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THE PENDULUM VIBRATION ABSORBER FOR THE NONLINEAR PRIMARY DUFFING SYSTEM

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Abstract
In practical cases for radical operating conditions some systems exhibit slight nonlinear behavior. The autoparametric vibration absorber based on a pendulum has been widely studied and employed to quench the resonant excitation response of a primary linear spring and mass viscous damped system. This work presents an analysis of nonlinear primary systems modeled by the Duffing equation and coupled to a pendulum vibration absorber. The multiple scales method is used to determine that, for external and internal resonances, the excitation force and the cubic nonlinearity are absorbed by the pendulum. In both cases the steady state amplitude response of the primary system, with linear and nonlinear springs, is the same. It is observed that, in contrast to linear primary systems, a Duffing primary system leads to three real fixed points for the frequency response. In fact, the nonlinearity on the primary system is transferred to the pendulum dynamics, modifying the steady state response and bending the jumps. The fixed points, amplitude and phase response and the stability are given in terms of the nonlinearity. To ensure the performance of the absorber some constraint between the nonlinearity and the external force must be accomplished. The system performance is illustrated by means of numerical simulations.

INTRODUCTION

The study of a dynamic vibration absorbers has been gradually increasing. For instance, the results obtained by Cartmell [1], using the techniques described in Nayfeh and Mook

[4], prove how in practical cases the tuning of the excited harmonically primary linear system, with a pendulum, bring to the quenching of the first one. Other example is given in the work by Song *et. al.* [5], where optimal parameters of the system are computed to obtain vibration absorption and stability. Moreover, in Zhu *et. al.* [7] is considered a passive vibration control scheme, including nonlinear damping and nonlinear springs. The nonlinearities can appear also in the absorber dynamics as in the pendulum absorber (see also Mikhlin and Reshetnikova [3] and references therein).

This work considers a mechanical system consisting of a primary system, including non-linear elastic terms and dynamically coupled to a pendulum vibration absorber. A similar system is analyzed by Hsieh and Shaw [2] as a chaotic system. The multiple scales method is employed to determine the frequency response of the overall nonlinear system, using similar approximations as those in Woafo [6]. The resulting solution enables us to conclude that, the non-linear cubic term improves the absorption performance of the pendulum for greater excitation forces. The stability is also affected by the nonlinear term.

SYSTEM EQUATIONS

The system under study consists of an oscillating mass m_1 attached to a pendulum vibration absorber as shown in Fig. 1. All the devices in the primary system are linear, except for the elastic stiffness.The equations of motion describing the dynamics of the

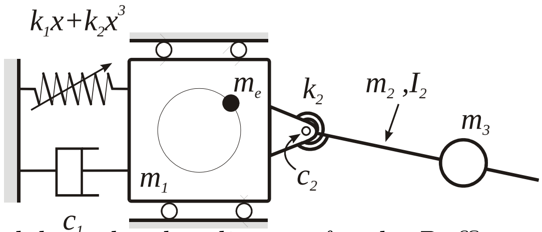

Figure 1: Pendulum absorber diagram for the Duffing primary system

horizontal displacement of m_1 and the angular displacement of m_3 are given by

$$(m_1+m_2+m_3)\,\ddot{x}+c_1\dot{x}+k_1x+k_2x^3-\left(\frac{1}{2}m_2l_b+m_3l\right)\left(\ddot{\theta}\theta+\dot{\theta}^2\right) = F_0\cos\Omega t \quad (1)$$

$$\left(\frac{1}{4}m_2l_b^2+I_2+m_3l^2\right)\ddot{\theta}+c_2\dot{\theta}+k_3\theta-\left(\frac{1}{2}m_2l_b+m_3l\right)\ddot{x}\theta = 0 \quad (2)$$

