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ANALYSIS OF WAVE PROPAGATION IN FLUID-FILLED VISCOELASTIC PIPES

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Abstract

This paper describes the investigation of the propagation wave speed and wave attenuation in viscoelastic fluid-filled pipes. Relatively predictable for metal pipes, these are largely unknown for plastic pipes, since they depend on the pipe wall properties. Wave number measurements, encompassing both wave speed and wave attenuation, were carried out on different water-filled plastic pipes using three hydrophones. The frequency dependent wave speed and attenuation were calculated from the transfer function between three pressure measurements. Experimental results for different pipe wall materials, particularly those with applications in water supply installations, are presented. The purpose of this paper is to present a method of analysis in the frequency domain that can be used to determine the acoustical properties of fluid-filled plastic pipes.

INTRODUCTION

The complex wave speed (complex-valued and frequency-dependent) is used in the standard impedance or transfer matrix method [1] and an impulse response method has been proposed to compute the non-periodic transients [2]. A similar method applies the concept of transmission loss instead of the concept of wall impedance [3]. The Poisson-coupled vibrations in an elastic pipe and the fluid in the pipe could potentially yield transfer matrix relations relating the amplitude quantities of motion transmitted through a length of pipe [4]. The amplitude relation is good for modal analysis and useful for deriving the transmission loss matrix of pipe, but is not suitable for complete transmission of the sound pressure wave (amplitude and phase). An extended method, which uses the static mechanical properties and frequency-dependent mechanical properties of the pipe wall, has been proposed [5].

In the presented work, a transfer function method is proposed. Compared to the transfer matrix method, the transfer function method is based on an analysis of the transfer function relationships between three sound pressure measurements in a pipe [6, 7]. We show that it is possible to calculate the wave speed and attenuation from the transfer function using simple wave propagation formulations.

WAVE PROPAGATION

The acoustic wave number of plane waves propagating in a pipe with unknown termination is determined from pressure measurements at three separate locations along the pipe. If we consider a fluid-filled pipe of arbitrary length and a plane acoustic pressure excitation P_0 applied through fluid on one side of the pipe:

$$P_0(\omega) = A_0 \cdot e^{j\omega t} \tag{1}$$

where A_0 is the pressure amplitude, $j = \sqrt{-1}$, ω is the angular frequency and t is time. The pressure at any point along the pipe can be represented by two waves traveling in opposite directions. The wave number is defined as $k = \omega/c$ or $k = 2\pi/\lambda$, where c and λ represent wave speed and wavelength. Omitting the time dependence for convenience, the pressure P at an arbitrary position x is defined as:

$$P(\omega, x) = A \cdot e^{-jkx} + B \cdot e^{jkx} \tag{2}$$

If we consider a pipe along which three pressure transducers are mounted and are equally spaced (as shown in Fig. 1), such that $\Delta x_1 = \Delta x_2 = \Delta x$ and $x_2 = 0$. From Eq. (2) it follows that:

$$P_1 = A \cdot e^{jk\Delta x} + B \cdot e^{-jk\Delta x}; \quad P_2 = A + B; \quad P_3 = A \cdot e^{-jk\Delta x} + B \cdot e^{jk\Delta x} \tag{3}$$

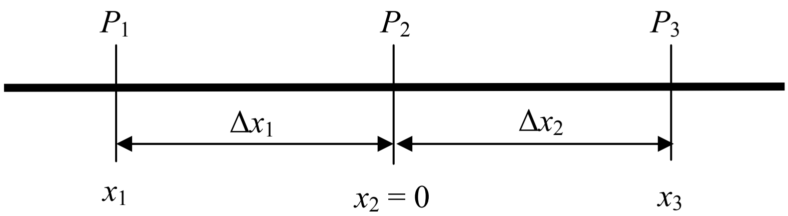

Figure 1 – Locations of the pressure transducers along the pipe.

