ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

THE OPTIMAL POLE-ZERO MODEL FOR A ROOM TRANSFER FUNCTION AND ITS SPATIAL INTERPOLATION AND EXTRAPOLATION

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Abstract

With the advent of digital signal processing technique, noticeable development was achieved in the field of room acoustics. Many applications, acoustic feedback (echo) cancellation, virtual acoustics, sound field control and so on, require exact and efficient model of room transfer function. However, there are several problems to be solved for the manipulation of these room transfer functions. Several signal processing methods which deal with room transfer functions are faced with high complexity and computational burden so that they cannot be implemented in real-time systems. In this paper, the pole-zero model approximation is examined for the efficient representation of room transfer function. This paper discusses about the optimal number of the pole and zero which minimize the modeling error. With the optimized room model, the interpolation and extrapolation of room transfer function are investigated by observing the movement of poles and zeros in z-plane.

INTRODUCTION

Room transfer functions describe the combined effect of multi-path sound transmission and reflection characteristics between a source and a receiver in a room. The exact modeling method for room transfer functions is required in many applications such as room acoustic equalizer, echo canceller, de-reverberator, noise canceller and sound field generator. Considerable advances in the field of room acoustics have been achieved with the development of signal processing techniques. However, there are still significant problems for manipulating room transfer functions. Several models are introduced to handle room transfer functions. This paper proposes a method for choosing the optimum number of poles and zeros and discusses about its ability for the spatial interpolation and extrapolation.

THEORETICAL CONCEPT OF ROOM TRANSFER FUNCTION

The room transfer function is the sound transmission characteristics in a room. Every acoustical space has its own sound field. The sound field can be described through addition of all reflected sound components. The sound field can be represented by 'Wave Theory'. It is assumed that the room under consideration has locally reacting walls and ceiling, the acoustical properties of which are completely characterised by impedance. When the source signal is a point source which has an angular frequency ω, the sound field can be obtained by equation (1).

$$p(r) = iQc^2\omega\rho_0 \sum_n \frac{p_n(r_0)p_n(r)}{(\omega^2 - {\omega_n}^2 - 2i\delta_n\omega_n)K_n} \tag{1}$$

where Q is volume velocity, c is speed of sound, ρ_0 is gas density, r and r_0 are coordinates of receiver and sound source, and δ_n is damping constant [1]. In addition, p_n is the eigenfunction when the eigenfrequency is $f_n = \omega_n / 2\pi$. K_n is a constant which can be obtained by equation (2). Equation (2) represents the characteristics between two eigenfunctions in a room which has volume V. As shown in this equation, eigenfunctions can be considered as members of orthogonal set.

$$\iiint_V p_n(r)p_m(r)dV = \begin{cases} K_n & \text{for} \quad n = m \\ 0 & \quad\quad\; n \neq m \end{cases} \tag{2}$$

When the source signal is a point source of angular frequency ω, the equation (1) represents the sound pressure at any location in a room. This is called, 'Green's function' of the room. This equation is symmetric in the coordinates of the sound source and of the receiver. If all the eigenvalues and eigenfunctions are known, we can evaluate any desired acoustical property of rooms.

If the source signal is direc-delta function, equation (1) represents the transfer function of a room between the two points r_0 and r. The corresponding frequency, $f_n = \omega_n / 2\pi$, is called as eigenfrequency or resonance frequency. The eigenfrequency and damping constant of this function, which are directly connected with poles in general pole-zero model, are dependent on the room shape and acoustical properties of walls, ceiling and other component which is already installed. The eigenfunction p_n, which has connection with zeros in pole-zero model, is highly dependent on the source and receiver locations.

CONVENTIONAL MODELS FOR ROOM TRANSFER FUNCTION

The conventional models for estimating room transfer function are explained in this chapter. Room transfer function can be regarded as a non-parametric model. This non-parametric model is represented with some parameters such as poles and zeros. The object of room transfer function modeling is to represent the acoustic characteristics of a room given as size, shape, and source and receiver locations. If acoustic properties in a room are regarded as a system, modeling room transfer function can be regarded as a system identification problem.