The following system parameters are defined as

$h=\frac{1}{m_1+m_2+m_3}$	$g=\frac{1}{\frac{1}{4}m_2l_b^2+I_2+m_3l^2}$	$\varepsilon=\frac{1}{2}m_2l_b+m_3l$
$\omega_1^2=\frac{k_1}{m_1+m_2+m_3}$	$\omega_2^2=\frac{k_3}{\frac{1}{4}m_2l_b^2+I_2+m_3l^2}$	$k_2'=\frac{k_2}{m_1+m_2+m_3}$
$\xi_1=\frac{c_1}{2\omega_1(m_1+m_2+m_3)}$	$\xi_2=\frac{c_2}{2\omega_2\left(\frac{1}{4}m_2l_b^2+I_2+m_3l^2\right)}$	

where ε represents a small perturbation parameter associated to the coupling between the pendulum and the primary system, among other kinds of perturbations like damping, nonlinearities and the exogenous excitation force. The equations of motion (1)-(2)

are then normalized as follows

$$\begin{aligned} \ddot{x}+2\xi_1\omega_1\dot{x}+\omega_1^2x+k_2'x^3-h\varepsilon\left(\ddot{\theta}\theta+\dot{\theta}^2\right) &= hF_0\cos\Omega t & (3)\\ \ddot{\theta}+2\xi_2\omega_2\dot{\theta}+\omega_2^2\theta-g\varepsilon\ddot{x}\theta &= 0 & (4) \end{aligned}$$

In order to specify the influence of the damping over the overall system, we define $\xi_1=\varepsilon\zeta_1$ and $\xi_2=\varepsilon\zeta_2$. The cubic term, associated to the Duffing equation, is described by $k_2'=\varepsilon\alpha$, representing the effects of a hard spring. In addition, the exogenous excitation is related with the amplitude perturbation by $F=hF_0$ and $f=\varepsilon F$. Then, the normalized system equations are expressed as

$$\begin{aligned} \ddot{x}+2\varepsilon\zeta_1\omega_1\dot{x}+\omega_1^2x+\varepsilon\alpha x^3-h\varepsilon\left(\ddot{\theta}\theta+\dot{\theta}^2\right) &= \varepsilon f\cos\Omega t & (5)\\ \ddot{\theta}+2\varepsilon\zeta_2\omega_2\dot{\theta}+\omega_2^2\theta-g\varepsilon\ddot{x}\theta &= 0 & (6) \end{aligned}$$

Frequency analysis

The multiple scales method is used to find an approximated solution for the perturbed system equations. Then, the perturbed solutions to the system equations (5)-(6) are proposed to be (see also Woafo *et. al.* [6])

$$\begin{aligned} x\left(T_0,T_1\right) &= x_0\left(T_0,T_1\right)+\varepsilon x_1\left(T_0,T_1\right)+\ldots & (7)\\ \theta\left(T_0,T_1\right) &= \theta_0\left(T_0,T_1\right)+\varepsilon\theta_1\left(T_0,T_1\right)+\ldots & (8) \end{aligned}$$

where T_0 is the fast time scale, T_1 is the slow time scale and both time scales are related by the perturbation as $T_n=\varepsilon^nT_0$, with $n=0,1,2\ldots$ and $T_0=t$. Time scales derivatives leads to the operators $\frac{d}{dt}=D_0+\varepsilon D_1+\ldots$and $\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0D_1+\ldots$.

Moreover, the external and internal resonance conditions are defined by

$$\begin{aligned} \Omega &= \omega_1+\varepsilon\rho_1 & (9)\\ \omega_1 &= 2\omega_2+2\varepsilon\rho_2 & (10) \end{aligned}$$

where ρ_1 and ρ_2 denote two detuning parameters.

