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LOW SPEED JET NOISE PREDICTION USING THE VORTEX SOUND FORMULATION

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Abstract

The basic sound radiation generated by large-scale structures is calculated for circular and elliptical single jets. A hybrid method is used in which the hydrodynamics is computed using an incompressible Large-Eddy Simulation (LES) procedure and the generated sound field is calculated using Powell's vortex sound formulation of Lighthill's analogy. For the implementation of the acoustic analogy, boundary corrections are required to account for the finite axial length of the computational domain. The simulations are performed for circular and elliptical jets with the same effective diameter and with a Reynolds number (Re) based on the diameter of the jet that is 6000. The inflow velocity profile provides a four-frequency excitation which is randomized in its phase and has a 2% peak amplitude. The circular jet shows a higher acoustic output for the lower range of the frequency spectrum which corresponds to the weaker mixing in the circular jet. The acoustic results based on the vortex sound formulation show a similar behaviour as those based on Lighthill's original velocity source formulation. However, the vortex sound formulation shows a higher sensitivity to the finite length of the computational domain. This sensitivity is minimised by the newly derived boundary conditions leading to good quantitative agreement between the two acoustic formulations.

INTRODUCTION

The commercialization of jet engines in the early 1950's raised the issue of jet noise and the importance of studying and understanding a previously less concerning topic. Since then, jet noise has been subject of considerable work perhaps initiated by Lighthill [1] with his famous theory on aerodynamically generated sound. In 1958 Lilley [2] paid particular attention to circular jets with his report on noise radiated from circular air jets. With jet engines becoming increasingly popular and with demand growing, jet aeroacoustics was more crucial in the effort to reduce the noise pollution generated by jets. In the last decade significant progress has been made in Computational Aeroacoustics (CAA) with the use of the direct and indirect approaches as suggested by Lighthill [3]. In this work the indirect approach is more suitable [3] and thus has been used; the aerodynamic source field was calculated and coupled with an acoustic analogy to find the sound field. In the direct approach the sound field is simulated using a specialized CAA scheme or the same Computational Fluid Dynamics (CFD) scheme used for the aerodynamic source field. This approach has been shown to be more suitable for high speed flows.

A widely accepted acoustic analogy used in the indirect approach is Lighthill's acoustic analogy. Lighthill's equation of motion can be seen as that of a fictitious acoustic medium acted upon by an external stress distribution. In this acoustic analogy the Navier-Stokes equations are transformed to generate an exact, inhomogeneous wave equation, which contains two space derivative terms on the right hand side that appear as a simple source. The source terms in this equation are of a quadrupole source nature and are only important in the turbulent region.

If the role of vorticity is taken into account in Lighthill's theory an alternative formulation may be used to predict the sound. Here, Lighthill's quadrupole source is approximated by means of the Biot-Savart induction formula [4]. The retarded time variations within the eddy are also neglected and a vortex sound equation may be obtained for isentropic flow. In this equation the source term is a vortex source, which vanishes in irrotational regions. It follows that in an unbounded irrotational fluid there are no sound waves propagating in the fluid. This formulation proposes that in low Mach number turbulent flows the component $div(\rho_0 \omega x v)$ of Lighthill's quadrupole is the principal source of sound.

The present work focuses on investigating the suitability of Powell's Vortex Sound formulation of Lighthill's acoustic analogy and comparing it to existing results that used Lighthill's compact velocity tensor source formulation [5]. This case study will focus on the differences between low-speed incompressible circular and low aspect ratio (2:1) elliptical jets at a Reynolds number $Re_{De} = 6000$. Emphasis is put on the effect of the large-scale structures on the noise and hence the LES is used. These large structures are responsible for the low frequency noise and the discrete peaks found in the sound field spectrum.

The sound radiation of subsonic jets has been studied experimentally, for example, by Crow and Champagne [6], Lush [7] and Moore [8]. One of the main conclusions was that the preferred direction for sound radiation in a circular jet was the streamwise direction. Large-scale structures were found to be generating the larger amount of sound radiation. Noncircular jets inherently reduce the presence of large-scale structures which is believed to cause a reduction in emitted acoustic power.

