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DETECTION OF BREATHING CRACKS IN A FRAME STRUCTURE USING DYNAMIC RESPONSE

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Abstract

In this paper, a technique for detection of breathing cracks in a planar frame structure is presented. First, the governing equations of a planar frame structure with breathing cracks are derived using the finite element method based on the mixed variational principle. Then a method for analysis of steady-state vibrations of the frame structure is presented. Based on the presented analysis method, a technique to determine the crack parameters is proposed.

INTRODUCTION

Many techniques for detecting cracks of a structure or a machine using its dynamical response have been proposed so far [1], [5], [6]. Most of the techniques detect cracks assuming that the cracks are open cracks which are always open during vibration. However, actual fatigue cracks are breathing cracks which open and close during vibration. In a previous report [2], [3], the authors proposed a technique for detection of breathing cracks in a beam using its dynamical response, and confirmed its applicability by numerical simulation and experiment. This paper presents a technique for detection of breathing cracks in a planar frame structure composed of beams which deforms in axial and lateral directions, as shown in Fig.1.

DERIVATION OF THE GOVERNING EQUATIONS AND ANALYSIS OF STEADY-STATE VIBRATION

Derivation of a finite element based on the mixed variational principle

As a preparation for developing a technique for detection of breathing cracks, we derive a finite element for a beam which deforms in the axial and lateral directions. As discussed later,

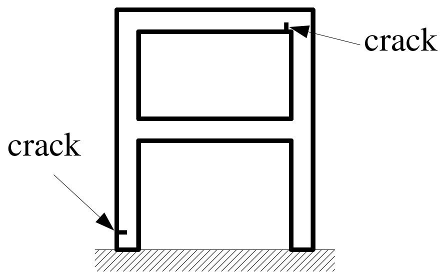


Figure 1: Planar frame structure with breathing cracks

we model the breathing crack in a form that flexibility of the element with the breathing crack changes depending on the sign of the tension and bending moment in the element. Hence, we use the mixed variational principle[4], which enables us to deal the tension and bending moment in the element explicitly. We assume that deformations in the axial and lateral directions are not coupled.

We consider a beam element of length L_e, line density ρ, Young's modulus E, cross sectional area A and the second moment of area I. As shown in Fig.2, we fix the origin O at one end of the element and take the x axis along the longitudinal axis. We denote the elongation, deflection, tension and bending moment of the element by u, v, T and M, respectively. We assume that the element is subjected to, in the axial and lateral directions, viscous damping forces with coefficients c_u and c_v, and distributed external forces $f_u(x,t)$ and $f_v(x,t)$. We also assume that the element is subjected to, at the both ends, tensions T_i, shear forces S_i and bending moments M_i $(i = 0, 1)$, where the subscripts 0 and 1 represent the ends $x = 0$ and $x = L_e$, respectively. Then, the mixed variational principle for this element is written as

$$\begin{aligned} \int_0^{L_e} \bigg(& \rho \frac{\partial^2 u}{\partial t^2}\delta u + c_u \frac{\partial u}{\partial t}\delta u + T\frac{\partial}{\partial x}\delta u - f_u(x,t)\delta u + \frac{\partial u}{\partial x}\delta T - \frac{1}{EA}T\delta T \\ & + \rho\frac{\partial^2 v}{\partial t^2}\delta v + c_v\frac{\partial v}{\partial t}\delta v - M\frac{\partial^2}{\partial x^2}\delta v - f_v(x,t)\delta v - \frac{\partial^2 u}{\partial x^2}\delta M - \frac{1}{EI}M\delta M \bigg) \mathrm{d}x \\ & - T_1\delta u_1 + T_0\delta u_0 - S_1\delta v_1 + S_0\delta v_0 + M_1\frac{\partial \delta v_1}{\partial x} - M_0\frac{\partial \delta v_0}{\partial x} = 0, \end{aligned} \tag{1}$$

where δu, δv, δT and δM are variations of u, v, T and M in the element, δu_i and δv_i $(i =$

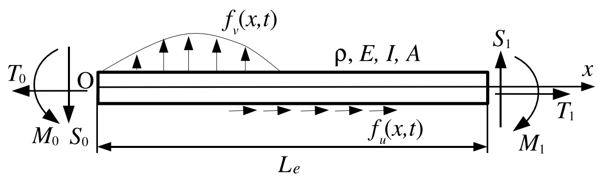


Figure 2: Beam element

$0,1)$ variations of u and v at the ends of the element.

