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# SEMIACTIVE BALANCING CONTROL OF A JEFFCOTT-LIKE ROTOR SYSTEM SUPPORTED ON MR DAMPERS

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**Abstract**

In this paper is addressed the problem of semiactive balancing control of a Jeffcott-like rotor system on journal bearings, one of them supported on radial Magneto-Rheological (MR) dampers. The mathematical model of the rotor system results from a Jeffcott model and the dynamics associated to the MR dampers, whose properties depend on the current inputs (control actions). There are different models for MR dampers proposed in the literature (Bingham, Bouc-Wen, Spencer, Choi-Lee-Park, etc.), most of them experimentally validated. For control purposes we use the Choi-Lee-Park polynomial model, which is quite consistent with the nonlinear and complex hysteresis damper models and also simplifies the physical implementation. The semiactive control scheme is then analyzed and synthesized to manipulate the unbalance response of the rotor system, by means of a proper modification of the rotordynamics coefficients (damping and stiffness). It is performed a controllability and stability analysis of the overall rotor system, in order to propose a suitable semiactive control strategy based on sliding-mode control techniques. Some numerical simulations are included to illustrate the dynamic performance and robustness when the rotor is started and operated over the first critical speed.

## INTRODUCTION

Vibrations caused by mass unbalance are a common problem in the rotating machinery. The unbalance occurs when the principal axis of inertia of the rotor is not coincident with its geometric axis. This is a result of inevitable imperfections in manufacturing and assembly of rotors. These vibrations can cause high levels of noise and wear, and these may lead to failures or lost of the machine. See, e.g., Vance [8], Wowk [10], Vance [8], Zhou and Shi [11].

Active vibration control has been an area of theoretical and experimental research in rotating machinery, providing many advantages for the attenuation of vibration amplitude during run-up and coast-down through critical speeds, and minimization of sudden transient behavior due to rotor unbalance or parametric uncertainty. This problem has been investigated using different devices such as magnetic bearings, active squeeze film dampers, lateral force actuators, pressurized bearings, etc. (see, e.g., Carmignani *et al.* [1], El-Shafei [4], Guozhi [3], Lum *et al.* [5], Zhou and Shi [11]).

Recently, many types of semiactive electrorheological (ER) or magnetorheological (MR) dampers have been used for vibration attenuation on rotor bearing systems (Carmignani *et al.* [1], El-Shafei [4], Guozhi *et al.* [3]). Different hysteresis models for the ER or MR dampers have been proposed in the literature (Bingham, Bouc-Wen, Spencer, Choi-Lee-Park, etc.), most of them theoretically and experimentally validated. In practice the MR fluids are more attractive than ER fluids, mainly because of the employment of low voltages and inherent higher yield strength (Spencer *et al.* [6]).

The objective of the present paper is to propose a semiactive balancing control scheme for a Jeffcott-like rotor system on journal bearings, one of them supported on two radial MR dampers. The mathematical model of the rotor system results from a Jeffcott model and the dynamics associated to the MR dampers, whose properties depend on the current inputs (control actions). The damping force provided by the MR dampers is modelled via the Choi-Lee-Park polynomial model (Choi *et al.* [2]), which is quite consistent with the nonlinear and complex hysteresis damper models and also simplifies the controller design and physical implementation. The semiactive control scheme is analyzed and synthesized to manipulate the unbalance response of the rotor system, by means of a proper modification of the rotordynamics coefficients (damping and stiffness), using a control scheme based on sliding-mode control techniques.

# ROTOR-BEARING SYSTEM

The rigid rotor bearing system consists of a planar and rigid disk of mass $m$ mounted on the midspan of a shaft of negligible mass and supported on a journal bearing (left) and a journal bearing (right) with two radial MR dampers and spring bearings (see Fig. 1). Because of the rotor unbalance, the mass center is not located at the geometric center of the disk $S$ but at the point $G$ (center of mass of the unbalanced disk) and the distance between these two points is denoted by the eccentricity $e$. The angular speed of the rotor is represented by $\omega$.

