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DYNAMIC CHARACTERIZATION OF FLEXIBLE MATRIX COMPOSITE DRIVESHAFT

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Abstract
This paper presents the dynamic characterization of a flexible matrix composite (FMC) driveshaft. A primary objective is to experimentally validate the analytic modelling of a FMC shaft based on the equivalent modulus beam theory (EMBT). A test rig is developed which consists of a FMC shaft, a foundation beam and bearings, a driving motor and a shaker. Then, experimental characterization of the FMC shaft is performed by measuring responses of both the FMC shaft and the foundation beam under the base excitation with a frequency sweep up to 130 Hz and imbalance loadings while spinning up the FMC shaft continuously near the 1st resonance. Results show that the measured responses around vertical shaft modes are observed in a fairly good correlation with the predicted from the EMBT modelling. Consequently, the shaft modelling based on EMBT can be utilized to estimate the dynamic behaviours of a FMC shaft. This is further validated by spin-up results, where a good agreement in the rate of response increase is observed between the measured and the predicted.

INTRODUCTION

Composite materials are increasingly employed in the design of high speed rotating shafts due to their lightness and a wide range of strength properties. Flexible matrix composite (FMC), which includes extremely flexible matrices, is a new advanced material with a potential application to the design of driveshafts under large bending deformation. A helicopter tailrotor driveshaft is one of such applications. However, the enhanced lateral flexibility reduces the bending mode natural frequencies and subsequently causes whirling instability under the supercritical speed operating conditions. In addition, FMC has higher internal damping than conventional metals, which has a destabilizing effect on a supercritical shaft. Accordingly, it is essential to estimate the dynamic characteristics of FMC shafts in a reliable manner.

Many researchers have rigorously investigated the modelling of composite shafts through analytical and experimental approaches. Zinberg and Symonds[1] applied an equivalent modulus beam theory (EMBT) to estimate the critical speeds of a helicopter shaft and Singh and Gupta[2] developed a layerwise beam theory (LBT) for applying to unsymmetric composite shafts. Also, many others[3-5] modified EMBT and LBT to compensate for coupling effects and stacking sequence of plies.

The purpose of this paper is to experimentally validate EMBT for a FMC driveline. Although EMBT has been proved to be effective for a composite shaft with symmetric balanced laminates, it has never been applied to characterize a FMC driveshaft with high internal damping. A test rig is developed which consists of a FMC shaft, a foundation beam and bearings, a driving motor and a shaker. Then, frequency response functions (FRFs) and transient responses are obtained under external excitation.

ANALYTIC FORMULATION

A test rig of a FMC driveline in this study is shown in Figure 1. A FMC shaft is supported by a foundation beam through the bearings and further constrained by external dampers. The FMC shaft is made of Carbon fiber T700 and Polyurethane Adiprene L100, whose properties are shown in Table 1, and has a symmetric balanced laminate: $[\pm 45°/\pm 90°]_S$. The shaft is segmented into two pieces and connected through rigid couplings.

Table 1 – Material properties of T700/L100

Properties	Modulus (GPa)	Loss factor
Longitudinal	115	0.011
Transverse	0.139	0.114
Shear	0.250	0.112

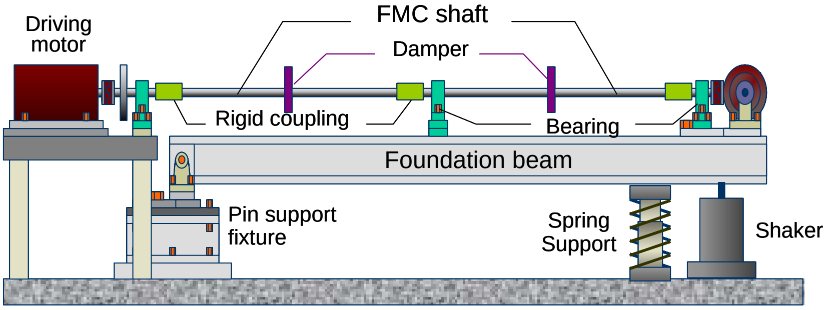


Figure 1 – Schematic of a test rig

The finite element method is applied to develop an analytic model. The FMC shaft and the foundation beam are modeled by 2-node beam elements and each node has six degrees of freedom. On the other hand, the bearings are modeled using spring elements with six degree-of-freedom nodes in consideration of coupling between the foundation beam's twisting/bending motion and the shaft's bending/axial motion. The motor/gearbox are modelled as concentrated mass elements.