Next we consider to approximate u, v, T and M. Here we approximate T by constants, u and M by a linear function and v by a cubic function. We take nodes at the both ends of the element, and denote the nodal elongations, deflections slopes of the deflection and bending moments by u_i, v_i θ_i and m_i $(i = 0, 1)$, respectively, and the tension in the element by $\hat{t}$. Using these quantities, we approximate u, v, T and M as

$$u = \boldsymbol{L}_w^{\mathrm{T}} \hat{\boldsymbol{w}}, \quad v = \boldsymbol{N}^{\mathrm{T}} \hat{\boldsymbol{w}}, \quad T = \hat{t}, \quad M = \boldsymbol{L}_m^{\mathrm{T}} \hat{\boldsymbol{m}}, \tag{2}$$

where $\boldsymbol{w}$ and $\boldsymbol{m}$ are vectors given by

$$\hat{\boldsymbol{w}} = \{u_0 \; v_0 \; \theta_0 \; u_1 \; v_1 \; \theta_1\}^{\mathrm{T}}, \quad \hat{\boldsymbol{m}} = \{m_0 \; m_1\}^{\mathrm{T}}. \tag{3}$$

In addition, $\boldsymbol{L}_w$, $\boldsymbol{L}_m$ and $\boldsymbol{N}$ are vectors given by

$$\begin{aligned} &\boldsymbol{L}_w = \{L_1(x) \; 0 \; 0 \; L_2(x) \; 0 \; 0\}^{\mathrm{T}}, \quad \boldsymbol{L}_m = \{L_1(x) \; L_2(x)\}^{\mathrm{T}}, \\ &\boldsymbol{N} = \{0 \; N_1(x) \; L_e N_2(x) \; 0 \; N_3(x) \; L_e N_4(x)\}^{\mathrm{T}}, \end{aligned} \tag{4}$$

where $L_1(x)$, $L_2(x)$, $N_1(x)$, $N_2(x)$, $N_3(x)$ and $N_4(x)$ are functions defined by

$$\begin{aligned} &L_1(x) = 1 - \frac{x}{L_e}, \quad L_2(x) = \frac{x}{L_e}, \\ &N_1(x) = 1 - 3\left(\frac{x}{L_e}\right)^2 + 2\left(\frac{x}{L_e}\right)^3, \quad N_2(x) = \frac{x}{L_e} - 2\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3, \\ &N_3(x) = 3\left(\frac{x}{L_e}\right)^2 - 2\left(\frac{x}{L_e}\right)^3, \quad N_4(x) = -\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3. \end{aligned} \tag{5}$$

We also express the variations δu, δv, δT and δM in Eq.(1) as

$$\delta u = \boldsymbol{L}_w^{\mathrm{T}} \delta\hat{\boldsymbol{w}}, \quad \delta v = \boldsymbol{N}^{\mathrm{T}} \delta\hat{\boldsymbol{w}}, \quad \delta T = \delta\hat{t}, \quad \delta M = \boldsymbol{L}_m^{\mathrm{T}} \delta\hat{\boldsymbol{m}}, \tag{6}$$

where $\delta\hat{\boldsymbol{w}}$ and $\delta\hat{\boldsymbol{m}}$ are arbitrary vectors, and $\delta\hat{t}$ an arbitrary scalar.

Substituting Eqs.(2) and (6) into Eq.(1), and considering that $\delta\hat{\boldsymbol{w}}$, $\delta\hat{t}$ and $\delta\hat{\boldsymbol{m}}$ are arbitrary, we have equations of the form

$$\begin{aligned} &\hat{\boldsymbol{M}}\ddot{\hat{\boldsymbol{w}}} + \hat{\boldsymbol{C}}\dot{\hat{\boldsymbol{w}}} + \hat{\boldsymbol{T}}_u \hat{t} - \hat{\boldsymbol{T}}_v \hat{m} = \hat{\boldsymbol{f}}(t) + \hat{\boldsymbol{b}}, \\ &\hat{\boldsymbol{T}}_u^{\mathrm{T}} \hat{\boldsymbol{w}} - \hat{A}_u \hat{t} = 0, \\ &\hat{\boldsymbol{T}}_v^{\mathrm{T}} \hat{\boldsymbol{w}} + \hat{\boldsymbol{A}}_v \hat{\boldsymbol{m}} = \boldsymbol{0}, \end{aligned} \tag{7}$$

where $\hat{\boldsymbol{M}}$, $\hat{\boldsymbol{C}}$, $\hat{\boldsymbol{T}}_u$, $\hat{\boldsymbol{T}}_v$ and $\hat{\boldsymbol{A}}_v$ are matrices determined from ρ, c_u, c_v, $\boldsymbol{L}_w$, $\boldsymbol{N}$, EI and $\boldsymbol{L}_m$. The quantity $\hat{A}_u$ is the inverse of EA multiplied by L_e. The quantities $\hat{A}_u$ and $\hat{\boldsymbol{A}}_v$ represent flexibility of the element. In addition, $\hat{\boldsymbol{f}}(t)$ is a vector determined from $f_u(x,t)$, $f_v(x,t)$, $\boldsymbol{L}_w$ and $\boldsymbol{N}$, and $\hat{\boldsymbol{b}}$ a vector given by

$$\hat{\boldsymbol{b}} = \{-T_0 \; -S_0 \; M_0 \; T_1 \; S_1 \; -M_1\}^{\mathrm{T}}. \tag{8}$$