Therefore,

$$\frac{P_1 + P_3}{P_2} = \frac{(A+B) \cdot e^{-jk\Delta x} + (A+B) \cdot e^{jk\Delta x}}{2 \cdot (A+B)} = \frac{1}{2}(H_{12} + H_{32}) = \cos(k \cdot \Delta x) \tag{4}$$

where H_{12} is the pressure transfer function between points 1 and 2, defined as P_1/P_2 , and H_{32} as P_3/P_2. The wave number (the last term in Eq. (4)) is thus defined from the three pressures as:

$$k = \frac{1}{\Delta x} \cdot \arccos\left(\frac{P_1 + P_3}{2P_2}\right) \tag{5}$$

In the case of a viscoelastic pipe, the wave number is complex and encompasses both the wave speed and wave attenuation. Therefore, one can introduce the complex form of the wave number as:

$$k = k_r + j \cdot k_i \tag{6}$$

where k_r is the real part of the wave number and k_i is the imaginary part of the wave number, referred to as the frequency-dependent attenuation coefficient $\alpha(f) = \mathrm{Im}(k)$. Substitution of the complex wave number into Eq. (2) yields the following explicit expression for wave amplitude:

$$P(\omega, x) = A \cdot e^{-\alpha x} \cdot e^{-jk_r x} + B \cdot e^{-\alpha x} \cdot e^{jk_r x} \tag{7}$$

and for the corresponding transfer function:

$$H(\omega, \Delta x) = e^{-j\omega k_r} \cdot e^{-\alpha\omega\Delta x} \tag{8}$$

With this expression for the pressure, Eq. (4) could be rewritten as:

$$\frac{1}{2}(H_{12} + H_{32}) = \cos(k \cdot \Delta x) \cdot \cosh(\alpha \cdot \Delta x) - j \cdot \sin(k \cdot \Delta x) \cdot \sinh(\alpha \cdot \Delta x) \tag{9}$$

In this case, when the viscoelasticity of the pipe wall is included in wave equation, the wave number is defined as $k = \omega/c_p$ (or $\mathrm{Re}(k) = \omega/c_p$ in the case of the complex wave number), where c_p is the phase velocity. The experimentally determined wave number (or wave speed) and attenuation coefficient can be calculated from Eq. (9) using the measured transfer function.

VERIFICATION OF THE METHOD

The effectiveness of the proposed method presented here is assessed for different pipe materials. All pipes had a nominal diameter of 1" and were designed for use in water supply installations. To clarify the impact of the viscoelastic properties of the pipe material on wave propagation, the operating conditions, measurements, and calculation procedures were kept the same for all tests.

Experimental set-up and procedure

The hydraulic circuit for measuring the transfer function consisted of straight pipe sections, 2 m in length each, connected to a supply installation. The hydrophones were mounted at the each end of the section. In the tests, water at 9 °C was used as the fluid (c = 1482 m/s). The measured section was suspended on foam pads at a certain angle, ensuring that no air bubbles were trapped within the measured sections. Two different pipe materials were examined in the experiment: the pipe made of polybutylene pipe has a wall thickness 3 mm and the polyethylene (PE) pipe had a wall thickness of 4.4 mm. Signals taken from the hydrophones (Brüel&Kjær 8103) were simultaneously fed via charge amplifiers (Brüel&Kjær 2626) into the dual channel FFT analyzer (Brüel&Kjær 2032). The FFT analyzer sampling frequency was 65 kHz, while the chosen frequency span of 6.4 kHz included all relevant frequency information. The sampling interval was 61 μs (8 Hz) and the record length was 125 ms. The measurement was conducted by obtaining 100 spectral averages of the transfer function between the two hydrophones using the signal analyzer.

EXPERIMENTAL RESULTS

The hydrophone arrangement allows for two sets of three equidistant spaced pressure measurements to be analyzed, namely the transfer functions H_{12} and H_{32}. In the following figures (Figs. 2 and 3) the measurement results of the $\mathrm{Re}(1/2(H_{12} + H_{32}))$ are shown.