Finally, the equations of motion for the driveline are written as:

$$[M]\{\ddot{q}\}+\left([C]+\Omega[G]\right)\{\dot{q}\}+\left([K]+[K_R]\right)\{q\}=\{F\} \tag{1}$$

where, $[M]$, $[C]$, $[G]$ are the inertia, damping and gyroscopic matrices and $[K]$, $[K_R]$ are the stiffness and rotating internal damping matrices, respectively. Also, $\{q\}$ is the displacement vector and Ω is the rotating speed of the shaft. As an external loading $\{F\}$, base excitation is applied.

Based on EMBT, equivalent modulus properties are derived in a complex form for incorporating internal damping. Or,

$$E_{eq} = E(1+i\eta_E), \qquad S_{eq} = S(1+i\eta_S) \tag{2}$$

In (2), E_{eq} and S_{eq} are the equivalent Young's and shear modulus, respectively, which are determined by material properties of the fiber and the matrix, laminate parameters such as ply orientations, number of layers[6].

EXPERIMENTAL VALIDATION

Experimental Set-up

The dynamic characteristics of the system are identified by measuring responses of both the FMC shaft and the foundation beam under two different excitation loadings: one is the base excitation by the shaker at the spring-supported end and the other is the imbalance loading during the shaft spin-up.

In order to measure the input excitation and the output responses, four types of sensors are installed on the testrig as shown in Figure 2: a force sensor for the shaker input, a tachometer for the shaft speed, accelerometers for the foundation response and optical probes for the shaft responses. Four accelerometers are placed along the foundation beam and each one measures the vertical and horizontal acceleration of the foundation beam. On the other hand, two sets of optical probes are located on the top surface of the foundation beam and measure the relative displacements between the shaft and the foundation beam. Each set has two optical probes: one for the vertical displacement and the other for the horizontal.

Calibration of the Optical Probes

The optical probes in this work need to be calibrated to estimate the sensitivity onto the FMC surface. In addition, the coupling exists between the horizontal and vertical directions since the shaft surface is not flat. Hence, the following procedure was applied to calibrate the optical probes. First, a lateral loading was applied to the shaft in the vertical direction such that the vertical displacement at the probe location should be 0.01″. The output signals from both the vertical and horizontal probes were measured. Second, the shaft is further deflected vertically up to 0.05″ with an increment of 0.01″ and the probe signals were recorded at the end of each increment. Third, the shaft is released with a decrement of 0.01″ and the probe signals were recorded at the end of each decrement. Fourth, the relationships between the vertical displacement and the probe signals were derived:

$$\kappa_{VV} = mean\left(\frac{O_{VV}^{\Delta}}{\delta_V^{\Delta}}\right); \qquad \kappa_{HV} = mean\left(\frac{O_{HV}^{\Delta}}{\delta_V^{\Delta}}\right) \tag{3}$$

Fifth, the above procedure was repeated in the horizontal direction and the displacement-signals relations were obtained:

$$\kappa_{HH} = mean\left(\frac{O_{HH}^{\Delta}}{\delta_H^{\Delta}}\right); \qquad \kappa_{VH} = mean\left(\frac{O_{VH}^{\Delta}}{\delta_H^{\Delta}}\right) \tag{4}$$

Finally, the sensitivity of the optical probes were determined as:

$$\begin{Bmatrix} \delta_V \\ \delta_H \end{Bmatrix} \equiv \begin{bmatrix} \gamma_{VV} & \gamma_{VH} \\ \gamma_{HV} & \gamma_{HH} \end{bmatrix} \begin{Bmatrix} O_V \\ O_H \end{Bmatrix} = \begin{bmatrix} \kappa_{VV} & \kappa_{VH} \\ \kappa_{HV} & \kappa_{HH} \end{bmatrix}^{-1} \begin{Bmatrix} O_V \\ O_H \end{Bmatrix} \tag{5}$$

The calibration signals in (3), (4) are illustrated in Figure 3 and the resulting sensitivities are summarized in Table 2.