Model of the breathing crack

The breathing crack opens and closes during vibration. When the breathing crack is closed, rigidity of the beam element remains the same as that of the element without the crack. When the the breathing crack is open, however, rigidity of the beam element is reduced, or flexibility is increased, locally. Here, we express the increment of the flexibility due to the crack by uniform increment of the flexibility in the element with the crack as shown in Fig.3. The crack may occur on the upper surface or lower surface of the beam. For axial deformation, there is no difference between those two cases. For lateral deformation, however, the opening and closing of the crack is opposite in the two cases. In the following, we denote the increment ratio of the flexibility for axial deformation by α, that due to the crack on the upper surface for lateral deformation by α^{U} and that due to the crack on the lower surface by α^{L}. It is clear from the physical consideration that α, α^{U}, α^{L} must satisfy the conditions

$$\alpha \geq 0, \quad \alpha^{\mathrm{U}} \geq 0, \quad \alpha^{\mathrm{L}} \geq 0. \tag{9}$$

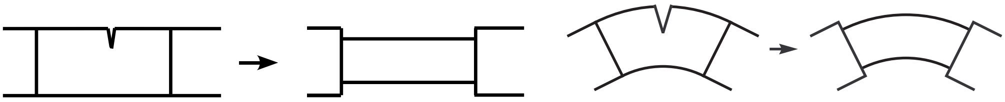
Using the above model, the governing equations for the element with a breathing crack can be written as

$$\begin{aligned}
&\hat{\boldsymbol{M}}\ddot{\hat{\boldsymbol{w}}} + \hat{\boldsymbol{C}}\dot{\hat{\boldsymbol{w}}} + \hat{\boldsymbol{T}}_u \hat{t} - \hat{\boldsymbol{T}}_v \hat{m} = \hat{\boldsymbol{f}}(t) + \hat{\boldsymbol{b}} \\
&\hat{\boldsymbol{T}}_u^{\mathrm{T}} \hat{\boldsymbol{w}} - \hat{A}_u \left(1 + \alpha H(\hat{t})\right) \hat{t} = 0 \\
&\hat{\boldsymbol{T}}_v^{\mathrm{T}} \hat{\boldsymbol{w}} + \hat{\boldsymbol{A}}_v \left(\boldsymbol{I} + \alpha^{\mathrm{U}} \sum_{i=0}^{1} H(m_i)\boldsymbol{I}_i + \alpha^{\mathrm{L}} \sum_{i=0}^{1} H(-m_i)\boldsymbol{I}_i \right) \hat{\boldsymbol{m}} = \boldsymbol{0},
\end{aligned} \tag{10}$$

where $H(\cdot)$ is the Heaviside function, $\boldsymbol{I}$ the identity matrix, $\boldsymbol{I}_i$ $(i = 0, 1)$ the matrices obtained by replacing the second and first columns of the identity matrix by the zero vector, respectively.

Assemble of the elements

Finally we assemble the elements to obtain the governing equations for the total frame structure. This can be done in the same way as that in the usual finite element method. We take the global coordinate system $\mathrm{O}^G - x^G y^G$, and express the nodal elongation u_i, deflection v_i, slope of the deflection θ_i, tension T_i, shear force S_i and bending moment M_i in terms of the global coordinates. It is easily seen from the geometrical consideration that for the element inclined to the global coordinate system by an angle ϕ, as shown in Fig.4, the nodal variables



(a) Model for the axial deformation (b) Model for the lateral deformation

Figure 3: Model of the element when the crack is opened

in the element can be expressed as

$$\begin{aligned} &u_i^G = u_i \cos\phi - v_i \sin\phi, \quad v_i^G = u_i \sin\phi + v_i \cos\phi, \quad \theta_i^G = \theta_i \\ &T_i^G = T_i \cos\phi - S_i \sin\phi, \quad S_i^G = T_i \sin\phi + S_i \cos\phi, \quad M_i^G = M_i. \end{aligned} \tag{11}$$

where u_i^G and v_i^G the displacements in x^G and y^G direction, θ_i^G the slope of the deflection, T_i^G and S_i^G the forces in x^G and y^G direction and M_i^G the moment. Applying the transformation (11) to all of the nodal variables, and imposing the condition that the global nodal variables at the nodes common to elements are the same, we obtain the governing equations for the frame structure. In addition, we specify, at the boundary nodes, three of the quantities u_i^G, v_i^G, θ_i^G, T_i^G, S_i^G, M_i^G following the given boundary conditions. At the fixed or simply supported ends, unknown reaction forces or moments occur. We remove the equations involving these unknown reaction forces or moments from the governing equations. Then, we obtain