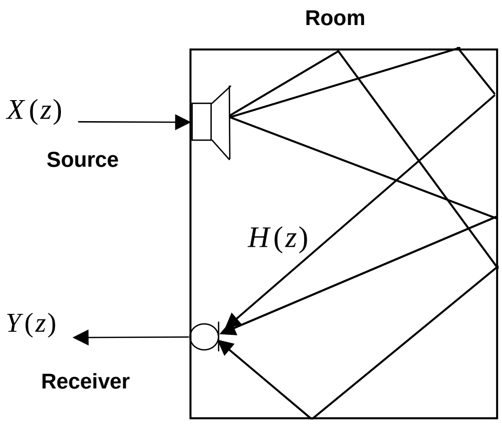


Figure 1. Schematic diagram of room transfer function

Room transfer function describes the sound transmission characteristics between a sound source and a receiver as shown in figure 1. The room transfer function, $H(z)$, can be represented by

$$H(z) = Y(z)/X(z) \tag{3}$$

where $X(z)$ is the source signal and $Y(z)$ is the receiver signal.

All-zero room models

All-zero model is the usual model for a room transfer function. This model can be represented as MA (Moving Average) model which is written in equation (4).

$$H(z) = Cz^{-Q_1}\prod_{i=1}^{Q_2}(1-q_i z^{-1}) = \sum_{i=0}^{Q_1+Q_2} b_i z^{-i} \tag{4}$$

where C is gain constant, Q_1, Q_2 are order of zeros, q_i is zero, and b_i is MA coefficient [2]. The MA coefficients in this model correspond to the impulse response of the room impulse response in time domain. While all-zero model is easily estimated due to its simple structure, it is difficult to use this model for reverberant and resonating room. Because all zeros or MA coefficients of this model should be changed when the room transfer function varies due to changes in the source and receiver positions.

All-pole room models

The all-pole model considerably reduces the filter order and the complexity of transfer function model. It is a computationally efficient method for the case of reverberant room. This model can be expressed with AR (Auto Regressive) model.

$$H(z) = \frac{C}{\prod_{i=1}^{P}(1 - p_i z^{-1})} = \frac{1}{1 + \sum_{i=1}^{P} a_i z^{-i}} \tag{5}$$

where P is order of poles, p_i is pole, and a_i is AR coefficient [3]. However, all-pole model is limited in modeling ability because of their non-minimum phase property. The modeling responses exhibit inherent latencies in their frequency components. Consequently, if is difficult to estimate the exact transfer function with all-pole model.

Pole-zero room models

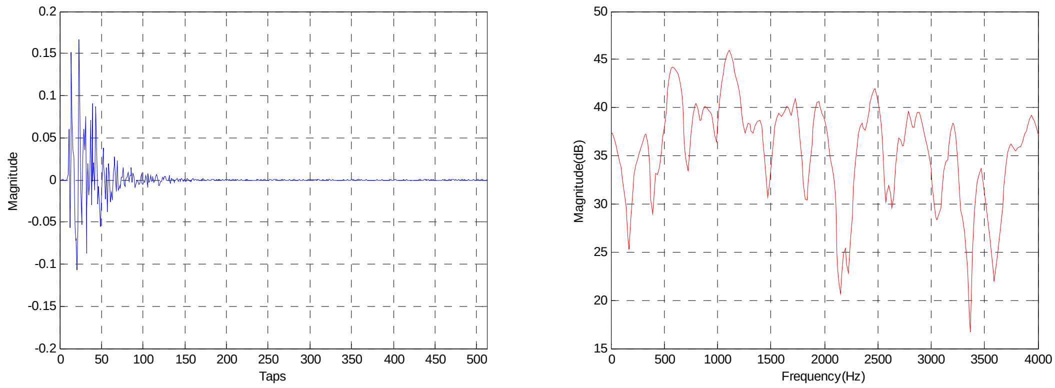
The room transfer function can also be modelled by the pole-zero model which is equivalent to the ARMA (Auto Regressive Moving Average) model. The pole-zero model is the most general linear time invariant model.

$$H(z) = Cz^{-Q_1} \frac{\prod_{i=1}^{Q_2}(1 - q_i z^{-1})}{\prod_{i=1}^{P}(1 - p_i z^{-1})} = \frac{\sum_{i=0}^{Q_1+Q_2} b_i z^{-i}}{1 + \sum_{i=1}^{P} a_i z^{-i}} \tag{6}$$

where C is gain constant, Q_1, Q_2 are order of zeros, q_i is zero, b_i is MA coefficient, P is order of poles, p_i is pole, and a_i is AR coefficient [2, 4]. As shown in equation (6), poles correspond to resonances, and zeros represent time delay and anti-resonances. Poles in pole-zero models can easily describe the long impulse response due to the resonances with few parameters. Zeros help pole-zero model to improve the estimation accuracy. When the room transfer function changes, all parameters of the pole-zero model also should be changed like other models. However, the number of changing parameters is much smaller than all-zero model and the estimation accuracy is much higher than all-pole model. Consequently, pole-zero models seem to be matched with room transfer function in physical point of view.