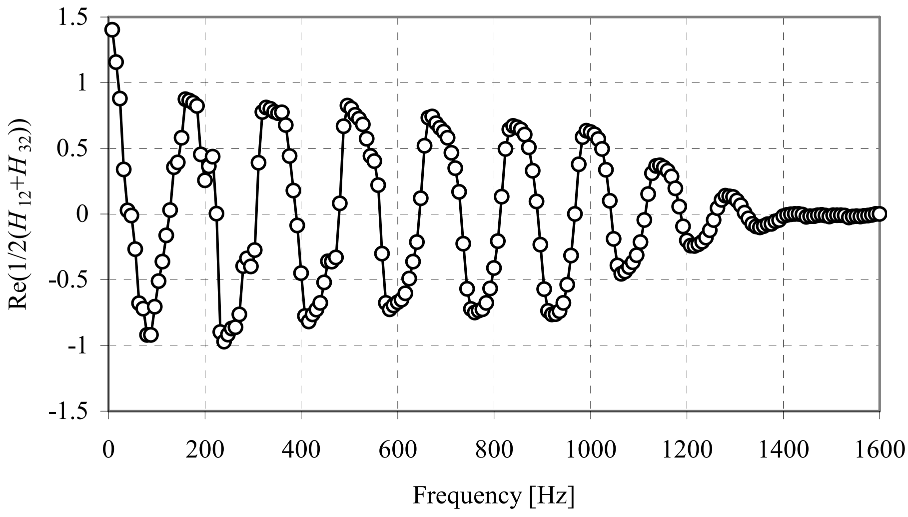

Figure 2 – Real part of the transfer function for the PE pipe.

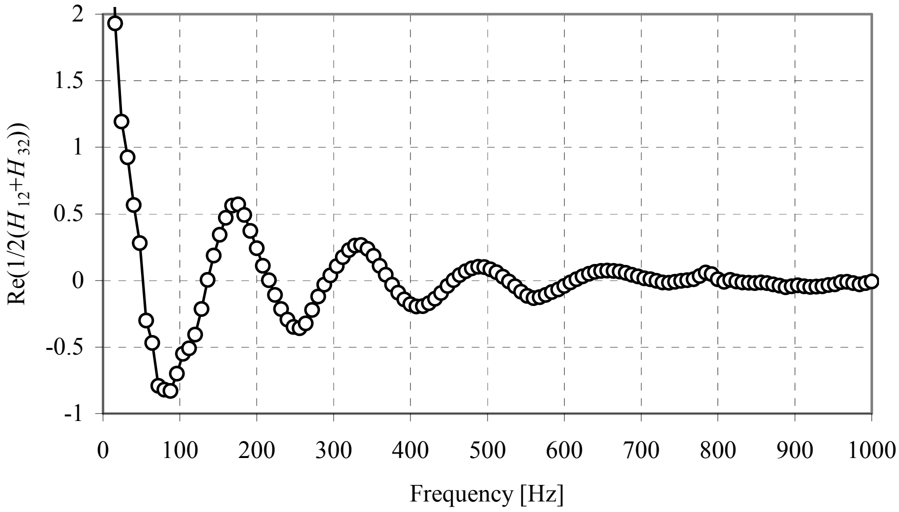

Figure 3 – Real part of the transfer function for the PB pipe.

The results show the functional dependence as indicated by the real part of the right side of Eq. (9). The argument of the cosine function represented the influence of the wave speed, while the attenuation expressed as the argument of the hyperbolic cosine caused the amplitude attenuation.

Wave speed

The wave number was calculated using Eq. (5) with each of the two sets of measurements. In this case, the real part of the transfer function (Eq. (9)) was used. Phase speed is calculated according to the wave number as follows: $c_p = \omega/\mathrm{Re}(k)$. Using the least-squares linear regression in that range, the obtained phase speed is 340 m/s. Above that frequency the phase speed increases approximately linearly with frequency, implying a frequency-dependent phase speed. This linear dependence can be approximated by the relationship with the following form: $c_p = 366 - 0{,}0334 \cdot f$, where f is the frequency in Hz and c_p the phase speed in m/s.

