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NUMERICAL MODELING OF SOUND SIGNAL PROPAGATION THROUGH A LIQUID WITH BUBBLE AREA

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Abstract
Numerical modeling of pressure wave propagation is considered in a liquid with air bubbles, which are in the rectangular channel. It is assumed that the liquid is the acoustically compressible one and besides that the Herring-Flynn state equation is used for the bubble description. It is shown that properties of the pressure wave propagation depend on the gas volume content and on the bubble area width. Regimes when amplitude of the pressure wave is much more than initial one are found.

INTRODUCTION

Pressure wave propagation in a liquid with a bubble area is an important problem. The similar phenomena arise in many natural and technical processes. Gas bubbles in a liquid change acoustic properties of fluid. This influence can have various characters. Plane wave propagation via an air bubbling has been investigated fairly comprehensively (see [1,2]). Effects of waves damping in air bubbling were carefully studied (see [3-5]). Attempts of numerical modeling of the two-dimensional pressure wave propagation via an air bubbling of square section were presented in [6], where the pressure wave amplification in comparison with the initial disturbance was revealed.

In the given work we study the mechanism of pressure wave increase at their propagation via a channel with the bubble. We have shown that the cumulative flow is formed as a result of interaction of the wave with the bubble zone. This phenomenon leads to the pressure splash in the bubble area. Formation of such flow can be explained by the pressure gradient directed inside of the bubble zone.

This paper outline is as follows. The mathematical model of the problem considered is presented in Section II. The method of the computational solution of the problem is given in Section III. We discuss the results of the numerical modeling of the problem considered in Section IV.

II. FORMULATION OF THE PROBLEM

Let the wave disturbance propagate in the tank of rectangular section (Fig. 1). Let us also suppose that the tank is filled by water with a bubble area. We assume that the initial disturbance does not depend on z, therefore the problem will be two-dimensional.

The mathematical model is considered at following assumptions. Water is assumed to be acoustically compressible liquid and gas in bubbles is described by the state equation for the ideal gas. All the bubbles have the spherical form and equal radius for each elementary volume. We do not take into account processes of bubble fragmentation and adhesion. Let us assume the distance between bubbles be much greater than the bubble radius and the interaction between bubbles be due to pressure changes only. We consider that fluid viscosity is significant only in the processes of interaction between the phases.

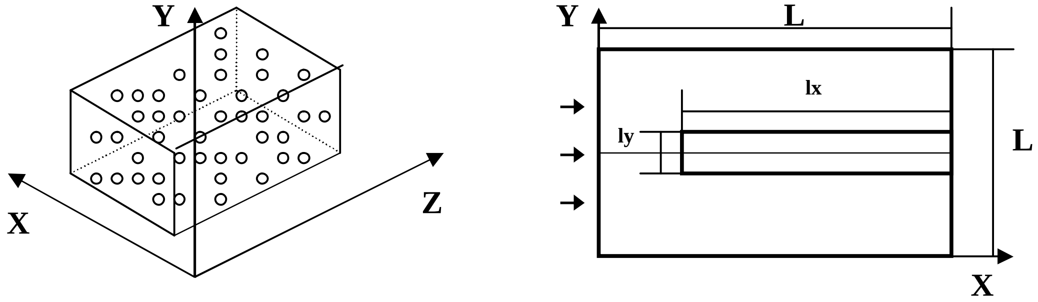

Figure 1 - Estimated area.

The mathematical model with the above mentioned assumptions corresponds to assumptions which are taken into account in [2]. The system of differential equations describing our problem can be written as

$$\frac{d\rho}{dt}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0, \quad \frac{dn}{dt}+n\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0, \tag{1}$$

$$\rho\frac{du}{dt}+\frac{\partial p_l}{\partial x}=0, \quad \rho\frac{dv}{dt}+\frac{\partial p_l}{\partial y}=0, \tag{2}$$

$$\frac{dp_g}{dt}=-\frac{3\gamma p_g}{R}w-\frac{3(\gamma-1)}{R_0}q, \tag{3}$$

$$w=\frac{dR}{dt}, \quad \left(\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right), \tag{4}$$

$$\alpha_l + \alpha_g = 1,\ \ \alpha_g = \tfrac{4}{3}\pi R^3 n\ , \tag{5}$$

$$\rho = \rho_l + \rho_g\ ,\ \ \rho_i = \rho_i^0 \alpha_i\ , \tag{6}$$

where ρ is the average density of a liquid and bubbles, ρ_i^0 is the real density of each phase, u and v are the components of velocity in x and y direction, accordingly, n is the number of bubbles per unit volume, p_i is the pressure of phases, R is the bubble radius, γ is the adiabatic exponent, w is the radial velocity of bubble, q is the heat transfer rate, α_i is the volume content of phases. The subscripts i=l, g correspond to liquid and gas phases.