OPTIMAL POLE-ZERO MODEL FOR A ROOM TRANSFER FUNCTION

This chapter explains optimal pole-zero model for a room transfer function and the method how to get the optimal model. The room transfer function depicted in figure 2 is used to explain about the optimal pole-zero model.



(a) Room impulse response *(b) Room transfer function*

Figure 2. Impulse response and transfer function of a room

Figure 2 represents impulse response and transfer function of a room. This room transfer function is measured in a real room with 8000Hz sample rate [5]. The pole-zero model is used to estimate the room transfer function in figure 2. Poles and zeros in equation (6) are estimated using an iterative Gauss-Newton algorithm which minimizes a robustified quadratic prediction error criterion [6].

As explained in previous section, the number of poles and zeros is crucial to estimate the room transfer function because of computational burden. The optimal pole-zero model means the pole-zero model which has minimum poles and zeros with minimize the modeling error criterion. The modeling error criterion defined in [5] is used to evaluate the performance of estimated room transfer function.

$$J(\%) = \frac{E[e^2(t)]}{E[d^2(t)]} \times 100 \qquad (7)$$

where $d(t)$ is desired signal, $e(t)$ is an error signal, and $E[\circ^2]$ denotes variance.

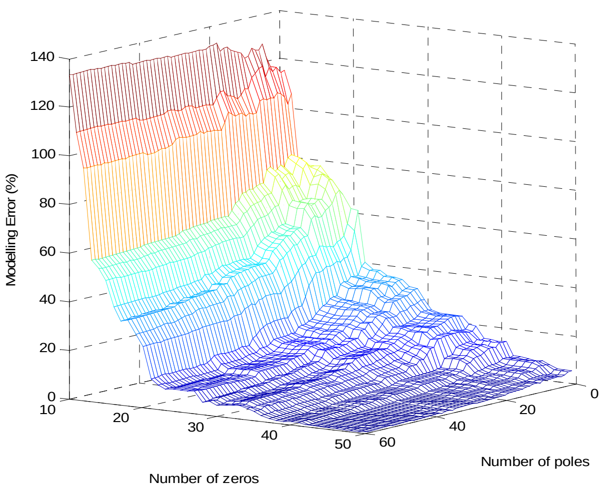
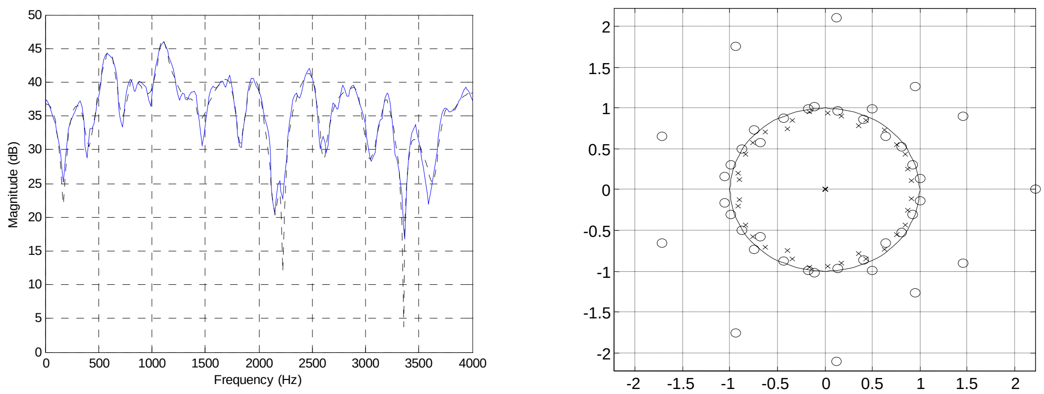