The same procedure applied to the measurement results for the PB pipe gives a linearly frequency-dependent phase speed for the whole observed frequency range. In this case the phase speed could be approximated as: $c_p = 350 - 0{,}027 \cdot f$. From the obtained results it is evident how the wave speed is influenced by the pipe material. Despite the fact that the wave speed is very similar for both pipe materials, there is a noticeable distinction in frequency dependence. Wave speed for the PB pipe is linear frequency-dependent for the whole frequency range, while for the PE pipe this dependence is distributed in two regions: low frequency, where speed is constant, and high frequency with strong frequency dependence. Regarding the connection between the phase speed and the attenuation according to the Eq. (9), similar results could also be expected for the attenuation coefficient.

Attenuation

The measurement of the loss factor is based on the attenuation of sound pressure with distance along the length of the pipe. There are two basic mechanisms by which a wave is attenuated: geometric attenuation and material attenuation. Geometric attenuation is the phenomenon by which the amplitude of a wave decreases as the wavefront spreads out over a wider area. In the case of a fluid-filled viscoelastic pipe, this attenuation is the consequence of axisymmetric inflating and deflating of pipe wall caused by the fluid-borne pressure pulsations. Material attenuation can be classified as either intrinsic (absorption) or extrinsic (scattering).
The attenuation (loss) of the amplitude of the propagating wave results in a negative value of the imaginary part of the wave number. This loss is given in dB/m as:

$$Loss[dB/m] = -\frac{20 \cdot \mathrm{Im}(k)}{\ln(10)} = 8{,}67 \cdot \alpha \cdot \omega \qquad (10)$$

The wave attenuation can also be derived from the amplitude of the transfer function between sensor signals [8]. The rate of the attenuation is:

$$Loss[dB/m] = -\frac{20 \cdot \ln|H(\omega, \Delta x)|}{\Delta x \cdot \ln(10)} \qquad (11)$$

Implementation of this method assumes that one can measure the decay in amplitude along the length of the pipe through the measurement of the transfer function. After performing a calculation of wave speed, based on the real part of the transfer function, these values were used to calculate the wave attenuation. For this purpose, the imaginary part of the transfer function (Eq. (9)) was used. Once a purely real valued initial guess is determined for the wave number, the method adjusts the value of both the real and imaginary parts of the wave number until the computed wave number most closely matched the measured value of the transfer function. Since the relationships used to find the complex wave number for wave propagation are known (Eq. (9)), the method essentially finds a complex wave number such that an analytical description of the transfer function at each frequency most closely approximates the measured transfer function at the same frequency. The results of this calculation are shown in Figs. 4 and 5. For the PB pipe, the wave attenuation is linearly frequency dependent and could be approximated as *Loss* = – 0,035·*f* [dB/m]. For the PE pipe (Fig. 4), the attenuation for the frequency range between 0 and 1 kHz was approximated as *Loss* = – 0,7442 – 0,035·*f* [dB/m] and for the range between 1 and 1,5 kHz as *Loss* = 29,26 – 0,033·*f* [dB/m]. The influence of viscoelastic pipe material is clearly evident when this result is compared with the frequency-dependent phase speed.