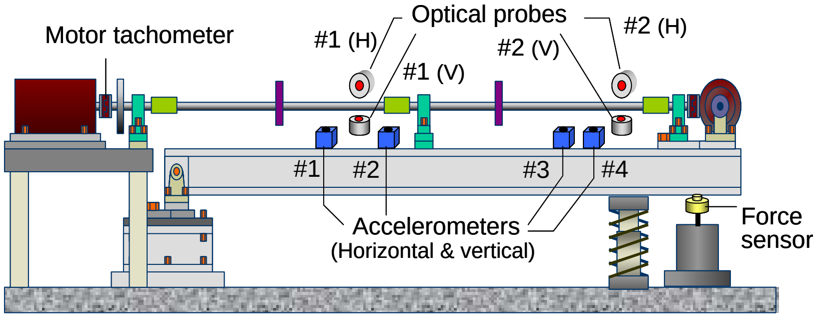
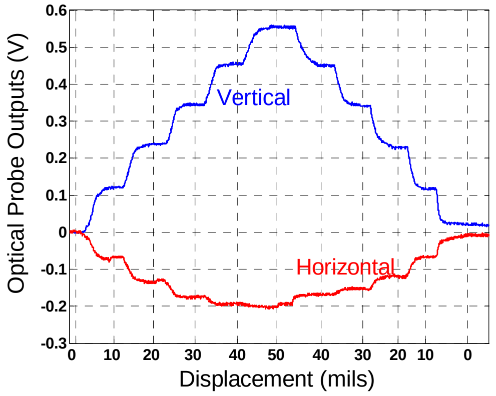


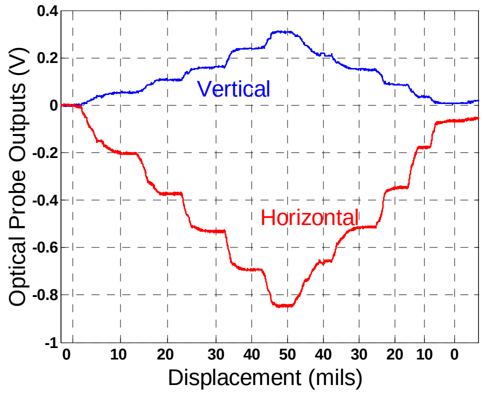
Figure 2 – Experimental set-up

Table 2 Sensitivities of the optical probes

Optical probe	Sensitivity (mils/N)			
	γ_{VV}	γ_{VH}	γ_{HV}	γ_{HH}
1	104.6	39.8	-27.6	-71.8
2	109.1	-37.3	37.3	-94.0



(a) Vertical



(b) Horizontal

Figure 3 – Calibration of the optical probes

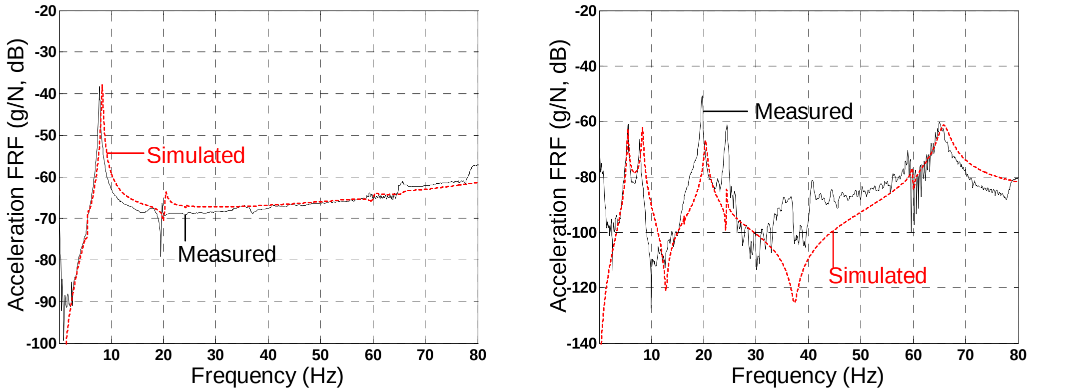
RESULTS AND DISCUSSIONS

Frequency response functions under base excitation

For the base excitation, the FRFs of both the measured and the simulated are illustrated in Figures 4 and 5. Also, the modal characteristics and shapes are shown in Table 3 and Figure 6, respectively.