To the best of the authors' knowledge, not many publications are available where low Mach number elliptical jets' sound radiation has been studied computationally. Although Lighthill's acoustic analogy is widely used not many attempts to implement the Vortex Sound approach have been found. One of the challenges in this approach is the complexity in the implementation of the necessary boundary corrections.

NUMERICAL FORMULATIONS

The in-house code *Lithium* was used for the simulation of the full incompressible Navier-Stokes (N-S) equations using a finite difference discretisation on a staggered Cartesian grid. The projection method was used together with a compact third order Runge-Kutta method. The pressure equation resulting from the projection method was solved used a Fast Fourier Transform (FFT) scheme [9]. The convection terms were calculated using a one point biased fourth-order upwind scheme based on a five point stencil, where a fourth order interpolation was used between the staggered properties when required. The diffusion terms were calculated using a fourth order staggered scheme. An explicit SGS model was then used, which was based on the MTS model of Inagaki [10].

The computational domain was of the size (50, 30, 30)R_e, where R_e is the initial effective radius of the jet. The grid size was of 320x193x193 points with a moderate clustering of points near the jet centre line.

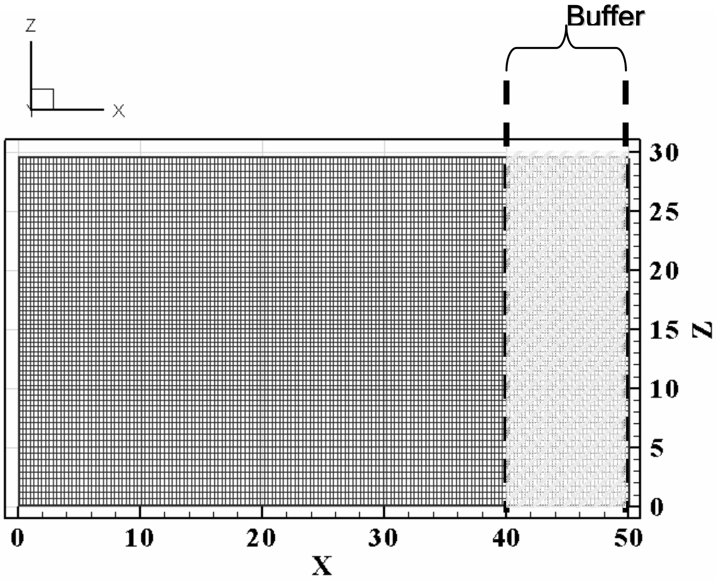

Figure 1 – Typical mesh used of dimensions 50Rx30Rx30R containing 320x193x193 points in the x, y and z directions. Note that the figure above is actually showing 160x86x86 points.

The jet was subjected to a disturbance with a randomized phase in space. Inflow conditions were specified as $v=w=0$ and the axial velocity u had a hyperbolic tangent shape. On the stream-normal and spanwise sides of the computational domain a free slip condition for the velocities was used along with a constant pressure condition. Convective boundary conditions were used at the outflow side of the domain with a buffer zone located in the last $10R_e$ as shown in Fig. 1 [5].

An acoustic analogy was coupled to the flow field solution to predict the sound radiation as described in the previous section. In the case of Lighthill's formulation (LF) the required volume integration was calculated using a second order scheme and performed during the flow simulation. The integral was limited to the physical zone of the flow, i.e. did not include the buffer zone. The surface integrals required by the boundary correction to the quadrupole source [11] were also calculated during the simulation using a second order scheme and added to the quadrupole later in the post processing stage.