## Rotor-bearing model

A simplified model for the rigid rotor bearing system can be obtained as a Jeffcott-like rotor system with MR dampers, as follows (see Vance [8])

$$\begin{aligned} m\ddot{x} + c_x\dot{x} + k_x x + F_{MRx} &= me\omega^2 \cos\omega t \qquad (1) \\ m\ddot{y} + c_y\dot{y} + k_y y + F_{MRy} &= me\omega^2 \sin\omega t \qquad (2) \end{aligned}$$

where $x$ and $y$ are the radial displacements of the disk, $m$ is the unbalance mass, $e$ the disk eccentricity, and $c_x$ and $c_y$ are radial viscous dampings on the journal bearings. Moreover, the radial stiffnesses $k_x$ and $k_y$ are computed as the equivalent stiffness of

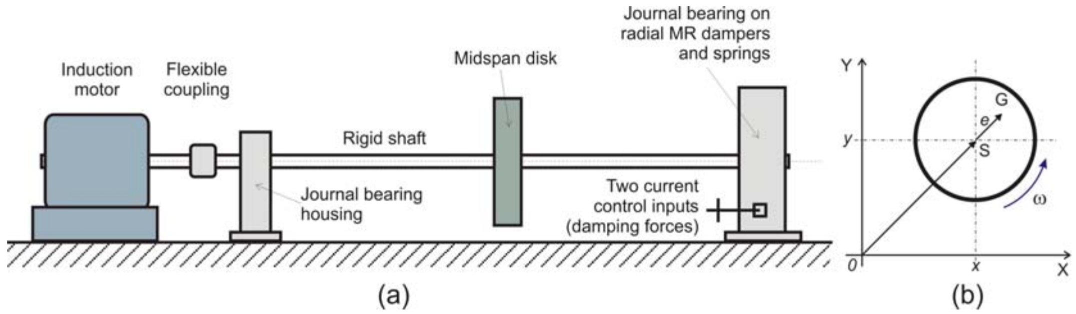


*Figure 1: Schematic diagrams. (a) Rotor bearing system supported on MR damper and springs. (b) Disk with unbalance.*

the shaft ($k_s = 48EI_s/l^3$) and linear springs $k_r$ used in series with each MR damper, to compensate the load equilibrium positions, in such a way that $k_i = k_s + k_{ri}$, $i = x, y$. The MR damping forces are described by $F_{MRx}$ and $F_{MRy}$, each corresponding to individual and independent radial control forces, which will be controlled through manipulation of their own electrical currents. Moreover, it is assumed that the angular speed $\omega$ is constant; otherwise, the dynamics associated to the angular speed leads to a three degrees-of-freedom nonlinear model.

The system parameters are given in Table I.

| **Table I. Simulation parameters for the rotor-bearing system.** | | |
|---|---|---|
| Shaft diameter | $D$ | 0.010 m |
| Disk mass | $m$ | 0.650 kg |
| Unbalance eccentricity | $e$ | $2.47 \times 10^{-5}$ m |
| Shaft length | $l$ | 0.600 m |
| Journal viscous dampings | $c_x, c_y$ | 10.4 N/(m/s) |
| Shaft stiffness | $k_s$ | 23.017 kN/m |
| Radial spring stiffness | $k_{rx}, k_{ry}$ | 801.87 N/m |
| Young's modulus (steel type 4140) | $E$ | 211G Pa |

## MR damper model

The MR fluids are smart materials that respond well to an applied magnetic field, leading to an important change in their rheological behavior (viscosity and stiffness). The viscosity and stiffness changes are continuous and reversible, which makes feasible the application of MR dampers for vibration control (Spencer *et al.* [6]). The passive nature of the MR dampers limit their practical use to semiactive vibration control, although this is sufficient to improve the rotor bearing system response and extend the stability thresholds. Specifically, via the application of a feedback control to manipulate the electrical currents of two radial dampers (directions $X$ and $Y$) one can control two independent damping forces to attenuate the unbalance system response.