We assume that bubbles can radiate a sound which leads to the energy dissipation at the wave propagation. All these effects are taken into account in the Herring-Flynn state equation [7]

$$R\frac{dw_{hf}}{dt} + \frac{3}{2}w_{hf}^2 + \left(4\nu_l - \frac{R^2}{\rho_l C_l}\frac{dp_g}{dR}\right)\frac{w_{hf}}{R} = \frac{p_g - p_l}{\rho_l^0},\ \ w = w_{hf}\ , \tag{7}$$

where ν_l is the liquid viscosity, C_l is the speed of sound in the pure liquid (C_l=1500 m/s).

Considering liquid as acoustically compressible and the gas as ideal we get the state equations for phases in the form

$$p_l = p_{l0} + C_l^2\left(\rho_l^0 - \rho_{l0}^0\right),\ \ p_g = \rho_g^0 \tilde{R} T_g\ , \tag{8}$$

where $\tilde{R}$ is the gas constant. Here and in what follows, the subscript *0* denotes the parameters related to the initial undisturbed state.

The heat flow q can be obtained from the following relations

$$q = Nu\lambda_g \frac{T_g - T_0}{2R}\ ,\ \ \frac{T_g}{T_0} = \frac{p_g}{p_0}\left(\frac{R}{R_0}\right)^3 \tag{9}$$

$$Nu = \sqrt{Pe}\ ,\ \ Pe \ge 100\ ,\ \ Nu = 10\ ,\ \ Pe < 100\ , \tag{10}$$

$$Pe = 12(\gamma - 1)\frac{T_0}{|T_g - T_0|}\frac{R|w|}{\chi_g},\ \ \chi_g = \frac{\lambda_g}{C_g \rho_g^0}\ , \tag{11}$$

where T_0 = const is the fluid temperature, λ_g is the thermal conductivity, Nu is the Nusselt number, C_g is the thermal capacity of gas.

Thus the mathematical model of our problem is described by the system of equations (1) - (11) with initial and boundary conditions. Using this system of equations we investigate wave propagation via bubble area.

III. NUMERICAL METHOD FOR THE PROBLEM

We do not have any possibility to find the analytical solution of our problem. Therefore we use the numerical method to solve the above mentioned problem.

Let us use the Lagrangian coordinates as in this case the bubble area will be fixed. Considered amplitudes of pressure waves do not lead to distortion of Lagrangian grids and this simplifies algorithm, which solves the problem.

Using the Lagrangian variables the system of equations (1)-(2) can be written as

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho J}\left(\frac{\partial p_l}{\partial x_0}\frac{\partial y}{\partial y_0}-\frac{\partial p_l}{\partial y_0}\frac{\partial y}{\partial x_0}\right),\quad \frac{\partial x}{\partial t}=u\ , \tag{12}$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho J}\left(\frac{\partial p_l}{\partial y_0}\frac{\partial x}{\partial x_0}-\frac{\partial p_l}{\partial x_0}\frac{\partial x}{\partial y_0}\right),\quad \frac{\partial y}{\partial t}=v\ , \tag{13}$$

$$J=\frac{\partial x}{\partial x_0}\frac{\partial y}{\partial y_0}-\frac{\partial x}{\partial y_0}\frac{\partial y}{\partial x_0} \tag{14}$$

where J is the Jacobian of the transformation from Lagrangian to Euler variables, x_0 and y_0 are the Lagrangian variables, which are taken equal to the initial values of the Eulerian coordinates.

