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MODAL IDENTIFICATION AND VIBRATION CONTROL USING MASS-SPRING AND CANTILEVER-BEAM ABSORBERS

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Abstract
The paper deals with the identification, analysis and synthesis of a passive and active vibration absorber connected to a mechanical system to attenuate harmonic vibrations with multiple frequency components. The primary system is coupled to a mass-spring vibration absorber and also to a cantilever beam absorber. The overall mechanical system is modeled through modal identification techniques, whose parameters are necessary for the passive designs. Because the overall system is made active with the addition of an external control force, then the open loop dynamics is properly modified with a feedback controller, synthesized via modal control techniques, to improve its robustness properties with respect to frequency variations, inclusive for simultaneous resonant frequencies. Some numerical and experimental results are provided to illustrate the dynamic performance of the passive and active vibration control system on a mechanical platform.

INTRODUCTION

Vibration absorbers are a valuable tool used to suppress or attenuate vibrations due to excitation in mechanical systems. There are basically three vibration control methods described as passive, semi-active and active vibration control. Passive vibration control relies on the addition of stiffness and damping to the system to reduce the steady state response, and it is useful for specific excitation frequency and stable operating conditions, however, it is not robust for variable excitation frequency and uncertainty on the system parameters. Semi-active vibration control deals with adaptive spring or damper characteristics, which are tuned according to the current operating conditions. Active vibration control achieves the better performance, by adding degrees of freedom (DOF) to the system and controlling with actuator forces according to feedback and feedforward information of the real system obtained from sensors. See [3], [2], [7].

This paper deals with tow passive and active vibration absorbers connected to a mechanical system to attenuate harmonic vibrations with multiple frequency components. The primary system is coupled to a mass-spring vibration absorber and also to a flexible cantilever beam absorber. The overall mechanical system is modelled and validated using modal identification techniques, for an equivalent three DOF, whose parameters are necessary for the passive designs. Because the overall system is made active with the addition of an external control force, then the open loop dynamics is properly modified with a state feedback modal control to improve its robustness properties with respect to frequency variations or simultaneous resonant frequencies.

SYSTEM DESCRIPTION

A model of a single DOF mechanical system coupled to a mass-spring passive vibration absorber and a flexible cantilever beam vibration absorber is shown in Fig. 1a. The experimental setup is a rectilinear plant (Model 210a) provided by Educational Control Products© with additional parts (see Fig. 1b). This mechanical system consists of two mass carriages (m_1,m_2) interconnected by bidirectional linear springs (denoted by k_1,k_2). Each mass carriage suspension has anti-friction ball bearing systems and, therefore, the linear dashpots (c_1,c_2) are included only to describe the small (linear) viscous dampings. The control force is represented by $u(t)$, which can directly push the mass m_2 to satisfy some desired control objectives (e.g., trajectory tracking and disturbance attenuation). This control force is obtained from a brushless-type servo motor connected to a pinion-rack mechanism. The underactuated mass carriage m_1 is directly affected by an exogenous force $F(t) = F_0 \sin(\omega t)$, generated from a small shaker connected to the carriage. In both mass carriages there exist high resolution optical encoders to measure their actual positions via cable-pulley systems. The velocities and accelerations are numerically approximated. The signal and control processing are obtained through a high-speed DSP board into a PC running under *Windows XP*© and *Matlab/Simulink*©.

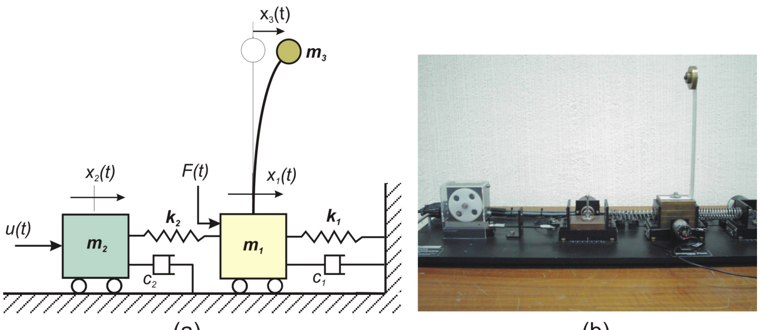

Figure 1: Mechanical system with mass-spring absorber and cantilever beam absorber. (a) Schematic diagram, and (b) Experimental platform ECP^{TM}.