Applying the differential operators to the solutions (7)-(8), and constructing each of the perturbed system equations (5)-(6), yields a set of differential equations in terms of powers of ε. The resulting equations for the first two powers of ε for the perturbed equations are expressed by

$$\begin{aligned} \varepsilon^0 &: \quad D_0^2x_0+\omega_1^2x_0=0 & (11)\\ \varepsilon^1 &: \quad D_0^2x_1+\omega_1^2x_1=-2\zeta_1\omega_1D_0x_0-2D_0D_1x_0-\alpha x_0^3+h\left(D_0\theta_0\right)^2 & (12)\\ &\qquad +h\left(D_0^2\theta_0\right)\theta_0+f\cos\left(\Omega T_0\right) & \\ \varepsilon^0 &: \quad D_0^2\theta_0+\omega_2^2\theta_0=0 & (13)\\ \varepsilon^1 &: \quad D_0^2\theta_1+\omega_2^2\theta_1=g\left(D_0^2x_0\right)\theta_0-2D_1D_0\theta_0-2\zeta_2\omega_2D_0\theta_0 & (14) \end{aligned}$$

The proposed solutions for (11) and (13) are of the form

$$\begin{aligned} x_0 &= A\left(T_1\right)e^{i\omega_1T_0}+\bar{A}\left(T_1\right)e^{-i\omega_1T_0} & (15)\\ \theta_0 &= B\left(T_1\right)e^{i\omega_2T_0}+\bar{B}\left(T_1\right)e^{-i\omega_2T_0} & (16) \end{aligned}$$

where the amplitudes depend on the fast time scale T_1 and the oscillations on the time scale T_0. Here $\bar{A}(T_1)$ and $\bar{B}(T_1)$ denote complex conjugates of the amplitudes $A(T_1)$ and $B(T_1)$, respectively. Substituting (15) and (16) in equations (12) and (14) result

$$\begin{aligned} D_0^2 x_1 + \omega_1^2 x_1 &= -2i\zeta_1\omega_1^2 A e^{i\omega_1 T_0} - 2i\omega_1 A' e^{i\omega_1 T_0} - \alpha A^3 e^{3i\omega_1 T_0} \\ &\quad - 3\alpha A^2 \bar{A} e^{i\omega_1 T_0} - 2h\omega_2 B^2 e^{2i\omega_2 T_0} + \frac{1}{2} f e^{i\Omega T_0} + C.C. \end{aligned} \tag{17}$$

$$\begin{aligned} D_0^2 \theta_1 + \omega_2^2 \theta_1 &= -g\omega_1^2 A B e^{(\omega_1+\omega_2)T_0} - g\omega_1^2 A \bar{B} e^{(\omega_1-\omega_2)T_0} \\ &\quad - 2i\omega_2 B' e^{i\omega_2 T_0} - 2i\zeta_2\omega_2 B e^{i\omega_2 T_0} + C.C. \end{aligned} \tag{18}$$

where $C.C.$ stands for complex conjugate terms. Canceling secular terms from equations (17)-(18) and using the polar forms for $A(T_1) = \frac{1}{2} a(T_1) e^{i\delta(T_1)}$ and $B(T_1) = \frac{1}{2} b(T_1) e^{i\gamma(T_1)}$ yields

$$-i\zeta_1\omega_1^2 a - i\omega_1 a' + \omega_1 a\delta' - \frac{3}{8}\alpha a^3 - \frac{1}{2} h\omega_2^2 b^2 e^{i\phi_1} + \frac{1}{2} f e^{i\phi_2} = 0 \tag{19}$$

$$-\frac{1}{4} g\omega_1^2 a b e^{-i\phi_1} - i\omega_2 b' + b\omega_2\gamma' - ib\zeta_2\omega_2^2 = 0 \tag{20}$$

where the phases are defined by

$$\phi_1 = \rho_1 T_1 - \delta, \qquad \phi_2 = 2\gamma - \delta - 2\rho_2 T_1 \tag{21}$$