In the case of Powell's Vortex Sound Formulation (PF), the basic far sound field pressure fluctuation can be given by [4]:

$$p(\overline{x},t) = \frac{-\rho_0 x_i x_j}{4\pi c_0{}^2 |\overline{x}|^3} \frac{\partial^2}{\partial t^2} \int y_i \left(\overline{\omega} \times \overline{v}\right)_j d\overline{y} \tag{1}$$

Clearly when the integral volume in (2) is defined over a finite volume, an unphysical dependence on the origin of the co-ordinates of y can occur. This particularly holds for the axial direction (i=1) where the vorticity is not small at the inlet and outlet of the computational domain. Following the derivation that removed a similar dependency in the axisymmetric version of Möhring's formulation, the following general boundary correction is proposed;

$$\ddot{Q}_{ij} = \frac{\partial^2}{\partial t^2} \int \left(y_i - y_{0i}\right)\left(\overline{\omega} \times \overline{v}\right)_j d\overline{y} + \int \left(y_i - y_{0i}\right) f_j \Big|_{y_1 = L_x} d\overline{y}_h - \int \left(y_i - y_{0i}\right) f_j \Big|_{y_1 = 0} d\overline{y}_h \;, \tag{2}$$

where y_{0i} explicitly marks the origin of the co-ordinate y_i and $d\overline{y}_h$ is a surface element in the inlet (y_1=0) and outlet (y_1=L_x) of the computational domain. To require independence of y_{0i} one gets,

$$\frac{\partial^2}{\partial t^2} \int \left(\overline{\omega} \times \overline{v}\right)_j d\overline{y} + \int f_j \Big|_{y_1 = L_x} d\overline{y}_h - \int f_j \Big|_{y_1 = 0} d\overline{y}_h = 0 \,. \tag{3}$$

Introducing the vector identity:

$$\nabla(|v|^2 / 2) - \left(\overline{v} \times \overline{\omega}\right) = \left(\overline{v} \cdot \nabla\right)\overline{v} \;, \tag{3}$$

one gets

$$f_1 = -\frac{1}{2} \frac{\partial^2 \left(u^2 - v^2 - w^2\right)}{\partial t^2} \,, \; f_2 = -\frac{\partial^2 (uv)}{\partial t^2} \text{ and } f_3 = -\frac{\partial^2 \left(uw\right)}{\partial t^2} \,. \tag{4}$$

The boundary corrections were implemented as part of the *indirect* approach to calculate the sound field, where the acoustic analogy is coupled to the velocity field in the simulations. The time history, sound directivities and frequency spectra were calculated during the post-processing stage.

HYDRODYNAMIC AND ACOUSTIC RESULTS

After the initial transient structures were allowed to leave the domain, statistical data was gathered. A detailed description of the mean flow can be found in Alonso and Avital [5]. A snapshot of vorticity at normalized time 340 is shown in Fig. 2. Spanwise slices are also shown at the inflow, 5*R*, 10*R*, 15*R* and 20*R*. It can be observed that the circular jet has larger structures and that the elliptical jet shows a stronger mixing due to its inherent eccentricity, which induces the appearance of smaller-scales.