The *primary* and the two *secondary* subsystems are composed by the linear elements (m_2, c_2, k_2), (m_1, c_1, k_1) and (m_3, k_3), respectively. The undesirable vibrations on the mass carriage m_2 are produced by the action of the harmonic force $F(t) = F_0 \sin \omega t$. The cantilever beam absorber is rigidly mounted on the primary system, and is used in combination with the mass-spring passive absorber to simultaneously attenuate several frequency components on the excitation force as well as improve the overall system dynamics. Moreover, the inclusion of the control force $u(t)$ leads to an active vibration control scheme, which will be properly designed through modal identification and control techniques.

MODELLING OF THE MECHANICAL SYSTEM

A flexible cantilever beam is a distributed absorber modelled by partial differential equations. This kind of beam absorber has been analyzed as a discretized one DOF nonlinear vibration absorber to compensate harmonic forces acting on a single DOF mass-spring primary system ([6], [1]). The first mode of the cantilever beam can be easily modelled as an equivalent mass-spring system and, more realistically, with the application of modal analysis techniques. Here it is considered a single DOF mass-spring-dashpot primary system (m_1, c_1, k_1) connected to a mass-spring-dashpot passive absorber (m_2, c_2, k_2). Over the primary system is mounted a flexible cantilever beam absorber, with equivalent tip mass m_3 and equivalent stiffness $k_3 \approx 3EI/l^3$, to describe approximately the first linear mode or horizontal motion of the mass m_3. The cantilever beam absorber can contribute with additional nonlinear coupling terms and more interesting phenomena.

The equations of motion of the discretized three DOF mechanical system, using Euler-Lagrange equations, are obtained as follows

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_3(\dot{x}_1 - \dot{x}_3) + k_1x_1 + k_2(x_1 - x_2) + k_3(x_1 - x_3) = F(t) \quad (1)$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 - k_2(x_1 - x_2) = u(t) \quad (2)$$

$$m_3\ddot{x}_3 - c_3(\dot{x}_1 - \dot{x}_3) - k_3(x_1 - x_3) = 0 \quad (3)$$

where x_1, x_2 and x_3 are horizontal displacements of the masses m_1, m_2 and m_3, respectively. The exogenous harmonic force is described by $F(t) = F_0 \sin \omega t$, with amplitude F_0 and excitation frequency ω. There is an external force control $u(t)$ acting on the second subsystem (2), which will serve as an active control force to improve the robustness of the system dynamics against parameter and frequency variations. The overall system (1)-(3) can be clearly distinguished as a multi-mass vibration absorber, able to attenuate external forces F with multiple excitation frequencies. It can be shown that system (1)-(3) is completely controllable from u and, therefore, stabilizable with linear control laws, as well as completely observable from the displacement x_1. Moreover, in presence of viscous dampings the open loop dynamics is asymptotically stable.

The system equations (1)-(3) can be compacted in the standard form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) + \mathbf{u}(t) \quad (4)$$

where $\mathbf{x} \in R^3$ denotes the generalized coordinate vector, and $\mathbf{f} \in R^3$ and $\mathbf{u} \in R^3$ are the perturbation and control force vectors, in such a way that

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \ \mathbf{f} = \begin{bmatrix} F(t) \\ 0 \\ 0 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 0 \\ u(t) \\ 0 \end{bmatrix} t, \ \mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_3 & 0 & -c_3 \\ 0 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_1 + k_2 + k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively.

PARAMETER IDENTIFICATION AND VALIDATION

In order to estimate the open-loop system parameters and validate the mathematical model, a linear sine sweep was performed on the experimental platform by applying a sine sweep with force $F(t) = 4 \sin \omega t \,\mathrm{N}$ and linear frequency sweep from $0.25\,\mathrm{Hz}$ to $5\,\mathrm{Hz}$ during $30\,\mathrm{s}$ and sampling time of $1\,\mathrm{ms}$. The displacements x_1 and x_2 are processed to compute the corresponding FFT and the frequency response functions (FRF) for their transfer functions $G_1(s) = X_1(s)/F(s)$ and $G_2(s) = X_2(s)/F(s)$, respectively. In Fig. 2 are shown both experimental FRFs.The modal parameters are obtained using SDOF