Steady state response

From system equations (19)-(20) are determined the steady state response amplitudes and their stability. Therefore, taking real and imaginary parts of (19) and (20), respectively, and reflecting the steady state on the phases (21) like $\delta' = \rho_1$ and $\gamma' = \frac{\rho_1}{2} + \rho_2$, with $a' = 0$, $b' = 0$ for the fixed points, results that

$$\omega_1 a\rho_1 - \frac{3}{8}\alpha a^3 - \frac{1}{2} h\omega_2^2 b^2 \cos\phi_1 + \frac{1}{2} f\cos\phi_2 = 0 \tag{22}$$

$$-\zeta_1\omega_1^2 a - \frac{1}{2} h\omega_2^2 b^2 \sin\phi_1 + \frac{1}{2} f\sin\phi_2 = 0 \tag{23}$$

$$-\frac{1}{4} g\omega_1^2 a b\cos\phi_1 + b\omega_2\left(\frac{\rho_1}{2} + \rho_2\right) = 0 \tag{24}$$

$$\frac{1}{4} g\omega_1^2 a b\sin\phi_1 - b\zeta_2\omega_2^2 = 0 \tag{25}$$

Now, solving the above equations are determined the approximated amplitude responses for the primary system and pendulum, that is,

$$a = \frac{4\omega_2^2}{\varepsilon g\omega_1^2}\sqrt{\left(\left(\frac{\Omega}{2\omega_2} - 1\right)^2 + \xi_2^2\right)} \tag{26}$$

$$b^4 + Qb^2 + R = 0 \tag{27}$$

where

$$\begin{aligned} Q &= \left[12\frac{\omega_2(\varepsilon\alpha)(\Omega - 2\omega_2)^2}{(\varepsilon h)\varepsilon^3 g^3\omega_1^6} + 48\frac{(\varepsilon\alpha)\omega_2^3\xi_2^2}{(\varepsilon h)\varepsilon^3 g^3\omega_1^6} - 8\frac{(\Omega - \omega_1)}{(\varepsilon h)(\varepsilon g)\omega_2\omega_1}\right](\Omega - 2\omega_2) \\ &\quad + \frac{16\xi_1\xi_2}{(\varepsilon h)(\varepsilon g)} \end{aligned} \tag{28}$$

$$
\begin{aligned}
R \;=\; & \frac{36\left(\varepsilon\alpha\right)^2\omega_2^2}{\left(\varepsilon^2 h^2\right)\left(g^3\varepsilon^3\right)\omega_1^6}\left[\left(\frac{\Omega}{2\omega_2}-1\right)^2+\xi_2^2\right]^{3/2} \\
& -\frac{768\left(\varepsilon\alpha\right)\omega_2^4\left(\Omega-\omega_1\right)}{\left(\varepsilon^2 h^2\right)\left(g^4\varepsilon^4\right)\omega_1^7}\left[\left(\frac{\Omega}{2\omega_2}-1\right)^2+\xi_2^2\right]^2 \\
& +\frac{64\left[\left(\Omega-\omega_1\right)^2+\omega_1^2\xi_1^2\right]}{\left(\varepsilon^2 h^2\right)\left(\varepsilon^2 g^2\right)\omega_1^2}\left[\left(\frac{\Omega}{2\omega_2}-1\right)^2+\xi_2^2\right]-\frac{F^2}{\left(\varepsilon h\right)^2\omega_2^4}
\end{aligned} \tag{29}
$$

Note that, the primary system response (26) does not depend on the cubic term. In fact, the amplitude is the same as that determined by Cartmell [1], when there are only linear elements. The pendulum response is affected by the cubic nonlinearity in (28)-(29), that is, the cubic term has been transferred from the primary system to the pendulum. Observe, in addition, that if the cubic term is neglected, $\alpha = 0$, the response coincides also with that in Cartmell [1]. Therefore, it is concluded the robustness of the pendulum vibration absorber with respect to some nonlinearities on the primary system, cancelling the effect of the cubic nonlinearity.