The Herring-Flynn state equation for adiabatic compressed gas takes the form

$$R\frac{dw_{hf}}{dt}+\frac{3}{2}w_{hf}^2+\left(4\nu_l+\frac{R^2}{\rho_l C_l}\frac{3\gamma P_g}{R}\right)\frac{w_{hf}}{R}=\frac{p_g-p_l}{\rho_l^0},\ w=w_{hf} \tag{15}$$

$$\frac{\partial R}{\partial t}=w\ ,\ \ \alpha_g=S_0/\left(4\!/\!_3\,\pi R^3 N\right), \tag{16}$$

$$\rho_g^0=m_g/\left(4\!/\!_3\,\pi R^3 N\right),\ \ \rho_l^0=m_l/\left(Vol-\left(4\!/\!_3\,\pi R^3 N\right)\right), \tag{17}$$

where S_0 is the "volume" of a cell in Euler variables, N is the quantity of bubbles in a cell. The state equations for phases are the following

$$\frac{\partial p_g}{\partial t}=-\frac{3\gamma p_g}{R}w-\frac{3(\gamma-1)}{R_0}q\ ,\ \ p_l=p_{l0}+C_l^2\left(\rho_l^0-\rho_{l0}^0\right), \tag{18}$$

The system of equations (12) - (18) is solved numerically using an implicit scheme. We use a rectangular grid for approximation of the differential equations

$$
\begin{gathered}
\left(x_{0i}, y_{0i}, t_k\right),\ \left(x_{0i+1/2}, y_{0i+1/2}, t_k\right),\\
x_{0i+1} = x_{0i} + h_{x0},\ \ x_{0i+1/2} = x_{0i} + 0.5 h_{xo},\ \ i = 0,1,...,N_1 - 1,\\
y_{0j+1} = y_{0j} + h_{y0},\ \ y_{0i+1/2} = y_{0j} + 0.5 h_{y0},\ \ j = 0,1,...,N_2 - 1,\\
x_{00} = 0,\ \ x_{0N_1} = L_x,\ \ y_{00} = 0,\ \ y_{0N_2} = L_y,\\
t_{k+1} = t_k + \tau,\ \ t_k = k\tau,\ \ k = 0,1,2...,
\end{gathered}
\tag{19}
$$

where h_{x0}, h_{y0} , and τ are the steps in the X, Y coordinates and time, respectively.

For numerical simulation of our problem we choose the time step using the Kurant's condition [8].

IV.RESULTS OF NUMERICAL MODELING

Let us present some results of numerical modeling of wave propagation in a liquid containing bubble area. At numerical modeling we assume that pressure on the wall at x=0 has dependence on time in the form

$$
p^0(t, y, x=0) = p_0 + \Delta p_0 \exp\left[\psi(t_*)\right],\ \ \psi(m) = -\left(\frac{t - m/2}{m/6}\right)^2 \tag{20}
$$

At other walls we take a solid wall into account. As initial condition we use undisturbed liquid with bubble area.

For all calculations we apply the following values for the impulse parameters: p_0 = 0.1MPa, Δp_0 = 0.3MPa, t* = 1ms, T_0 = 300 K, where T_0 is the liquid temperature.

Figure 2 shows evolution of the pressure wave in the first variant of calculation. In this variant we observe the strong increase of the pressure splash. Parameters of this variant are the following: l_x = 4m, l_y = 0.01m are the length and the width of bubble zones, R =10^{-4}m is the bubble radius. The tank has the size 5m×1m, the volume content of gas equals 10^{-2}. We can see that the splash is formed on the crest of wave, and this one propagate together with liquid, but it moves slowly in time. We can notice that the amplitude of wave is increasing in time up to the value of 30 atmospheres which is 9 times as much as initial one. After that we observe the destruction of the splash because the conditions supporting its existence are broken. The pressure near walls falls up to 2 atmospheres. The velocity of the splash distribution is 1000 m/s and the velocity of the wave is 1500 m/s.

In the second variant of calculation the volume content of gas is 10^{-3} what is less than in prevision case. Other parameters of calculation are the same. We observe long existence of the pressure splash in this variant of calculation. The pressure splash appears gradually and is formed before the wave, where it exists long enough time and propagates with velocity equal to sound speed in a pure liquid. Evolution of the splash is presented in figure 3.