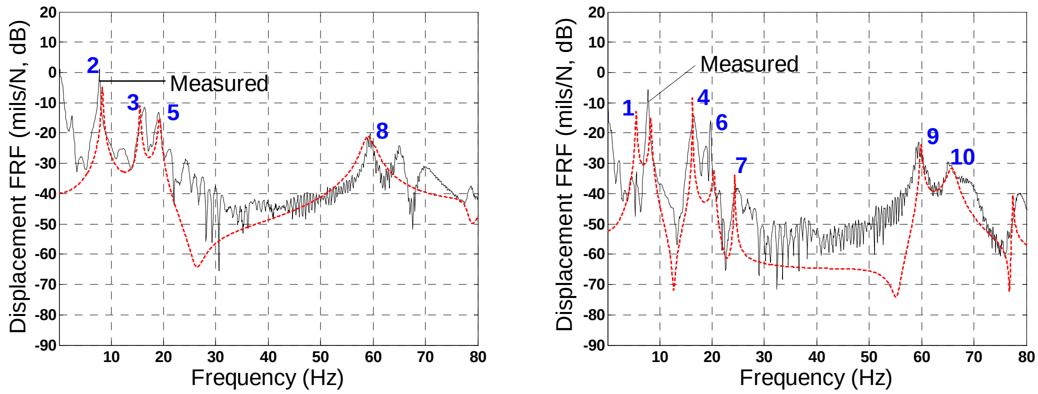
As summarized in Table 3, three types of the test rig modes exist: the shaft modes, the foundation beam modes and the coupled modes between the shaft and the foundation beam. In the shaft modes, only the shaft motion is observed. In the foundation beam modes, however, the foundation beam motion is dominant and the shaft behaves similarly to the foundation beam. Also, in the coupled modes, both the shaft and the foundation beam respond to the excitation, but the shaft motion is different from the foundation beam’s.

The measured results of the shaft modes are in good agreement with the simulation results as shown in Figures 4 and 5. The vertical shaft modes, especially, are well estimated by the simulation model. However, the experimental results do not well match the simulated ones around the coupled and the foundation beam modes. There are several factors responsible for the difference.



(a) Vertical response of Accelerometer #1 *(b) Horizontal response of Accelerometer #1*

Figure 4 – Frequency response functions of the foundation beam



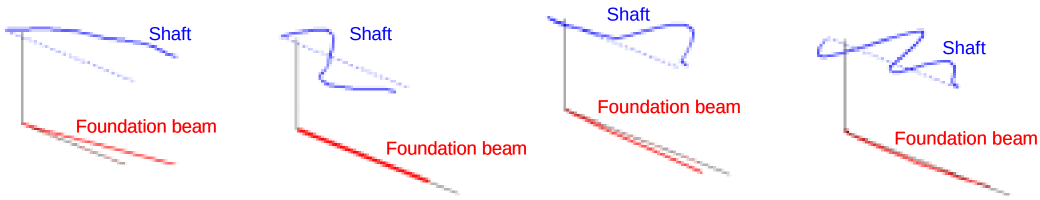
(a) Vertical response of Optical probe #1 *(b) Horizontal response of Optical probe #1*

