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ADAPTIVE DESIGN OF LINEAR-PHASE MAXIMALLY FLAT FILTERS FOR DIGITAL AUDIO

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Abstract

Linear phase response and flat magnitude response are highly desirable characteristics of any filter in audio signal processing. This paper proposes a simple and efficient novel procedure for the design of finite impulse response (FIR) maximally flat filters with linear phase. The procedure is based on Butterworth polynomials, bilinear transformation and adaptive inverse modelling. Instead of using prescribed magnitude response through a set of discrete values, the designed filter has magnitude response equal to the squared magnitude response of a Butterworth filter. Instead of multisine excitation, random noise is used as the excitation signal for the adaptive process. The delayed noncausal impulse response component is approximated using the causal impulse response of a finite impulse response filter. Possible applications of newly proposed design method are in restoration of old analogue audio recordings, acoustical measurements or audio engineering.

INTRODUCTION

Linear phase response and flat magnitude response are highly desirable characteristics of any filter. Precise linear phase requires that poles and zeros exist in mirror-image pairs. The impulse response of a linear-phase finite impulse response (FIR) filter has to be symmetrical [1]. The standard method for the design of a linear-phase digital filter is the Park-McClellan algorithm [2] which employs the Remez exchange algorithm and Chebyshev approximation theory.

An adaptive process to the design of linear-phase FIR filters is given in [3]. Here, the coefficients are adjusted identically in pairs using the adaptive least mean squares (LMS) algorithm. This method is an extension of the application of adaptive modelling to FIR digital filter design, and uses the set of prescribed gain magnitude and phase characteristics. The input signal and desired response of the filter are the sums of sinusoids with prescribed amplitudes and phase shifts.

This paper proposes a novel method for realizing FIR filters with linear phase and flat magnitude characteristics. The procedure is based on Butterworth polynomials, bilinear transformation and adaptive inverse modelling. Instead of using prescribed magnitude response through a set of discrete values, the designed filter has magnitude response equal to the squared magnitude response of a Butterworth filter. Generally, instead of multisine excitation, random noise is used as the excitation signal for the adaptive process.

PROPOSED METHOD

Let us define the Butterworth function of order n ,

$$H(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_s} \right)^{2n}} \tag{1}$$

where n is filter order, $s = j\omega$ denotes complex frequency, $j = \sqrt{-1}$, ω is radian frequency and ω_s is Butterworth filter cutoff radian frequency [4].

By applying the bilinear transformation

$$s = \frac{2}{T}\frac{z-1}{z+1} \tag{2}$$

where T denotes sampling period and z is variable in the z-transform, we obtain

$$H(z) = \frac{1}{1 + \left(\frac{s}{j\omega_s} \right)^{2n}} = \frac{T^{2n} \cdot \omega_s^{2n} (1 + z^{-1})^{2n}}{T^{2n} \cdot \omega_s^{2n} (1 + z^{-1})^{2n} + 2^{2n} \cdot (-1)^n (1 - z^{-1})^{2n}} = \frac{P(z)}{Q(z)} \tag{3}$$

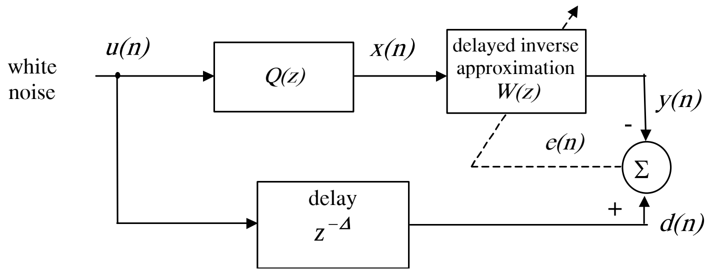


Figure 1 - Adaptive inverse modelling of Q(z)

$H(z)$ has poles in mirror-image pairs, where each pole inside the unit circle in the z-plane has its counterpart outside the unit circle. These poles outside the unit circle correspond to the left-handed, noncausal sequence in the symmetrical impulse response. This delayed left-handed sequence can be approximated by the causal impulse response of a FIR filter designed by an adaptive inverse modelling procedure that includes modelling delay. Application of an adaptive inverse modelling procedure [3] to the approximate determination of the inverse of $Q(z)$ can be described by Fig. 1. Here, $e(n)$ denotes the error signal, $u(n)$ is the training signal, $x(n)$ is the output signal from the plant that is to be modelled, and $y(n)$ is the adaptive filter output signal.
Adaptive filter output is defined by

$$y(n) = \mathbf{X}^T(n)\mathbf{W}(n) \;, \tag{4}$$

the error signal is

$$e(n) = d(n) - y(n) \;, \tag{5}$$

where

$$\mathbf{W}(n) = \left[w_0(n) \;\; w_1(n) \;\; \ldots \;\; w_{L-1}(n)\right]^T$$

is the weight vector of the adaptive finite impulse response (FIR) filter, and

$$\mathbf{X}(n) = \left[x(n) \;\; x(n-1) \;\; \ldots \;\; x(n-L+1)\right]^T$$

is the input signal vector. L is adaptive filter length, Δ is modelling delay, and $[\;]^T$ stands for vector transpose. $W(z)$ signifies delayed inverse approximation of the denominator in (3).