Fixed points. The real and imaginary parts are taken from equations (19)-(20) and the system is solved for a', b', ϕ_1' and ϕ_2', resulting the differential equations

$$a' = \frac{-\zeta_1\omega_1^2 a - \frac{1}{2}h\omega_2^2 b^2\sin\phi_1 + \frac{1}{2}f\sin\phi_2}{\omega_1} \tag{30}$$

$$b' = \frac{\frac{1}{4}g\omega_1^2 ab\sin\phi_1 - \zeta_2\omega_2^2 b}{\omega_2} \tag{31}$$

$$\phi_1' = \frac{\omega_1 a\rho_1 - \frac{3}{8}\alpha a^3 - \frac{1}{2}h\omega_2^2 b^2\cos\phi_1 + \frac{1}{2}f\cos\phi_2}{\omega_1 a} \tag{32}$$

$$\phi_2' = \frac{\frac{1}{2}g\omega_1^3 a^2\cos\phi_1 - \frac{3}{8}\omega_2\alpha a^3 - \frac{1}{2}h\omega_2^3 b^2\cos\phi_1 + \frac{1}{2}\omega_2 f\cos\phi_2 + \omega_1\omega_2 a2\rho_2}{\omega_1 a\omega_2} \tag{33}$$

The fixed points are computed for the external and internal resonance conditions, that is, when $\rho_1 = \rho_2 = 0$ and $a' = 0$, $b' = 0$, $\phi_1' = 0$ and $\phi_2' = 0$.

The system has one fixed point for the uncoupled or linear response, where the pendulum is not affected by the primary system ($b = 0$), and two fixed points for the coupled response or nonlinear interaction ($a \neq 0$, $b \neq 0$), where the absorption is being performed. Clearly, one is interested on the absorption conditions and the corresponding fixed points given in the following Table I:

Table I. Fixed points	
Fixed point 2	Fixed point 3
$a = \frac{4\zeta_2\omega_2^2}{g\omega_1^2}$	$a = -\frac{4\zeta_2\omega_2^2}{g\omega_1^2}$
$b = \frac{u}{g\omega_1^3\omega_2}$	$b = \frac{u}{g\omega_1^3\omega_2}$
$\phi_1 = \frac{1}{2}\pi$	$\phi_1 = -\frac{1}{2}\pi$
$\phi_2 = \arctan\left(\frac{s+q}{r}\right)$	$\phi_2 = \arctan\left(\frac{-(s+q)}{-r}\right)$

where $r = \frac{48\alpha\zeta_2^3\omega_2^6}{g^3\omega_1^6 f}$, $s = 8\zeta_1\zeta_2\omega_1^6 g\omega_2^2$ and $q = \frac{hu^2}{g^3\omega_1^6 f}$, u are the roots of the polynomial

$$2304\zeta_2^6\omega_2^{12}\alpha^2 + 64\zeta_2^2\zeta_1^2 g^4\omega_1^{12}\omega_2^4 - f^2 g^6\omega_1^{12} - 16\zeta_2\zeta_1 h\omega_1^6\omega_2^2 g^3 u^2 + h^2 g^2 u^4 = 0 \tag{34}$$

This polynomial has four roots for u,

$$u_{1,2,3,4} = \pm \frac{\sqrt{-gh\left(v \pm \sqrt{-w\alpha^2 + z}\right)}}{gh} \tag{35}$$

where $v = 8\zeta_1\zeta_2\omega_2^2\omega_1^6 g^2$, $w = 2304\zeta_2^6\omega_2^{12}$ and $z = f^2 g^6 \omega_1^{12}$. Note that all the involved parameters are positive. The roots for u must be real, implying one main constraint on the system parameters in order to get real values for b and ϕ_2, characterized by

$$\alpha \leq \sqrt{\frac{16}{9}\frac{f^2 g^6}{\zeta_2^6} - \frac{64}{9}\frac{\zeta_1^2 g^4}{\zeta_2^4}\omega_1^4} \tag{36}$$

This constraint limits the parameter α (cubic nonlinearity) with respect to the amplitude force f in order to satisfy the absorption conditions.
Stability analysis. The stability of the system response is determined using again the system equations (30)-(33). The stability of the solutions is evaluated using the Hurwitz criteria for the fixed points associated to the system parameters (see Table II) and actual operating conditions.