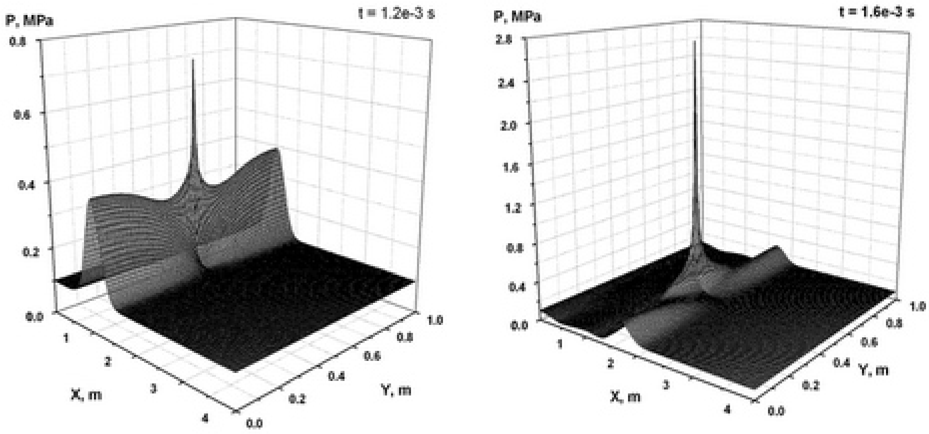

Figure 2 - Pressure diagram at the moments of time t = 1.2ms (left) and t = 1.6ms (right).

We can see three peaks on the oscillogram of this signal what tells us that the oscillatory splash mode appears. By the end of calculation the pressure of splash is 2.5 times greater then amplitude of an initial signal.

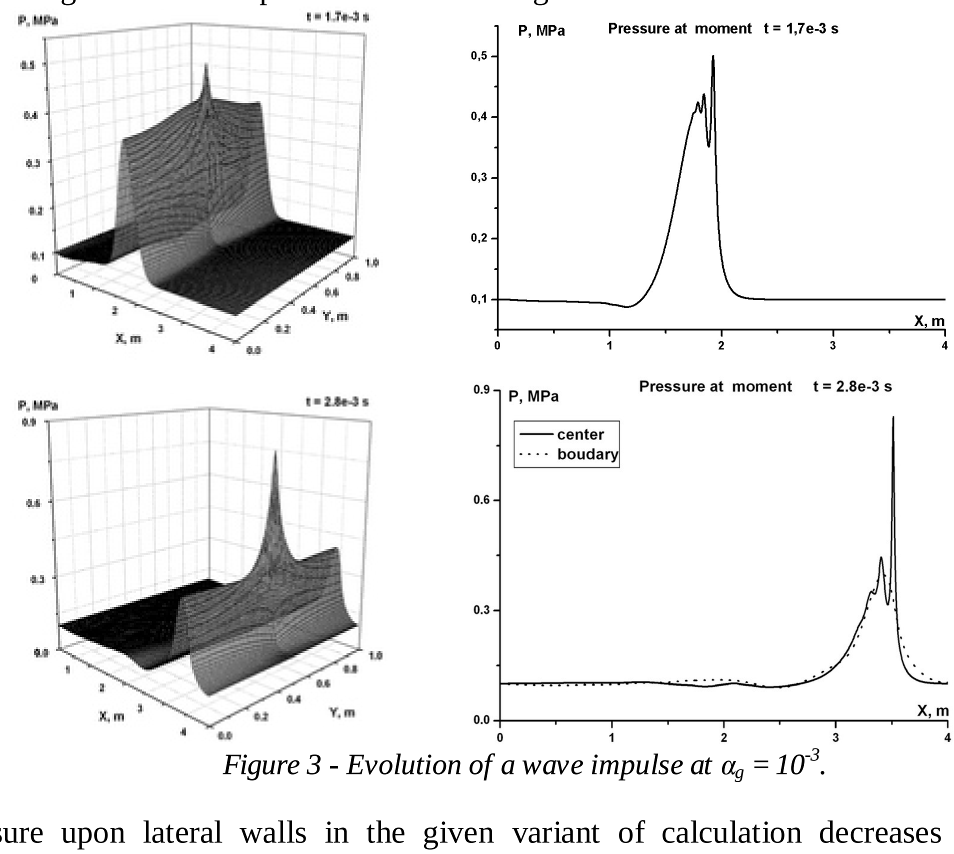

Figure 3 - Evolution of a wave impulse at $\alpha_g = 10^{-3}$.

Pressure upon lateral walls in the given variant of calculation decreases at the beginning, when wave enters the bubble zone. This decrease is not more than 0.3 atmospheres. There is a repressuring upon lateral walls and eventually it is recovered but the wave thus is narrowed.

In the third variant of calculation we change the width of the tank. More exactly it equals 0.4m. The given parameter changes the time of the existence of the pressure splash. This fact is illustrated in figure 4. In the given variant of calculation the pressure splash exists not a long time. The relation of the bubble channel width and the tank width influences the time of existence of the pressure splash. This relation is equal to 0,025. In this case the velocity of the splash distribution is about 840 m/s, a velocity of the wave is almost 2 times greater.