For simplicity in the control synthesis, we consider the Choi-Lee-Park polynomial model for the MR dampers (see Choi *et al.* [2] and references therein). The Choi-Lee-Park polynomial model is able to predict the field-dependent damping force and hysteresis behavior of MR dampers. An schematic diagram of the polynomial model is shown in Fig. 2. This simple model divide the hysteresis loop in two regions, for positive acceleration (lower loop) and negative acceleration (upper loop), and then the

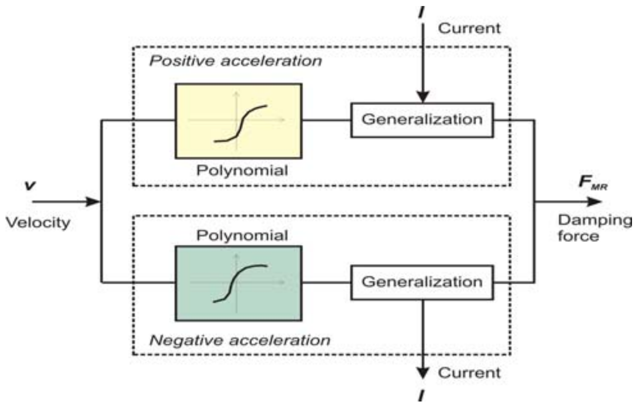


*Figure 2: Schematic diagram of the polynomial model of a MR damper proposed by Choi et al. [2].*

lower and upper loops are fitted by polynomials to experimental results. The damping force for the MR damper is now expressed by the polynomial

$$F_{MR}(v) = \sum_{i=0}^{n} a_i v^i, \qquad n = 6 \tag{3}$$

where $v$ is the piston velocity and $a_i$, $i = 1, ..., 6$, are coefficients determined from a proper curve fitting. The order of the polynomials depend on the measured hysteresis behavior as well as the size and excitation of the MR damper. The coefficients $a_i$ are described in terms of their linear approximation with respect to the intensity of the electrical current $I$ as follows

$$a_i = b_i + c_i I, \quad i = 1, ..., 6 \tag{4}$$

Therefore, the damping force (3) can be specified as

$$F_{MR}(v, I) = \sum_{i=0}^{n} (b_i + c_i I)\, v^i, \qquad n = 6 \tag{5}$$

where the coefficients $b_i$ and $c_i$ are independent from the input current $I$. In our case we consider two identical MR dampers RD-1097-01 and the rheonetic wonder box device controller kit RD-3002-03, both manufactured by *Lord Corporation*®. In this case the polynomials with best fitting to the experimental results are of 2nd order ($n = 2$), whose coefficients are given in Table II.

**Table II. Polynomial coefficients $b_i$ and $c_i$.**

| Index | Positive acceleration $\dot{v} > 0$ | | Negative acceleration $\dot{v} < 0$ | |
|---|---|---|---|---|
| $i$ | $b_i$ | $c_i$ | $b_i$ | $c_i$ |
| 0 | 0.403 | 2.928 | 0.5426 | −3.105 |
| 1 | −18.3 | 1156 | −18.549 | 1161 |
| 2 | 19.01 | −561.3 | 8.6212 | −372.5 |

In order to capture the switching values for the coefficients $b_i$ and $c_i$, in terms of the acceleration $\dot{v}$, the damping force (5) is rewritten in the following form

$$F_{MR}(v, I) = \sum_{i=0}^{n} (b_i^{\wedge} + c_i^{\wedge} I) v^i, \qquad n = 2 \tag{6}$$

where the general coefficients $(b_i^{\wedge}, c_i^{\wedge})$ are expressed with respect to the positive acceleration $(b_i^{+}, c_i^{+})$ and negative acceleration $(b_i^{+}, c_i^{+})$ coefficients,

$$c_i^{\wedge} = \frac{(c_i^{+} + c_i^{-}) + \left| c_i^{+} - c_i^{-} \right| \operatorname{sign}(\dot{v})}{2}, \quad b_i^{\wedge} = \frac{(b_i^{+} + b_i^{-}) + \left| b_i^{+} - b_i^{-} \right| \operatorname{sign}(\dot{v})}{2}$$

Note that, by assuming small displacements, the two radial MR dampers ($F_{MRx}$,$F_{MRy}$) in the rotor bearing system (1)-(2) can be independently controlled from their current inputs $I_x$ and $I_y$.