Table II. System parameters

$\omega_1 = 8.2368$	$\xi_1 = 0.0086$	$f = 0.2357$	$h = 0.1428$	$\alpha = 5$
$\omega_2 = 4.1183$	$\xi_2 = 0.0051$	$g = 4.2402$	$\varepsilon = 0.55$	

Linearizing (30)-(33) for the system parameters given in Table II and the fixed point 2 in Table I leads to the equilibrium $a = 0.002186$, $b = 0.710107$, $\phi_1 = \frac{\pi}{2}$ and $\phi_2 = \frac{\pi}{2}$. The corresponding characteristic polynomial for this fixed point is *unstable*. It is still possible to obtain stable responses for some detuning parameters ρ_1 and ρ_2, that is, if we determine the fixed points as a function of ρ_1 and ρ_2 then, when this fixed point is re-evaluated, the stability can be guaranteed. For instance, for the tuning conditions $\rho_1 = 0.65$ and $\rho_2 = 0$ the fixed point 2 result the equilibrium solutions $a = 0.0187$, $b = 0.76539$, $\phi_1 = \{0.1169, -3.0246\}$ and $\phi_2 = \frac{\pi}{2}$. When $\phi_1 = 0.1169$ the fixed point is *unstable* but for $\phi_1 = -3.0246$ is *stable*.

SIMULATION RESULTS

Some numerical simulations are presented to qualitatively evaluate the overall system performance for some values of the parameter α. From (27) is plotted the frequency response amplitude of the pendulum b. In Fig. 2(a) is shown the response for the absorber in the case of $\alpha = 0$, which corresponds to the linear primary system (see [1]). As the nonlinear cubic term increases to $\alpha = 52$, as shown in Fig. 2(b), the absorber response modifies the jumps at both extremes, thus bending them and creating a saddle point at the primary resonance frequency.When $\alpha = 54$ the size of jumps are also reduced (see Fig. 3). As we mentioned, the frequency response of the primary forced system is described by the equation (26), which is identically to the linear case. The corresponding value of the force amplitude is $F_0 = 3\,\mathrm{N}$.

To correlate the frequency response in the time domain, some numerical simulations of the system (3)-(4) and parameters in Table II are depicted in Fig. 4. It is important to remark that, when $\alpha = 0$ leads to large amplitudes and unstable behavior

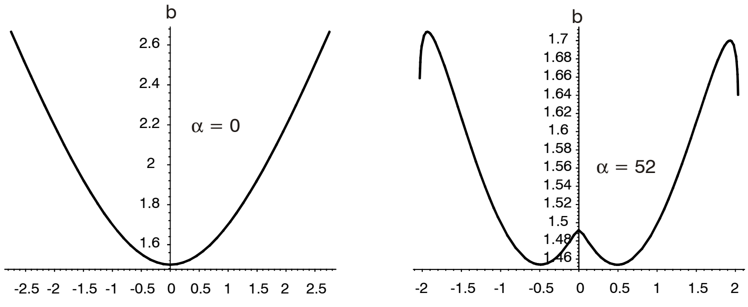

Figure 2: Absorber frequency response for $\alpha = 0$ and $\alpha = 52$.