Figure 5 – Frequency response functions of the FMC shaft

Table 3 – Modal characteristics

Mode no.[a]	Damped natural frequency (Hz)[b]	Mode descriptions	Remark[c]
1	5.51	Foundation beam, 1st horizontal	Fig. 6(a)
2	8.31	Foundation beam, 1st vertical	
3	15.51	Shaft, 1st vertical	
4	16.28	Shaft, 1st horizontal	
5	19.24	Shaft, 2nd vertical	Fig. 6(b)
6	20.34	Foundation-shaft coupled, 1st horizontal	Fig. 6(c)
7	24.32	Foundation-shaft coupled, 2nd horizontal	
8	59.04	Shaft, 3rd vertical	
9	59.91	Shaft, 3rd horizontal	Fig. 6(d)
10	65.84	Foundation-shaft coupled, 3rd horizontal	

a: Refer to Fig. 5, b: Simulation results, c: Mode shapes displayed



(a) Mode 1 *(b) Mode 5* *(c) Mode 6* *(d) Mode 9*

Figure 6 – Mode shapes of the test rig

First, the foundation beam motion was transmitted to the shaft through the bearing supports, which were placed on the flange of the foundation beam by clamps. Consequently, the beam motion was not completely transferred to the shaft. This is well observed for 1st horizontal beam mode. There is a good agreement in the beam responses as in Figure 4(b), but a big difference in the shaft motion in Figure 5(b). Hence, this is a strong indication that the bearings fails to fully transfer the motion from the beam to shaft. Second, the horizontal beam motion was found to accompany the axial beam rotation, which was sensitively affected by the stiffness characteristics of the pin support bolted onto the ground.

Although a detailed structural analysis was performed for deriving a proper model, some difference in bolting existed between the simulation and the experiment. This is responsible for the mismatch around the 1st and 2nd coupled modes in Figures 4(b) and 5(b). Third, the horizontal beam motion caused the vertical shaft due to the axial beam rotation and a vertical offset between the beam and the shaft. This coupled motion was largely affected by the stiffness and damping characteristics of the bearing and pin supports. The axial beam rotation from the simulation seems to be underestimated. Accordingly, some peaks observed in the experimental FRFs are not found in the simulation FRFs. The 3rd coupled mode in Figure 5(a) is a typical example.

Nevertheless, the base excitation experiment is mainly focused on verifying the FMC shaft modeling based on EMBT and the results above illustrate that the shaft model predicts the modal behaviors of the pure shaft modes fairly well. Thus, it can be stated that the developed shaft model based on EMBT is valid enough to be used for estimating dynamic characteristics of the FMC shaft.

Transient Spin-up Response

Transient responses were obtained while increasing the shaft speed continuously with a rate of 5.8 rpm/s up to near the 1st natural frequency of the shaft. Since the foundation beam motion did not much occur, the spin-up responses were dominantly affected by the shaft characteristics, especially the internal damping. The shaft response is directly related to the imbalance loading due to mass eccentricity, most of which was caused by the rigid couplings. As exhibited in Figure 7, the shaft response increased rapidly as the spinning speed approached to the 1st resonance. A good agreement in the rate of response increase was observed between the measured and the simulated, which were calculated by numerical integration based on the 4th order Runge-Kutta.

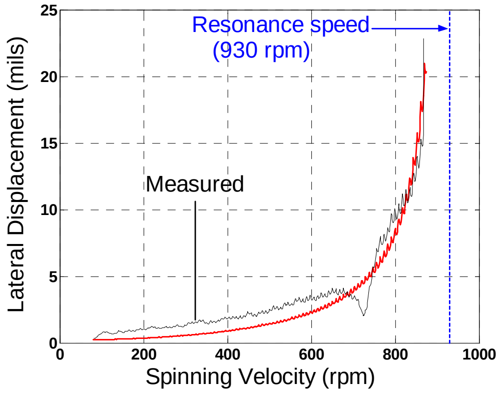


Figure 7 – Vertical spinup response of Optical probe #1

CONCLUSIONS

An analytic modelling of a FMC driveshaft based on EMBT has been validated experimentally in this paper. A FMC test rig was developed and the FRFs and transient spin-up responses were obtained. Results show that the measured FRFs around vertical shaft modes are observed in a fairly good correlation with the predicted from the EMBT modelling. Consequently, the shaft modelling based on EMBT can be utilized to estimate the dynamic behaviours of a FMC shaft. This is further validated by spin-up results, where a good agreement in the rate of response increase is observed between the measured and the predicted.

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