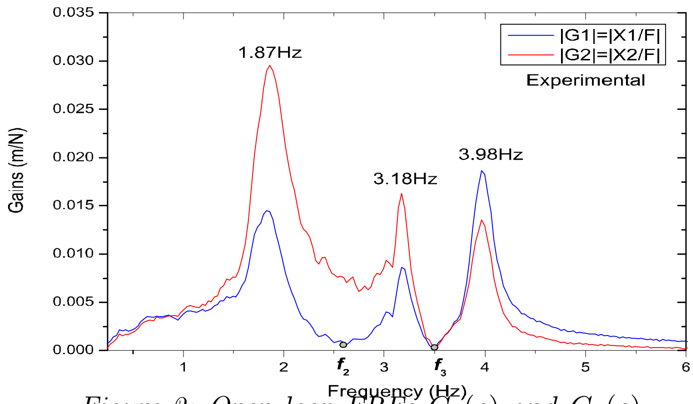

Figure 2: Open-loop FRFs $G_1(s)$ and $G_2(s)$.

(Peak Picking and Curve Fitting) methods for the experimental FRFs in Fig. 2. The resonant frequencies, damping ratios and mode shapes for the first three modes associated to the three DOF model (4) are shown in Table I.

Table I. Modal parameters from the experimental FRFs.			
Mode i	Resonant frequency ω_i [Hz]	Damping ratio ξ_i	Mode shape ψ_i
1	1.87	0.076	$\begin{bmatrix} 0.3929 & 0.7597 & 0.5182 \end{bmatrix}^T$
2	3.18	0.019	$\begin{bmatrix} -0.1935 & 0.3346 & -0.9223 \end{bmatrix}^T$
3	3. 98	0.022	$\begin{bmatrix} -0.1986 & 0.1412 & 0.9699 \end{bmatrix}^T$

The system parameters for the analytical model (4) are also obtained from other experiments, using time and sine sweep responses for the two excitation forces (F and u). In particular, it is assumed proportional damping and, therefore, the viscous damping coefficients c_i are approximated to satisfy this condition. The model validation is performed by comparing the numerical and experimental FRFs (see Fig. 3).The physical system parameters, validated to sufficiently match both numerical and experimental FRFs, are given in Table II.

Table II. Physical system parameters.		
$m_1 = 3.2\,\mathrm{kg}$	$m_2 = 1.35\,\mathrm{kg}$	$m_3 = 0.209\,\mathrm{kg}$
$k_1 = 740\,\mathrm{N/m}$	$k_2 = 340\,\mathrm{N/m}$	$k_3 \approx 105.07\,\mathrm{N/m}$
$c_1 = 2.3\,\mathrm{N/m/s}$	$c_2 = 0.97\,\mathrm{N/m/s}$	$c_3 \approx 0\,\mathrm{N/m/s}$
$l = 0.260\,\mathrm{m}$	$E = 71\,\mathrm{GPa}$ (Aluminum)	$I_{beam} = 8.6699 \times 10^{-12}\,\mathrm{m}^4$

The parameters l, E and I_{beam} denote the beam total length, Young's modulus (Aluminum alloy) and area moment of inertia (width $0.0254\,\mathrm{m}$ and thickness $0.0016\,\mathrm{m}$), res-

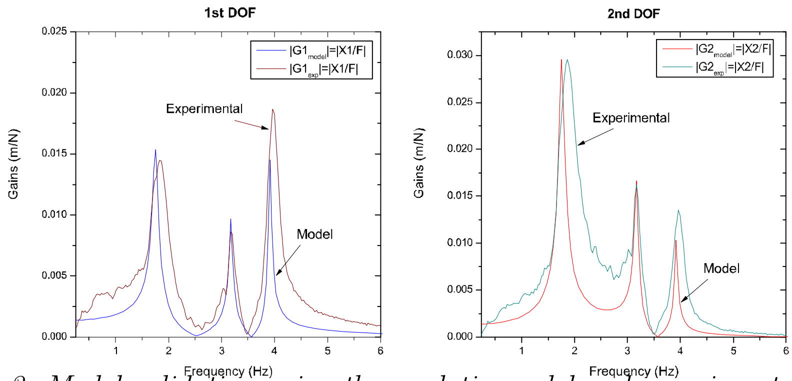

Figure 3: Model validation using the analytic model and experimental results.

pectively. The mass m_3 includes the tip mass and the equivalent beam total mass, such that beam can be considered as a massless cantilever beam.