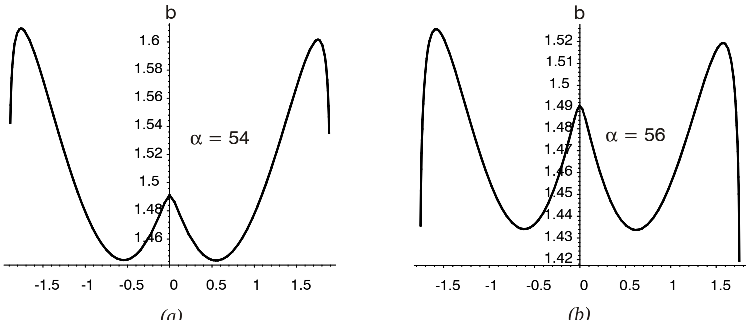

Figure 3: Absorber frequency response for $\alpha = 54$ and $\alpha = 56$.

with no absorption. In contrast, when the nonlinear cubic term is set to $\alpha = 5$ the absorption is achieved, that is, the nonlinearity improves the range of vibration absorption of the pendulum for greater excitation forces on the primary system.

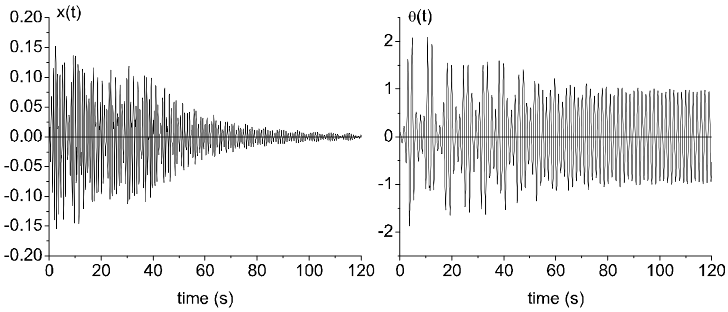

Figure 4: Time response for a Duffing primary system and pendulum absorber.

Finally, in Fig. 5 is shown the time response for $\alpha = 54$, when the external excitation has a detuning of $\rho_1 = 1$ and the absorption is still satisfied. At $t = 150\,\mathrm{s}$ the detuning is changed to $\rho_1 = 0$, where the absorption is still acomplished and improved. This behavior can be predicted from the frequency response (see Fig. 3a).

CONCLUSIONS

A realistic condition on which a primary excited Duffing system is passively controlled by one pendulum that acts as an absorber is considered. By the application of the multiple

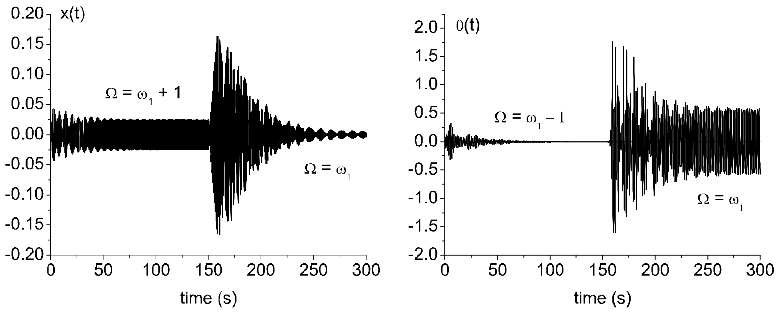

Figure 5: Time responses for $\alpha = 54$, $F_0 = 3$ and tuning variation on the excitation frequency from $\Omega = \omega_1 + 1$ to $\Omega = \omega_1$.

scales method is determined that, the nonlinear dynamic effect of the primary system is absorbed by the pendulum, then the fixed points of the overall system are modified with respect to a linear system. The frequency response of the absorber is modified by the cubic nonlinear term, while the primary system frequency response is the same as in the linear case. The transference of the nonlinear term improves the absorption, even for greater forces. Moreover, there exist approximated constraints between the force magnitude and the nonlinear cubic coefficient to obtain real values of the system response. The absorption capacity is robustly satisfied for excitation frequencies close to the internal resonance condition (tuning) but the system stability could be stable or unstable, depending on the phase of the fixed point. The pendulum can be considered as a cubic vibration absorber.

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