Since the numerator $P(z)$ is binomial and

$$\binom{n}{k} = \binom{n}{n-k} \tag{6}$$

the coefficients of $P(z)$ are symmetrical, and consequently $P(z)$ is a linear-phase transfer function. Since $H(z)$ is zero-phase, the delayed approximation of $1/Q(z)$ is also linear-phase.

The coefficients c_k of the resulting filter that approximates $H(z)$ can be achieved by multiplication of the polynomials (or convolution of the vectors of the coefficients):

$$C(z) = P(z) \cdot W(z) \tag{7}$$

The length of the resulting FIR filter is $M=L+2\cdot n$. If we want the resulting FIR filter to be a linear-phase one, its length has to be an odd number and the modelling delay has to be $\Delta=(M-1)/2$. This way the FIR approximation of the delayed inverse $1/Q(z)$ will also be linear phase, with the central coefficient $w_{\frac{L-1}{2}}$. Since M is an odd number and $2\cdot n$ is even for any n, L is also an odd number.
Due to misadjustment in the chosen adaptive process, the resulting filter is not exactly, but approximately linear phase. However, the difference is very small if residual error signal power is negligible compared to the input signal power.

As the candidates for adaptive inverse modelling, the least mean squares (LMS) [3], recursive least squares (RLS) and adaptive Kalman filter [5,6] were applied. All proposed methods use random white noise as the training signal. The adaptive Kalman filter showed greatest robustness for higher filter orders and lower cutoff frequencies.

The Butterworth filter cutoff frequency ω_s is related to the desired –3dB cutoff frequency ω_0 of the analogue prototype $H(\omega)$ as

$$\omega_s = \frac{\omega_0}{\sqrt[2n]{\sqrt{2}-1}} \tag{8}$$

Also, during the design, the frequency warping caused by the bilinear transformation has to be taken in account.

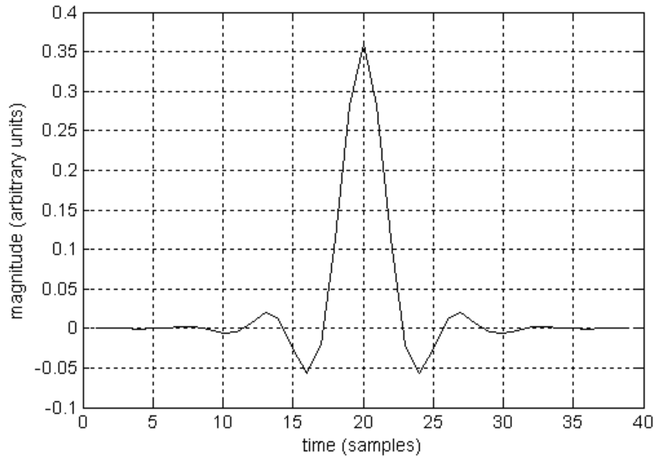


Figure 2 - Impulse response

FILTERS WITH LOW CUTOFF FREQUENCY

Since the magnitude frequency response of $Q(z)$ increases with frequency, and the bilinear transformation maps the frequency infinity to the Niquist frequency, for higher filter order and for lower cutoff frequencies the magnitude of $Q(z)$ very rapidly increases with frequency. In this case, a white-noise training signal causes $x(n)$ to be over-emphasized for higher frequencies, and results in an exceptionally high residual error signal at the end of the adaptive process. Even with the adaptive Kalman filter for n greater than 4 and continuous-time cutoff frequencies below 40% of Niquist frequency, the process fails. In these cases, a synthesized lowpass multisine signal with random phase can be used as the training signal:

$$u(t) = \sum_{l=1}^{N} \sin(2\pi f_l t + \phi_l) \tag{9}$$

where t denotes time and f_l is frequency. The proposed input signal contains additive random phase ϕ_l. This way the sharp spikes are avoided, and the excitation signal has a pseudo-random shape, in the time domain. This concept is described in [7]. The upper bound of the multisine signal must be high enough to cover the transition band. The phase response will be linear for the passband and transition band. The overall phase response will no longer be linear, but since the magnitude in the stop frequency band is very small, this is not significant.