VIBRATION CONTROL

The purpose of active vibration control is to eliminate some disadvantages of the passive vibration absorbers, adding DOFs or actuators that allow the application of a feedback/feedforward controller. Thus, one can find numerous active vibration control schemes like PID control, pole assignment, LQR control, H_∞ control, sliding-mode control and adaptive control. An *efficient* procedure to solve a vibration control problem consists of the simultaneous application of passive and active vibration absorbers, also called hybrid vibration absorbers, by means of which are combined efficiently the design of passive absorbers and a well-designed or optimal active controller, employing small control efforts, to provide robustness against variations on the excitation frequency and/or parametric uncertainty. See [2], [3], [5].

Modal vibration control techniques deserve much attention in real applications, which is somewhat equivalent to the pole assignment, although this relies more on cases where an estimated modal model is available from direct measurements and the control objectives consider explicitly the manipulation of the system properties through its critical modes. Thus, one can move the resonant frequencies or inject damping, in such a way to get a proper FRF.

A state space description of the three DOF system (4) is represented by

$$\dot{\mathbf{z}}(t) = \mathbf{A}\dot{\mathbf{z}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{f}(t), \quad \mathbf{z} \in R^6,\ \mathbf{u} \in R^3,\ \mathbf{f} \in R^3 \tag{5}$$

$$y(t) = \mathbf{H}\mathbf{z}(t), \quad y \in R \tag{6}$$

where the state vector, the active control force and harmonic perturbation are defined by $\mathbf{z} = [\mathbf{x}, \dot{\mathbf{x}}]^T$, $\mathbf{u} = [0, u, 0]^T$ and $\mathbf{f} = [F, 0, 0]^T$, respectively. The system parameters are compacted into matrices of proper dimensions

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with $\mathbf{0}$ and $\mathbf{I}$ denoting the zero and identity matrices. As mentioned before, the underactuated system (5)-(6) is completely controllable from the only control input u and completely observable from the output $y = z_1$ (displacement of the primary system).

Our vibration control objectives can be formulated in two steps as follows:

1. Design two passive vibration absorbers, one consisting of a mass-spring-viscous damper absorber and other a flexible cantilever beam absorber, both connected to the primary system.
2. Design a robust modal control (active vibration absorbers) to quickly stabilize the primary system response in presence of harmonic forces, possibly with multiple excitation frequencies.

Passive vibration absorbers

By assuming zero dampings, one can compute the excitation frequencies compensated by the design of the two passive vibration absorbers. That is, the two absorbers can attenuate harmonic forces $F(t)$, containing up to two different excitation frequencies into a narrow band close to the external resonance conditions: 1) $\omega_2 = \sqrt{k_2/m_2} \approx 15.207\,\mathrm{rad/s} = 2.42\,\mathrm{Hz}$, and 2) $\omega_3 = \sqrt{k_3/m_3} \approx 22.43\,\mathrm{rad/s} = 3.57\,\mathrm{Hz}$. Both tuning conditions are shown in Fig. 2 as points f_2 and f_3. It is evident, however, the lack of robustness of the passive scheme for excitation frequencies over a broader frequency range and even worse near the resonant frequencies 1.87 Hz, 3.18 Hz and 3.98 Hz.

Experimental results. The two passive vibration absorbers for the primary system are implemented on an experimental platform ECP$^{\mathrm{TM}}$ (see Fig. 1b). In order to show the dynamic performance of the two passive absorbers on the primary system, it is applied a harmonic force $F(t)$, with the following excitation frequency profile:

$$F(t) = \begin{cases} 4\sin(2\pi f_1 t)\ \mathrm{N} & \text{for } f_1 = 2.42\,\mathrm{Hz}\ \&\ t \in [0, 12.5)\,\mathrm{s} \\ 4\sin(2\pi f_2 t)\ \mathrm{N} & \text{for } f_2 = 3.57\,\mathrm{Hz}\ \&\ t \in [12.5, 25) \\ 4\sin(2\pi f_1 t) + 4\sin(2\pi f_2 t)\ \mathrm{N} & \text{for } f_1 = 2.42\,\mathrm{Hz},\ f_2 = 3.57\,\mathrm{Hz}\ \&\ t \in [25, 37.5)\,\mathrm{s} \\ 4\sin(2\pi f_3 t)\ \mathrm{N} & \text{for } f_3 = 1.87\,\mathrm{Hz}\&\ t \in [37.5, 50]\,\mathrm{s} \end{cases} \tag{7}$$

The experimental results are presented in Fig. 4. Clearly the passive absorbers are good enough for the first three time intervals but not for the case when $f_3 = 1.87\,\mathrm{Hz}$ (resonance).