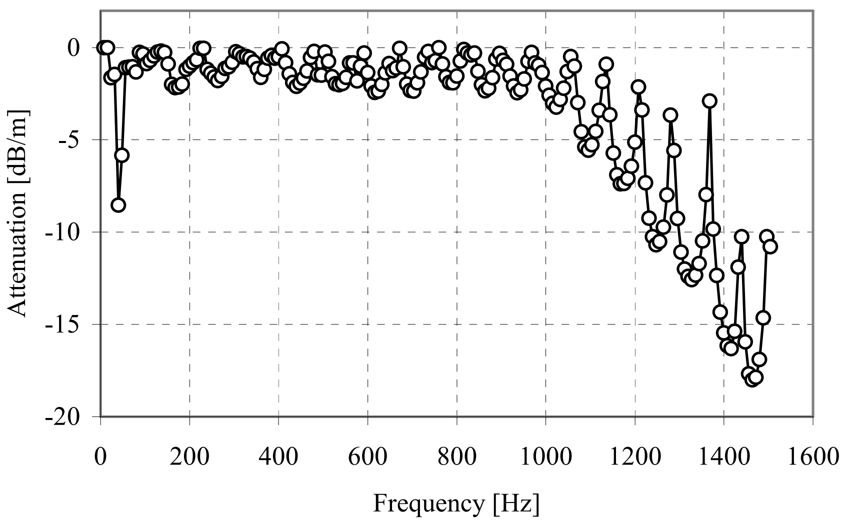

Figure 4 – Attenuation for the PE pipe

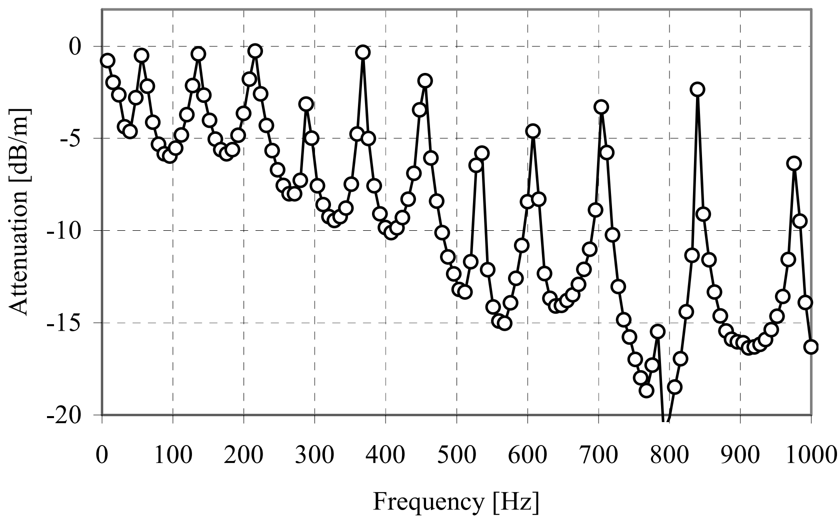

Figure 5 – Attenuation for the PB pipe

CONCLUSIONS

This study presents an experimental determination of wave propagation characteristics in fluid-filled viscoelastic pipes. The method evaluated is the complex wave number fitting method in which an analytical description of the wave propagation over the length of pipe is adjusted to match experimental measurements. The acoustic wave number of axisymmetric waves propagating in a finite pipe is determined from pressure measurements made at three locations along the pipe. Since the measurements were made in viscoelastic pipes, the wave number is complex and encompasses both the wave speed and wave attenuation. The real part of the measured transfer function was used to determine the wave number and the phase speed is obtained with the least-squares linear regression. These values were used further to calculate the wave attenuation. For this purpose, the imaginary part of the transfer function was used. This method is based on the adjustment of both the real and imaginary parts of the wave number until the computed wave number most closely matched the measured value of transfer function. Since the relationships used to find the complex wave number are known, this method essentially finds a complex wave number such that an analytical description of the transfer function at each frequency most closely approximates the measured transfer function at the same frequency.

The effectiveness of the proposed method is assessed for two types of pipe material. The fluid-borne sound signal was highly attenuated and the three-transducer method was found to be less sensitive to excessive noise. Even in these circumstances the proposed calculation method enables the determination of the acoustical properties of fluid-filled plastic pipes within a wide frequency range. The measured attenuation exhibits less fluctuation in comparison with previous published results.

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