Figure 3. The modeling error with respect to the number of poles and zeros

Figure 3 represents the modeling error J with respect to the number of poles and zeros. The modeling error is reduced when the number of poles and zeros is increased. In this case, the modeling error is more sensitive to the number of zeros than that of poles. The modeling error converges when the number of zeros is over 40 and the increase of pole-number helps its reduction. If parameters of pole-zero model are increased, the modeling error should be decreased. The optimum number of poles and zeros is selected when the modeling error converges below 1% with the smallest number of parameters regardless of poles and zeros. In this case, the optimal number of poles and zeros for pole-zero room model is 42 for zeros and 34 for poles. The estimated transfer function is depicted with dashed lines in figure 4.



(a) Original room transfer function (blue line) and estimated room transfer function (dashed lines) *(b) Pole-zero map*

Figure 4. Optimal pole-zero model for given room transfer function

Figure 4 (b) describes the distribution of pole and zero in optimal pole-zero model. Most of poles and zeros are located on the line of circle. The eleven poles outside the unit circle explain peaks which are depicted in figure 4 (a).

SPATIAL INTERPOLATION AND EXTRAPOLATION OF ROOM TRANSFER FUNCTION

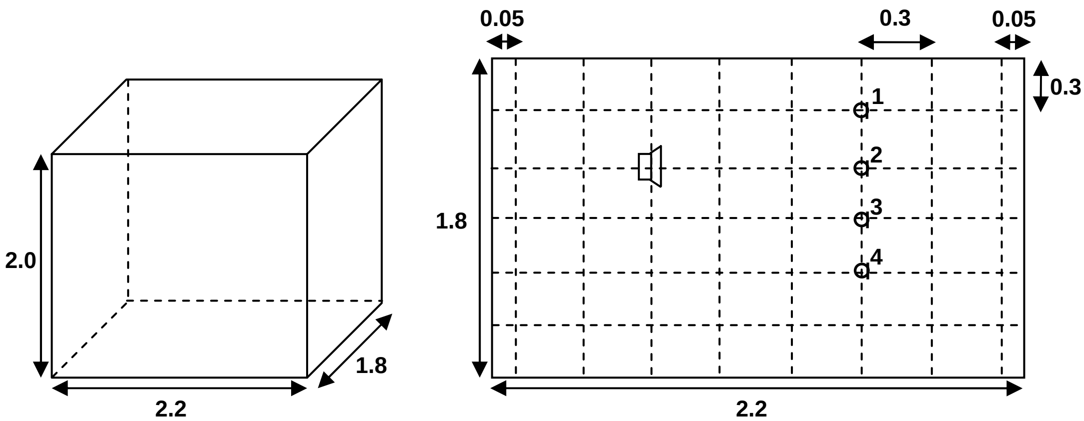
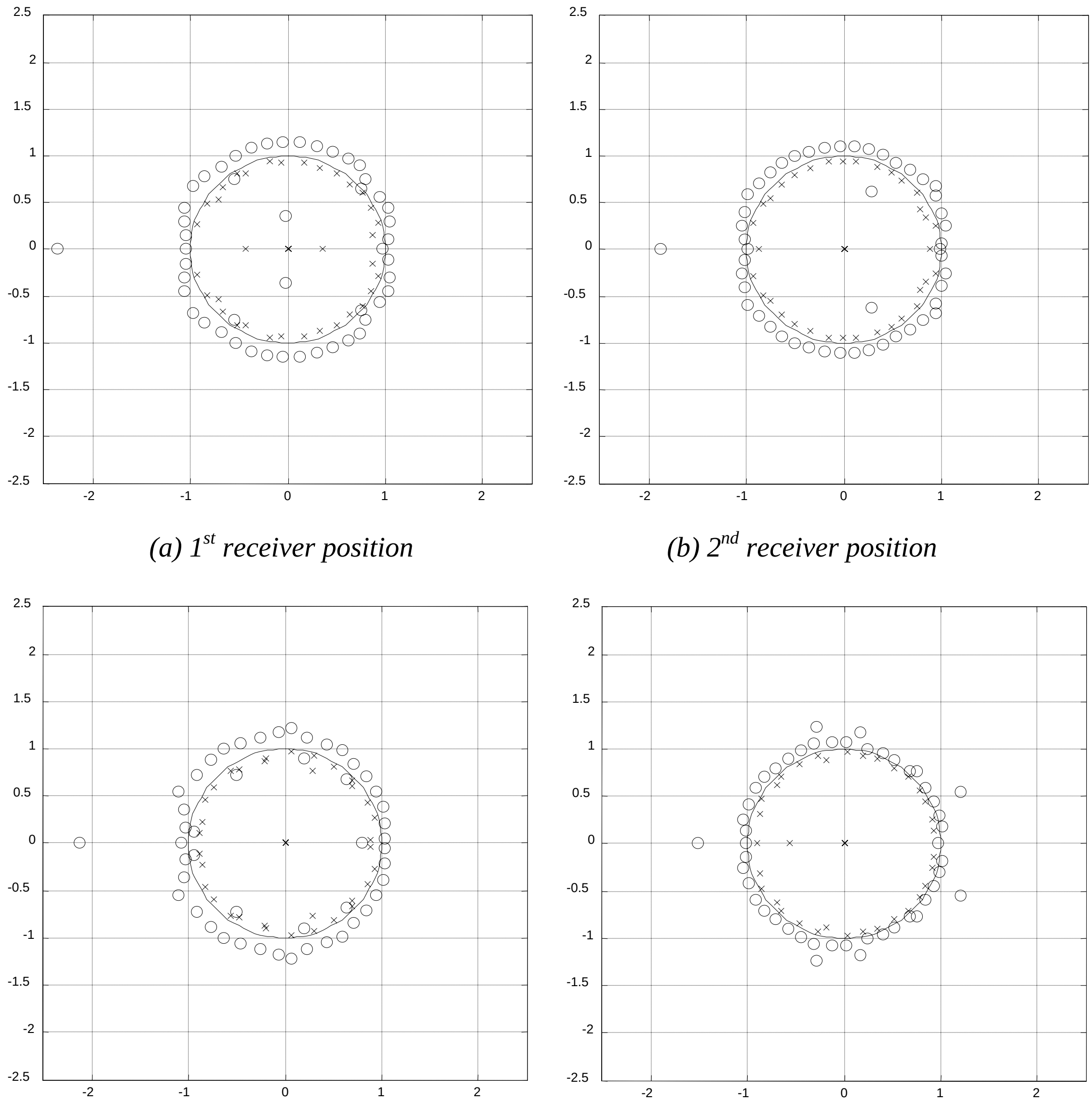