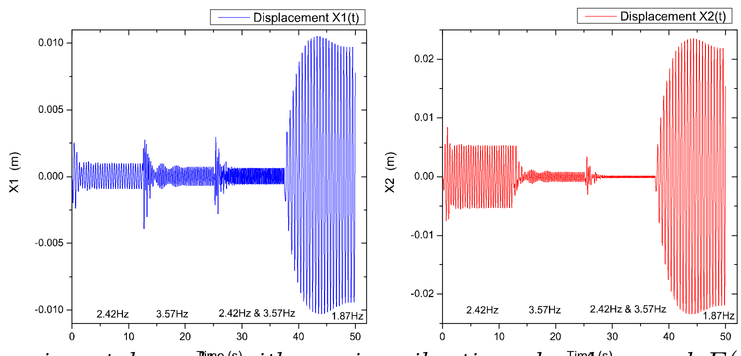

Figure 4: Experimental results with passive vibration absorbers and $F(t)$ given in (7).

Active vibration absorbers using modal control

Modal control methods are essentially the application of a systematic pole placement or eigenstructure assignment techniques. The modal controller is properly synthesized to reassign the system poles of the controlled system in desired values, thus modifying the eigenfrequencies, modal damping and modal shapes of the mechanical system ([3], [5]).

Consider the system (5)-(6) and assume that all the state variables are available for feedback and that all the system modes are controllable. Then, a static state feedback control law is given by

$$\mathbf{u} = -\mathbf{K}_f \mathbf{z} \tag{8}$$

where $\mathbf{K}_f \in R^{1\times 6}$ is the feedback gain matrix. The closed-loop system is then obtained as

$$\dot{\mathbf{z}} = (\mathbf{A} - \mathbf{B}\mathbf{K}_f)\,\mathbf{z} + \mathbf{E}\mathbf{f}, \quad \mathbf{z} \in R^6,\, \mathbf{f} \in R^3 \tag{9}$$

$$y = \mathbf{H}\mathbf{z}, \quad y \in R \tag{10}$$

where the closed-loop system poles of the matrix $(\mathbf{A} - \mathbf{B}\mathbf{K}_f)$ are placed on specific locations, thus reflecting the desired eigenfrequencies, modal dampings and modal shapes. It is well-known that the existence of the feedback gain matrix $\mathbf{K}_f$ is guaranteed by the controllability of the system, which is readily satisfied.

In modal coordinates, the 3-DOF system (4) can be expressed as a set of decoupled second order subsystems by using the state transformation $\mathbf{x} = \mathbf{\Psi}\mathbf{q}$, where $\mathbf{\Psi} = [\psi_1, \psi_2, \psi_3]$ is the modal matrix, containing the three modal shapes associated to each DOF and $\mathbf{q} = [q_1, q_2, q_3]^T$ is the vector of modal coordinates. Hence, system (4) can be transformed as

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{C}}\dot{\mathbf{q}} + \bar{\mathbf{K}}\mathbf{q} = \bar{\mathbf{f}}(t) + \bar{\mathbf{u}}(t) \tag{11}$$

where $\bar{\mathbf{M}} = \mathbf{\Psi}^T\mathbf{M}\mathbf{\Psi}$, $\bar{\mathbf{C}} = \mathbf{\Psi}^T\mathbf{C}\mathbf{\Psi}$ and $\bar{\mathbf{K}} = \mathbf{\Psi}^T\mathbf{K}\mathbf{\Psi}$ are (diagonal) modal mass, damping and stiffness matrices, respectively. Here $\bar{\mathbf{f}} = \mathbf{\Psi}^T\mathbf{f}$ and $\bar{\mathbf{u}} = \mathbf{\Psi}^T\mathbf{u}$ denote the modal force and control vectors. In normalized form, the uncoupled modal equations can be expressed by

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i = \alpha_i \bar{f}_i(t) + \beta_i \bar{u}_i(t), \quad i = 1, 2, 3 \tag{12}$$

where ζ_i and $\omega_i = \sqrt{\bar{k}_i/\bar{m}_i}$ describe the modal dampings and natural frequencies, respectively.