### Rotor-bearing system with two MR dampers

The 2 degree-of-freedom overall rotor-bearing system dynamics (1)-(2), assuming two identical radial MR dampers $F_{MRx}$ and $F_{MRy}$ (6), is described by

$$m\ddot{x} + c_x\dot{x} + k_x x + \sum_{i=0}^{n} b_i^{\wedge}\dot{x}^i = me\omega^2 \cos\omega t - \left( \sum_{i=0}^{n} c_i^{\wedge}\dot{x}^i \right) I_x \tag{7}$$

$$m\ddot{y} + c_y\dot{y} + k_y y + \sum_{i=0}^{n} b_i^{\wedge}\dot{y}^i = me\omega^2 \sin\omega t - \left( \sum_{i=0}^{n} c_i^{\wedge}\dot{y}^i \right) I_y \tag{8}$$

where $I_x$ and $I_y$ are the current control inputs. It is evident that, from the switching characteristics in the coefficients $(b_i^{\wedge}, c_i^{\wedge})$, the control system (7)-(8) is highly nonlinear and nonsmooth, which complicates the synthesis of vibration controllers. It is possible, however, to prove the local controllability, about equilibrium positions, of system (7)-(8) from the two current inputs.

An analysis of the rotor-bearing system (7)-(8) reveals that the average equilibrium displacements, for constant currents, are given by

$$\left( \bar{x} = -\frac{b_0^{\wedge} + c_0^{\wedge} I_x}{k_x},\ \bar{y} = -\frac{b_0^{\wedge} + c_0^{\wedge} I_y}{k_y} \right) \tag{9}$$

**Simulation results.** Consider the rotor-bearing system (7)-(8) with parameters in Tables I and II. The first critical speed is computed as $\omega_n = 191.43\,\mathrm{rad/s} = 1828$rpm. The initial conditions are set to $x(0) = y(0) = -10^{-4}\,\mathrm{m}$ and $\dot{x}(0) = \dot{y}(0) = 0\,\mathrm{m/s}$. In Fig. 3a is shown the dynamic behavior of the rotor-bearing system for three different constant current inputs $I_x \in \{0, 5, 10\}\,\mathrm{mA}$ and constant angular speed $\omega = 1.5\omega_n = 287.15\,\mathrm{rad/s} = 2742.1$rpm. It is evident the attenuation of the system response when the current input $I_x$ is increased, property used to control the balancing response. An orbit $x(t)$ vs $y(t)$ when $I_x = 100\,\mathrm{mA}$ is described in Fig. 3b.

## SEMIACTIVE VIBRATION CONTROL

Consider again the rotor-bearing system (1)-(2). For control purposes it is assumed the inverse model for the MR damping forces in (6), such that the current control inputs are obtained from

$$I_i = \frac{F_{MRd} - \sum_{i=0}^{n} b_i^{\wedge} v^i}{\sum_{i=0}^{n} c_i^{\wedge} v^i}, \qquad i = x, y, \quad n = 2 \tag{10}$$