Figure 5. Experimental setup - room size and arrangement of source and receiver

Figure 5 explains the experimental setup for room transfer function estimation and it interpolation and extrapolation. The measurement is conducted with 16 bit resolution and 16 kHz sample rate. Sound proof room is used to isolate the sound field from other background noise. We measured 4 impulse responses by using white noise sequences with fixed loudspeaker and numbered receiver positions. The optimal wiener FIR filter used for the estimation of room transfer functions [5].



(a) 1st receiver position *(b) 2nd receiver position*

(c) 3rd receiver position *(d) 4th receiver position*

Figure 6. Pole, zero maps with respect to the location of receiver

The optimal numbers for poles and zeros are obtained as 34 and 50 using proposed concept for this room transfer function. Figure 6 represents the pole-zero maps with respect to the location of receiver which is numbered in figure 5. The spatial interpolation and extrapolation of room transfer function is possible if we can make a relationship or a equation of pole-zero movement as shown in figure 6. The most poles and zeros are not easily movable. On the other hand, a few poles and zeros move dramatically with unknown regular pattern when the room transfer function is modelled using optimal pole-zero model. These patterns for movements of crucial poles and zeros make the spatial interpolation and extrapolation possible. Furthermore, the poles and zeros which have little movement are considered for development of the optimal pole-zero model.

SUMMARY

This paper investigates on the procedure to find the optimal number of poles and zeros in the pole-zero room model which minimize the modeling error and the number of changing parameters. This approach is another way to find the optimal room transfer function model. In addition, the possibilities about spatial interpolation and extrapolation using this optimal pole-zero model is considered. We measure transfer functions of real sound proof room between fixed source and varying receiver position. With optimal pole-zero model, moving poles and zeros which has a relation with the location of receiver are observed through the measurement. By using this relationship, we can get the concept of interpolation and extrapolation for the room transfer function.

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