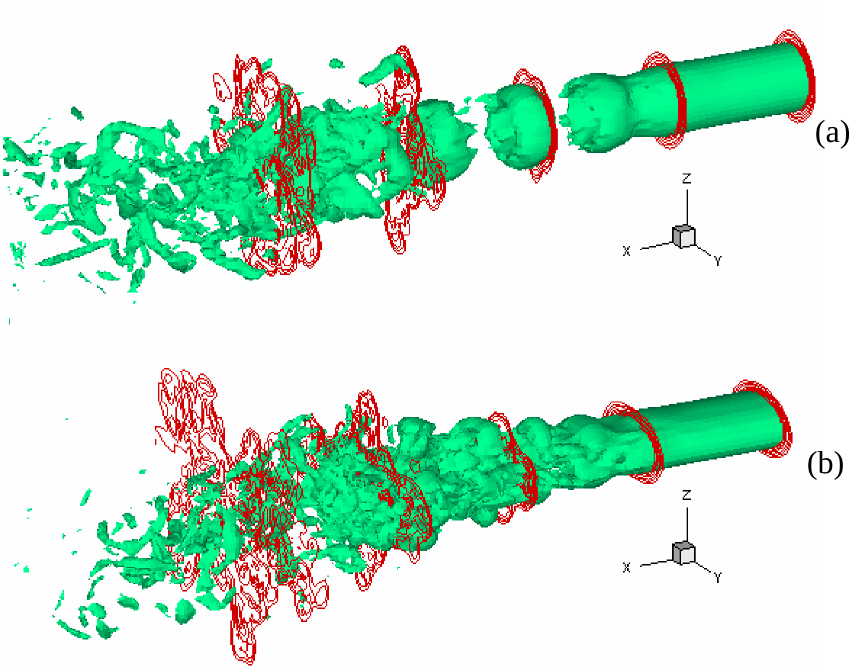

Figure 2 – Snapshot at normalized time 340 of vorticity magnitude iso-contour for the circular (a) and elliptical (b) jet. Slices shown at the inflow, 7.5R, 15R, 22.5R and 30R.

The time history of the terms that compose the quadrupole source shows that the circular jet has a stronger sound generation in overall. As an example, Fig. 3 shows the evolution of the dominant quadrupole Q_{11}. Very good agreement is seen in the amplitude between the LD and the PF formulations, when the boundary correction (2) was used, but not in the phase. Further analysis showed that boundary correction (2) managed to remove successfully the dependency on the co-ordinate origin. In Figs 3, Powell's time history seems to be 180^0 out of phase with Lighthill's. For example, at time 255 there is a peak of strength 0.15 in the circular case using LF; this peak occurs at the same instance with a value of -0.15 in the case of PF. It should be noted that Lighthill's quadrupole tensor source is symmetric and in Powell's the tensor is not symmetric. Thus caution should be applied when individual quadrupoles are compared.

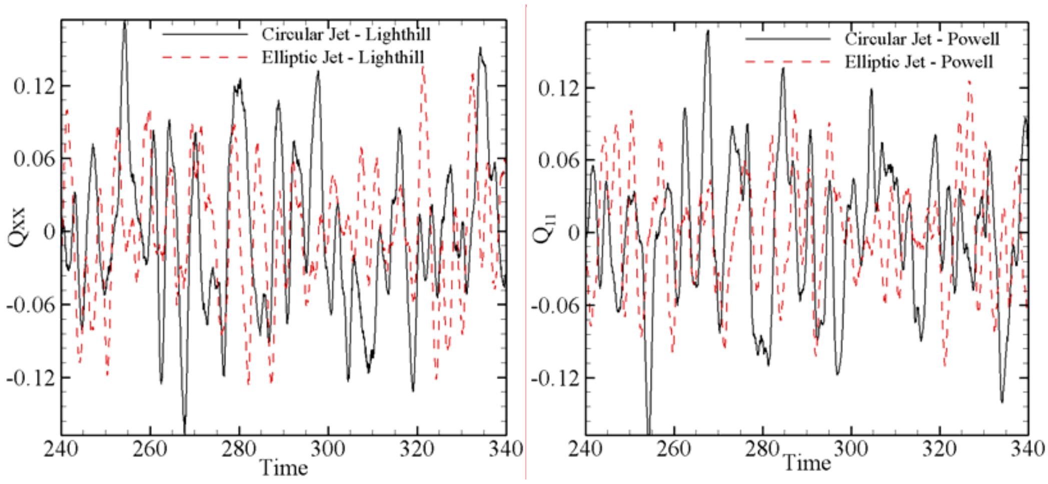

Figure 3 – Time history of Lighthill's Qxx and Powell's Q11 from normalized time 240 to 340. Results shown for the circular and elliptical jet, where the boundary correction of Eqn. (2) was used in Powell's formulation.