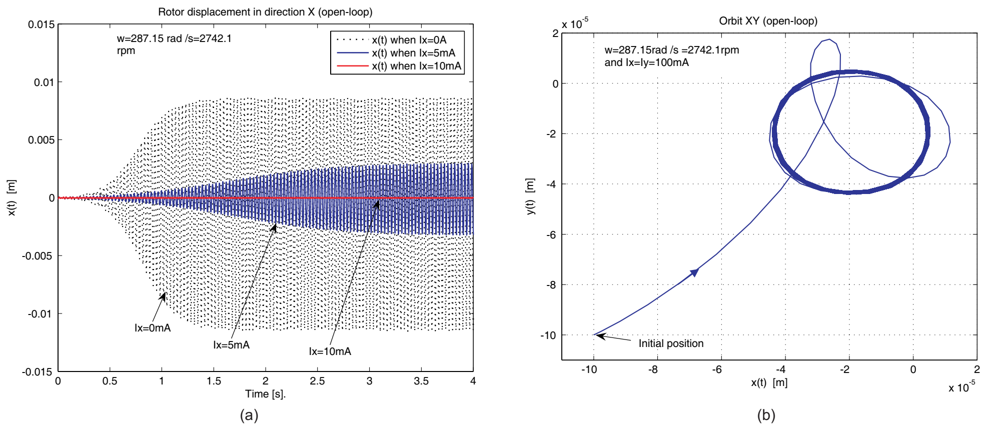


*Figure 3: Open-loop dynamics of the rotor-bearing system with MR dampers: (a) displacement $x(t)$ in direction X for $I_x \in \{0, 5, 10\}$ mA, (b) Orbit $y(t)$ vs $x(t)$ when $I_x = 100\,\mathrm{mA}$.*

It is important to remark that, in general, there is no singularity in (10) because $c_0^\wedge \neq 0$.

Now, the semiactive vibration control scheme is formulated as follows. First, it is designed a stabilizing vibration control law through the two independent MR damping forces, $F_{MRx}$ and $F_{MRy}$, for each degree-of-freedom in the rotor-bearing system (1)-(2). Second, the control forces are considered as the desired damping forces $F_{MRd}$ for the determination of the actual current control inputs from (10). This strategy is repeated until the unbalance response converges into a prespecified region.

Two sliding-mode controllers are then applied to achieve the vibration attenuation of the unbalance response (see Utkin [7]). This is a robust control strategy against exogenous perturbations and parameter uncertainties. Thus, the two sliding surfaces are defined by

$$\sigma_x(x - x_d) = \alpha_x(x - x_d) + \dot{x} \tag{11}$$
$$\sigma_y(y - y_d) = \alpha_y(y - y_d) + \dot{y} \tag{12}$$

where $x_d$ and $y_d$ are constant references (typically $x_d = y_d \equiv 0$ to center the orbit at the origin, but these can be used to compensate the average equilibrium (9)), respectively. Here, $\alpha_x$ and $\alpha_y$ are design control parameters to be selected to get the desired dynamic behavior. The sliding-mode controllers are synthesized as follows

$$F_{MRx} = (\alpha_x m - c_x)\dot{x} - k_x x - mM_x \operatorname{sign} \sigma_x(x - x_d) \tag{13}$$
$$F_{MRy} = (\alpha_y m - c_y)\dot{y} - k_y x - mM_y \operatorname{sign} \sigma_y(y - y_d) \tag{14}$$

where $M_x$ and $M_y$ are design parameters for the discontinuous control actions. These damping forces are considered as desired forces in (10). Due to physical limitations on the MR dampers, the resulting control currents are saturated to values between 0 A and 0.5 A and switching frequencies below 1 kHz.

# SIMULATION RESULTS

Consider the system parameters in Tables I and II. The design parameters for the sliding-mode controllers are selected as $\alpha_x = \alpha_y = 100$, $M_x = M_y = 10$ and $x_d = y_d = 0\,\mathrm{m}$. The initial conditions are set to $x(0) = y(0) = -10^{-5}\,\mathrm{m}$ and $\dot{x}(0) = \dot{y}(0) = 0\,\mathrm{m/\,s}$.

In Figs. 4 and 5 are shown the overall dynamic performance of the closed-loop system for a constant speed $\omega_n = 191.43\,\mathrm{rad/\,s} = 1828\mathrm{rpm}$. The unbalance response is stabilized and highly attenuated with small damping forces. It can be proved the robust behavior against variations on the operating speed $\omega$ and the unbalance parameters, even during run-up and coast-down through the first critical speed.