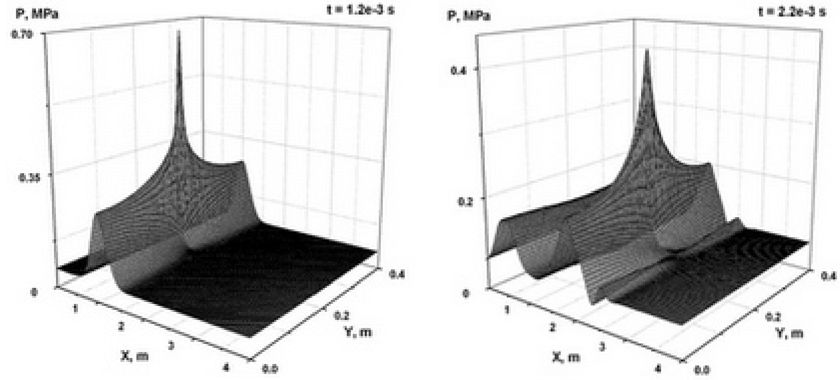

Figure 4 - Pressure diagram at the moments of time t = 1.2ms (left) and t = 2.2ms (right)

The fourth variant of calculation shows influence of value of the bubble channel width on the wave propagates evolution. In this case we increase the bubble channel width in 5 times in comparison with other variants of calculation. It equals 0.05m. Other parameters in this variant of calculation are the following: the tank is equal to 5m×3m, the volume content of gas equals 10^{-2}. The illustration of the wave evolution in this variant of calculation is given in figure 5. From figure 5 we see that the splash forms behind the pressure wave. The velocity of the splash in the bubble channel is equal to 400 m/s. The basic wave moves quickly and conditions for existence of the splash are broken. Late the splash collapses and gives a divergent wave.

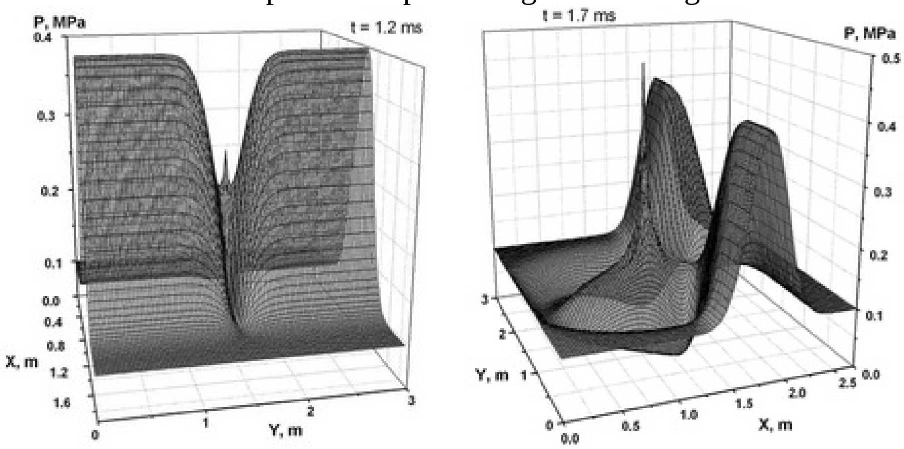

Figure 5 - Evolution of a wave impulse at $\alpha_g = 10^{-2}$ and $l_y = 0.05$ m. Pressure diagram at the moments of time 1.2 ms (left) and 1.7 ms (right)

V. CONCLUSIONS

Analysing the results of numerical modeling of the pressure wave propagation in a liquid with a bubble area we get that the pressure splash is formed in a bubble area. Also we can see that the value of the pressure splash depends on the bubble channel width, on the volume content of gas, and on the relation of the bubble channel width to the tank width.

We have found the value parameters of the problem when the pressure splash exists in the bubble channel for a long time. Two additional pressure splashes can be observed as well (the second variant of calculation).

We have got conditions when the pressure splash can appear behind, before and on initial wave (the first, the second and the fourth variants of calculation).

We have obtained that decreasing of the bubble channel width leads to increasing of the splash amplitude. The value of the splash amplitude can far exceed initial signal more than in 9 times (the first variant of calculation).

VI. ACKNOWLEDGMENTS

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