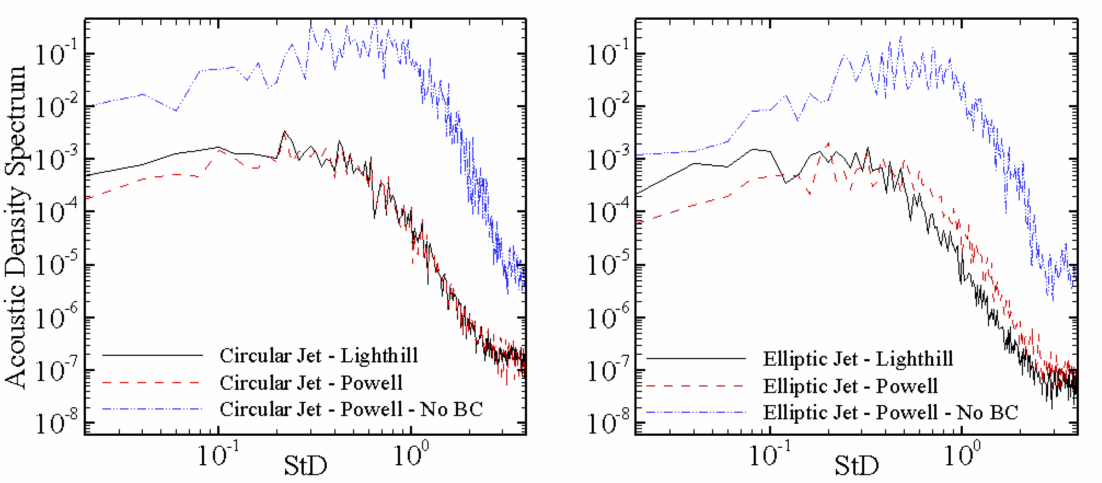

Figure 4 – Acoustic density spectrum for the circular and elliptic jet. Sampling time is 240 < T < 340.

As mentioned earlier, the contribution of the large structures to the acoustic density output is found to be dominant. In Fig. 4 the acoustic density spectrum shows that the low frequencies are dominant and responsible for a considerable part of the sound radiation. The effect of the boundary correction can be observed in Fig. 4. The dash-double-dot line represents the spectra when the boundary correction is not used. In this case the origin of the co-ordinates is set at the jet centre line near the end of the potential core; at 14 and 8 *x/R*, for the circular and elliptic jet respectively. It can be observed that the effect of the finite length of the computational box is very strong and PF overestimates the prediction of LF by 10dB to 30dB. On the other hand, the results show very good agreement with LF when boundary correction (2) was implemented, particularly for the circular jet and St_D>0.1 and less than 2dB difference in most of the other frequencies for both jets.

The directivity $D(\theta)$ is commonly used for circular jets, where the directivity is averaged azimuthally. However due to its nature it hides any asymmetric radiation patterns that may be of particular interest for noncircular jets. Although $D(\theta,\phi)$ is available [5], for the purpose of this work only $D(\theta)$ will be discussed. Figure 5 shows the directivity variation against the polar angle. In the case of PF the double peak feature can be observed, which is smoothed out in the LF prediction. This feature has been observed axisymmetric jets [11]. However, unlike the axisymmetric case, the directivity is almost flat for both formulations with a mild preference for the axial direction by about 2dB as compared to the normal direction. This can be attributed to the effect of the large scale low frequency structures [12]. The overall quantitative agreement between the two formulations is good again with less than 2dB difference.

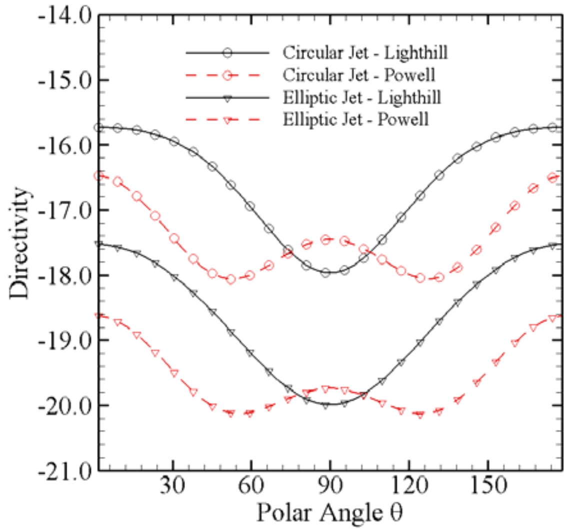

Figure 5 – Directivity for circular and elliptical jets in terms of polar angle. Sampling time is $240 < T < 340$, where the boundary correction (2) was used in Powell's formulation.

