parameter, the sensitivity of the cost function with respect to a change in the direction of the parameter of interest is regarded.

EXAMPLES

To verify presented sensitivity analysis two SEA models have been used. For practical demonstration in this paper one cost function and two assessment criterions have been taken into account.

Used Cost Function and Assessment Criterions

The formulation for the used cost function f relates to the quadratic difference of an energy within one subsystem of interest E_i and the desired energy $E_{desired}$ in the same subsystem.

$$f\left(E(x)\right) = \left(E_i - E_{desired}\right)^2 \tag{8}$$

The influence of assessment criterions on the sensitivity analysis is illustrated on two different examples. The first (a_{1_i}) only takes components of the gradient of the cost function (∇f_i) into account, while the second (a_{2_i}) additionally considers the distance vector h used in equation (6). For better comparison between the parameters, assessment values are calculated relatively to the maximal assessment. Hence, an assessment near to 1.0 shows that the parameter has big influence on the energy E_i.

$$a_{1_i}(\nabla f) = \frac{\nabla f_i}{\max\limits_{j\in\{1,\ldots,m\}} \nabla f_j} \qquad a_{2_i}(\nabla f, h) = \frac{h_i \nabla f_i}{\max\limits_{j\in\{1,\ldots,m\}} h_j \nabla f_j} \qquad i \in \{1,\ldots,m\} \tag{9}$$

Verification of the Sensitivity Analysis and its practical Application

To demonstrate the validity of the sensitivity analysis a very simple model consisting of ten steel plates and two separated cavities with an excitation in the engine compartment has been used; Figure 1 (left). For verification and application of the method 3 damping, 4 absorption and 2 insertion loss parameters have been chosen (Table 1). For all testing E_i represents the energy of the passenger compartment.

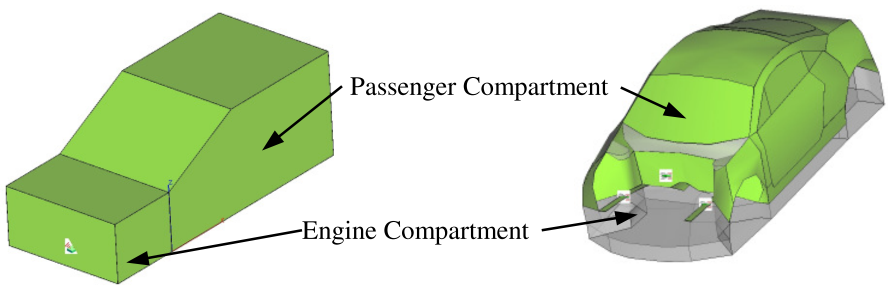

Figure 1 – Used SEA models for verification (Left: principle example; Right: Audi TT model)

Table 1 – Trim Parameters for Verification

Subsystem Name	*Damping*	*Absorption*	*Insertion Loss*
Firewall Engine Compartment	DAM_FW_EC	ABS_FW_EC	IL_FW_EC
Firewall Passenger Compartment	DAM_FW_PC	ABS_FW_PC	IL_FW_PC
Floor Passenger Compartment	DAM_Floor_PC	ABS_Floor_PC	
Roof Passenger Compartment		ABS_Roof_PC	

Applying assessment criterion one for the principle example, results recommend that an absolute change of the floor damping of the passenger compartment has the same effect as an absolute change of the floor absorption. This is only true if the domains for the parameter values have the same size.
In practical examples domain sizes for damping and absorption are different. Hence a scaling of the gradient vector is used to take this difference in domain sizes into account. This scaling is done in a_2 by using the distance vector h, where each component of the vector is dependent on the domain size of the related parameter. Considering the domain size of the parameters in the assessment criterion leads to more relevant results. Applying a_2 absorptions of floor, roof and on the engine compartment side of the firewall have the greatest influence on the energy E_i.

Table 2 – Sensitivities Assessment for Trim Parameters in Principle Example

Shortcut	a_1
DAM_Floor_PC	1.00
ABS_Floor_PC	0.98
ABS_FW_EC	0.77
ABS_Roof_PC	0.58
DAM_FW_EC	0.44
DAM_FW_PC	0.44
ABS_FW_PC	0.26
IL_FW_PC	0.00
IL_FW_EC	0.00

Shortcut	a_2
ABS_Floor_PC	1.00
ABS_FW_EC	0.78
ABS_Roof_PC	0.60
ABS_FW_PC	0.26
DAM_Floor_PC	0.03
IL_FW_PC	0.01
DAM_FW_EC	0.01
DAM_FW_PC	0.01
IL_FW_EC	0.00

For practical application a model of an Audi TT has been used; Figure 1 (right). The model was separately excited by an 1 Watt power in the engine compartment and a measured force at the trunk. Results show that independent of the excitation an increase of absorption on the roof and the floor of the passenger compartment have the highest influence in decreasing the energy in the passenger compartment.

Table 3 – Sensitivities Assessment for Trim Parameters in Practical Example

Engine Compartment Ex.	
Shortcut	a_2
ABS_Roof_PC	1.00
ABS_Floor_PC	0.79
ABS_FW_EC	0.41
ABS_FW_PC	0.35
IL_FW_EC	0.22
IL_FW_PC	0.07
DAM_Floor_PC	0.02
DAM_FW_EC	0.00
DAM_FW_PC	0.00

Trunk Floor Excitation	
Shortcut	a_2
ABS_Roof_PC	1.00
ABS_Floor_PC	0.79
ABS_FW_PC	0.35
DAM_Floor_PC	0.00
IL_FW_PC	0.00
ABS_FW_EC	0.00
DAM_FW_EC	0.00
DAM_FW_PC	0.01
IL_FW_EC	0.00

On the contrary, additional damping on the firewall and on the floor has little effect for an energy reduction. In practice this means that reducing the damping in these areas would save weight and costs without enlarging the energy of the passenger compartment.

CONCLUSIONS

A mathematically based sensitivity analysis for trim parameters has been introduced in this paper. A verification of the method has been shown on a principle example. Furthermore the application of the method on a passenger car points out areas where additional absorption material should be placed to reach maximal noise reduction. Additionally, it identifies areas where a reduction of trim parameters does not increase the energy in the passenger compartment. These promising results demonstrate that the implementation of the sensitivity analysis, presented in this paper, supports the car development process and reduces weight and costs while keeping the same quality of sound in the passenger compartment.
Finding a weight and cost optimal combination for trim parameters will still be a challenging task. Therefore, the development of a mathematical optimization of trim behaviour, which is based on presented sensitivity analysis, will be scope of future work.

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