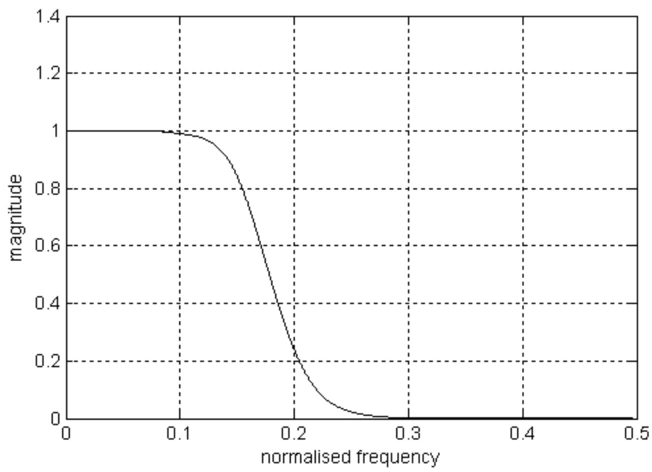


Figure 3 - Magnitude response

NUMERICAL RESULTS

Fig. 2 presents the impulse response of a linear-phase filter designed using the proposed procedure. The filter order is 38 and n=4, i.e. filter order of continuous-time prototype is 8. The continuous-time Butterworth filter cutoff frequency ω_s is equal to $0.4 \cdot \omega_N$, where ω_N denotes Niquist frequency. The length of the resulting filter is 39, and modelling delay Δ is 19. The adaptive Kalman filter was used for adaptive inverse modelling. All calculations were run in the MATLAB environment and the adaptive Kalman filter was realized using Filter Design Toolbox. Figures 3 and 4 present magnitude and phase responses of the same filter. The phase characteristic is highly linear, except for frequencies in the vicinity of the Niquist frequency. This nonlinearity is not important since those frequencies lie far outside the transition band. Figure 5 presents the error signal during the adaptation process. We observe a very low level of residual modelling error. About 1000 iterations were quite enough to accomplish satisfactory modelling error.

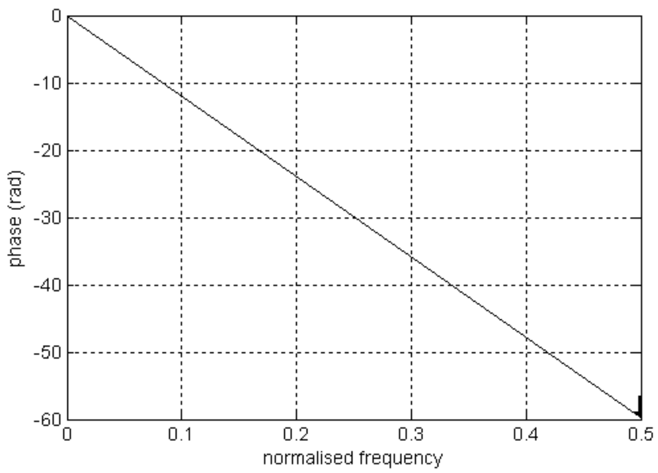
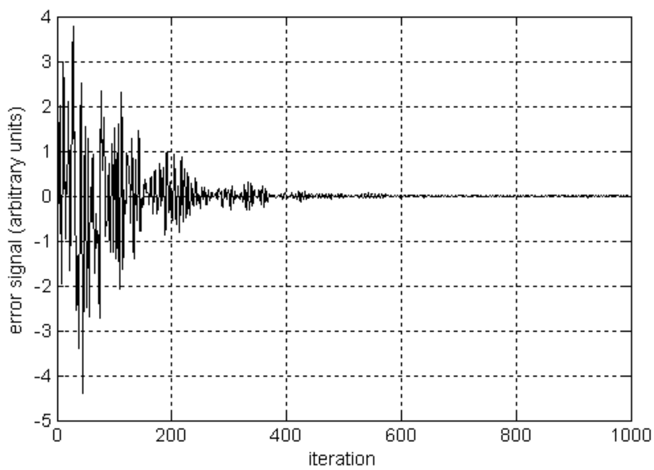


Figure 4 - Phase response



Figur 5 - Error signal during adaptation

CONCLUSIONS

In this paper, we have presented an adaptive method for the design of linear-phase FIR filters. The proposed technique is very simple, and it guarantees a magnitude response that is maximally flat in the passband, and monotonic overall. For higher cutoff frequencies and for lower filter orders, the method can be simplified even more, with the application of the LMS algorithm instead of the Kalman filter. Possible applications of newly proposed design method are in restoration of old analogue audio recordings, acoustical measurements or audio engineering.

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