SUMMARY

Basic sound radiation in circular and elliptical jets was investigated using an *indirect* CAA approach in which the velocity field was coupled to the sound field using the acoustic analogy approach.

The hydrodynamics were calculated using incompressible LES. In order to implement Powell's vortex sound formulation, a new type of boundary correction was developed to remove unphysical behaviour caused by the finite axial length of the computational domain. The effectiveness of the new boundary correction and the sound generation characteristics of circular and elliptic jet of moderately low Reynolds number where analysed using the new implementation of Powell's formulation and Lighthill's velocity tensor source formulation.

An analysis of the time history of the quadrupoles showed the dominance of the longitudinal *xx* quadrupole in both types of jets and the circular jet showing stronger amplitude. Powell's formulation showed a very good agreement with the predictions of Lighthill's formulation when the new boundary correction was used. The acoustic density spectrum showed almost a *line on line* agreement of both formulations for the circular jet and a difference of 2dB or less in most other cases. Such agreement was seen to be far from achieved if the boundary correction was not used in Powell's formulation. The directivity showed a flat pattern with a mild preference of the axial direction, and again a good agreement between the two acoustic formulations. Further work is being done to estimate the small scales contribution to the sound radiation and analyzing higher Reynolds number effect.

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REFERENCES

[1] Lighthill, M. J., "On sound generated aerodynamically I, general theory", Proc. Roy. Soc., **A211**, 564-587 (1952).

[2] Lilley, G. M., "On the noise from air jets, Aeronautical Research Council", A.R.C. 20376, 40, F.M. 2724 (1958).

[3] Lighthill, M. J., The final panel discussion, *Computational Aeroacoustics*. (J.C. Hardin and M.Y. Hussaini eds., Springer-Verlag, New York, 1993).

[4] Howe, M. S., *Theory of Vortex Sound*. (Cambridge Texts in App. Math., Cambridge University Press, 2003).

[5] Alonso, M., Avital, E. J., "Basic sound radiation of low speed coaxial jets", DLES-VI Workshop, Poitiers, France, (2005).

[6] Crow, S. C., Champagne, F. H., "Orderly structure in jet turbulence", J. Fluid Mech., **48(3)**, 547-591 (1971).

[7] Lush, P. A., "Measurements of subsonic jet noise and comparison with theory", J. Fluid Mech., **46(3)**, 477-500 (1971).

[8] Moore, C. J., "The role of shear-layer instability waves in jet exhaust noise", J. Fluid Mech., **80(2),** 321-367 (1977).

[9] Avital, E. J., "A **s**econd look at the role of the Fast Fourier Transform as an elliptic solver"**,** Int. J. Num. Meth. Fluids, **48(9)**, 909-927 (2005).

[10] Inagaki, M., Kondoh, T., Nagano, Y., "A Mixed-Time-Scale SGS model for practical LES", Transact. Japan Soc. of Mech. Eng., part b, **68(673)**, 2572-2579 (2002).

[11] Avital, E. J., Sandham, N. D., Luo, K. H., Musafir, R. E., "Calculation of Basic Sound Radiation of Axisymmetric Jets by Direct Numerical Simulation," AIAA J., **37(2)**, 161-168 (1999).

[12] Laufer, J., Yen, T-Ch., "Noise Generation by a Low-Mach Number Jet," J. Fluid Mech., **134**, 1-34 (1983).