Now, it is synthesized a modal controller by static state feedback to move the open-loop system poles from $\{-0.3442 \pm i11.02,\ -0.2172 \pm 19.94i,\ -0.1573 + i24.61\}$ to the desired locations $\{-3.608 \pm i7.77i,\ -0.5650 \pm i22.76,\ -3.3132 \pm i29.91\}$. That means that, the open-loop resonant frequencies and damping ratios are removed from the set $\{(1.76\,\text{Hz}, 0.031),\ (3.17\,\text{Hz}, 0.011),\ (3.92\,\text{Hz}, 0.006)\}$ to $\{(1.39\,\text{Hz}, 0.432),\ (3.62\,\text{Hz}, 0.026),\ (4.84\,\text{Hz}, 0.103)\}$. Note that the first mode is slightly moved to the left and its damping is increased, while the second and third modes are moved to the right with a small damping injection. Furthermore, the static state feedback is restricted to consider only measurements on the first and second modes (i.e., without any feedback from the cantilever beam). Thus, the corresponding feedback gain matrix in (8) leads to

$$\mathbf{K}_f = \begin{bmatrix} -1736.4 & 605.7 & 0 & 32.246 & 18.332 & 0 \end{bmatrix}$$

Experimental results. The closed-loop FRF is obtained by the application of a linear sine sweep with $F(t) = 4\sin\omega t$ N and linear frequency sweep from 0.25 Hz to 5 Hz, during 30 s and sampling time of 1 ms. The closed-loop experimental FRFs, with modal control, corresponding to the transfer functions $G_{1c}(s) = X_1(s)/F(s)$ and $G_{2c}(s) = X_2(s)/F(s)$ are shown in Fig. 5.The dynamic performance and robustness of the two passive and active vibration absorbers, when the harmonic force is provided in (7), are described by means of the experimental results in Fig. 6. As expected the primary system response is robust enough and much better than that obtained using the passive vibration absorbers (see Fig. 4).

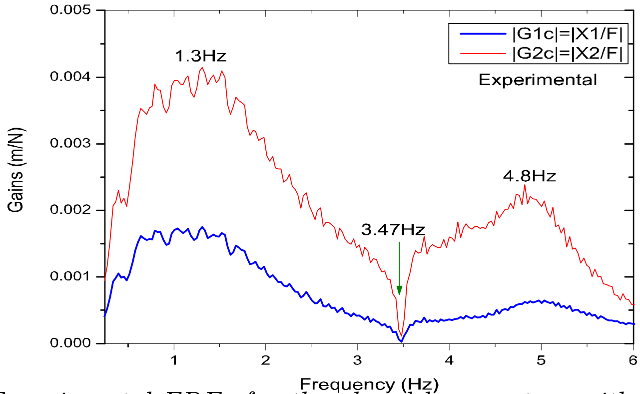

Figure 5: Experimental FRFs for the closed-loop system with modal control.

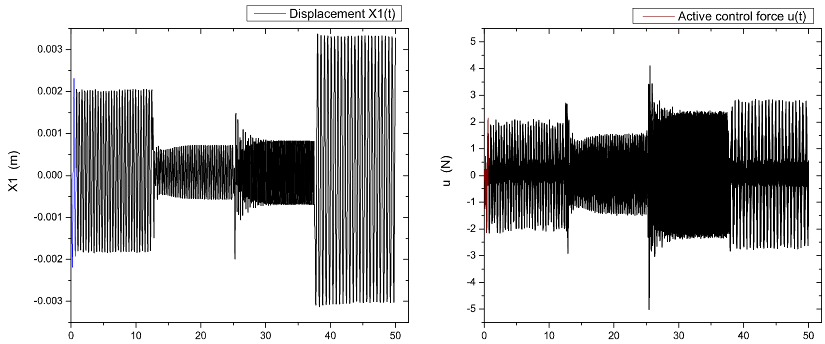

Figure 6: Experimental results of the closed-loop system using modal control.

CONCLUSIONS

Two passive vibration absorbers are used to robustly attenuate the system response of a primary system affected for a harmonic force with several excitation frequencies. The vibration control scheme is synthesized using modal analysis and modal control, leading to experimental results with good dynamic performance and robustness.

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