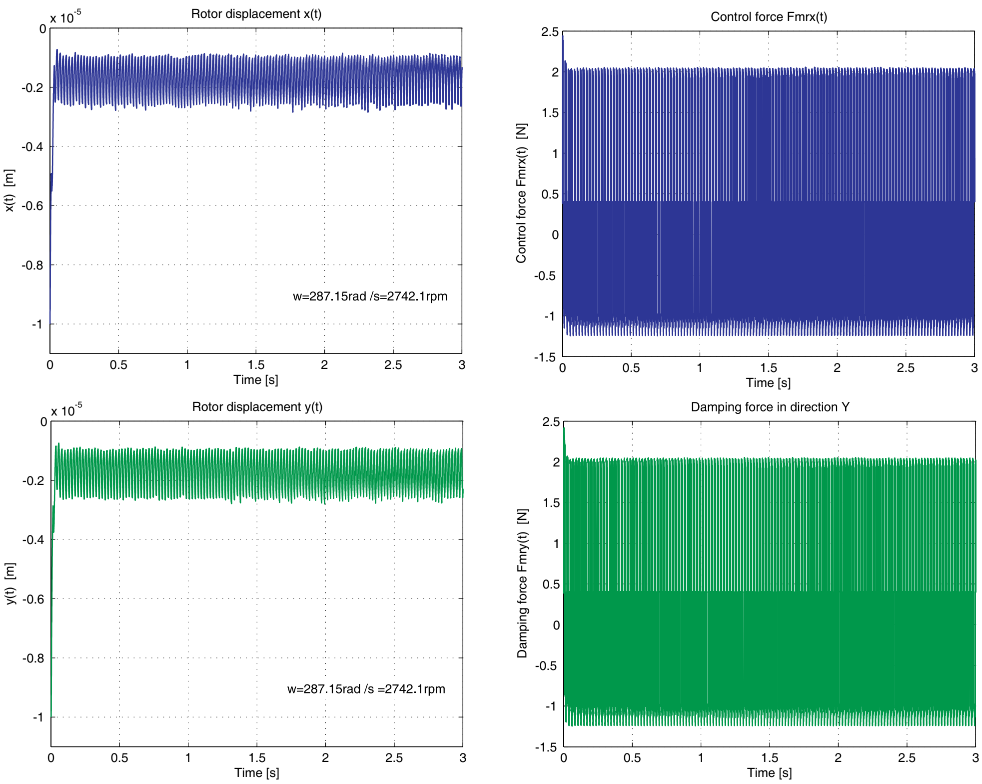


*Figure 4: Closed-loop system response using sliding-mode controllers with saturated control currents.*

# CONCLUSIONS

The semiactive balancing control of a Jeffcott-like rotor-bearing system with MR dampers is addressed. The proposed control scheme combines two radial MR dampers and springs to support one journal bearing of the rotor system. The rheological properties are controlled through current control inputs (electromagnetic field), using sliding-mode controllers, to get the desired stability and frequency response on the rotor-bearing system. Further work is being conducted to obtain the experimental validation on a realistic rotor-bearing system with MR dampers.

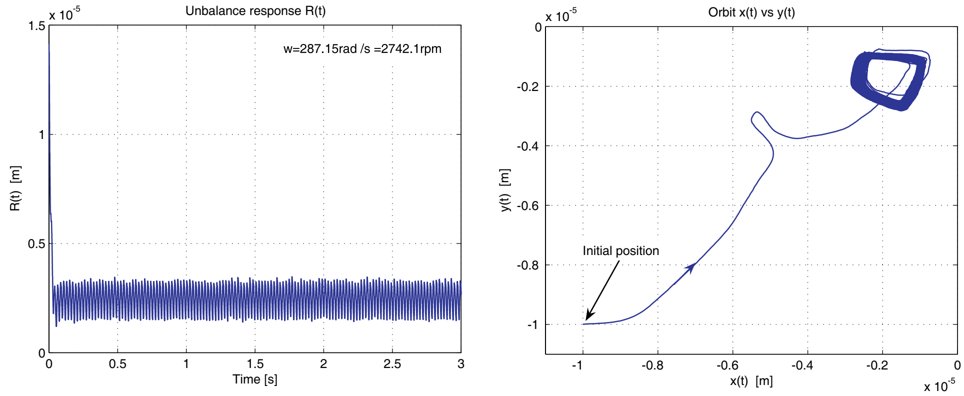


*Figure 5: Closed-loop unbalance response* $R = \sqrt{x^2 + y^2}$ *and orbit* $x(t)$ *vs* $y(t)$*.*

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