$$\begin{aligned} &\boldsymbol{M}\ddot{\boldsymbol{w}} + \boldsymbol{C}\dot{\boldsymbol{w}} + \boldsymbol{T}_u \boldsymbol{t} - \boldsymbol{T}_v \boldsymbol{m} = \boldsymbol{f}(t), \\ &\boldsymbol{T}_u^{\mathrm{T}} \boldsymbol{w} - \boldsymbol{A}_u(\boldsymbol{t};\boldsymbol{\alpha})\,\boldsymbol{t} = 0, \\ &\boldsymbol{T}_v^{\mathrm{T}} \boldsymbol{w} + \boldsymbol{A}_v\left(\boldsymbol{m};\boldsymbol{\alpha}^{\mathrm{U}},\boldsymbol{\alpha}^{\mathrm{L}}\right)\boldsymbol{m} = 0, \end{aligned} \tag{12}$$

where $\boldsymbol{w}$ is a vector obtained by arranging u_i^G, v_i^G and θ_i^G at every node except for the constrained ones, $\boldsymbol{t}$ and $\boldsymbol{m}$ vectors obtained by arranging the tension and bending moment in every element, and $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^{\mathbf{U}}$ and $\boldsymbol{\alpha}^{\mathbf{L}}$ vectors obtained by arranging the increment ratio of the flexibility of every elements. In addition, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{T}_u$, $\boldsymbol{T}_v$, $\boldsymbol{A}_u(\boldsymbol{t};\boldsymbol{\alpha})$ and $\boldsymbol{A}_v\left(\boldsymbol{m};\boldsymbol{\alpha}^{\mathbf{U}},\boldsymbol{\alpha}^{\mathbf{L}}\right)$ are matrices obtained by arranging appropriately the matrices $\hat{\boldsymbol{M}}$, $\hat{\boldsymbol{C}}$, $\hat{\boldsymbol{T}}_u$, $\hat{\boldsymbol{T}}_v$, $\hat{\boldsymbol{A}}_v\left(\boldsymbol{I} + \alpha^{\mathrm{U}}\sum_{i=0}^{1} H(m_i)\boldsymbol{I}_i + \alpha^{\mathrm{L}}\sum_{i=0}^{1} H(-m_i)\boldsymbol{I}_i\right)$ and the scalars $\hat{A}_u\left(1 + \alpha H(\hat{t})\right)$ for each element.

Note that the nodal tensions can be obtained by substituting the solution of Eq.(12) into the first equation of Eq.(10).

Analysis of the steady-state vibration by the harmonic balance method

Based on the governing equation(12), we analyze the steady-state vibration for a periodic excitation. In the following the fundamental frequency of the excitation is denoted by ω.

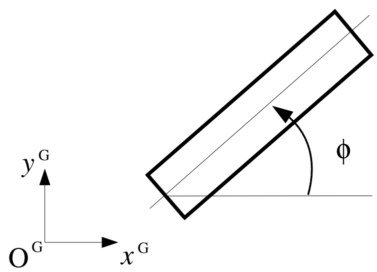


Figure 4: Element inclined to the global coordinate system by ϕ

Since Eq.(12) is nonlinear, we solve it by an iteration method. For this, we denote the solution for the kth iteration by $\boldsymbol{w}^k$, $\boldsymbol{t}^k$, $\boldsymbol{m}^k$, and express them using the solutions for the $(k-1)$th iteration as

$$\boldsymbol{w}^k = \boldsymbol{w}^{k-1} + \boldsymbol{\Delta w}, \quad \boldsymbol{t}^k = \boldsymbol{t}^{k-1} + \boldsymbol{\Delta t}, \quad \boldsymbol{m}^k = \boldsymbol{m}^{k-1} + \boldsymbol{\Delta m}, \tag{13}$$

where $\boldsymbol{\Delta w}$, $\boldsymbol{\Delta t}$, $\boldsymbol{\Delta m}$ are corrections from the solutions for the $(k-1)$th iteration. We assume that they are small. We substitute $\boldsymbol{w}^k$, $\boldsymbol{t}^k$, $\boldsymbol{m}^k$ in Eq.(13) for $\boldsymbol{w}$, $\boldsymbol{t}$, $\boldsymbol{m}$ in Eq.(12), and neglect the terms of $\boldsymbol{\Delta w}$, $\boldsymbol{\Delta t}$, $\boldsymbol{\Delta m}$ whose order is higher than one. Furthermore, considering that step-like change in the flexibility matrices $\boldsymbol{A}_u\left(\boldsymbol{t};\boldsymbol{\alpha}\right)$, $\boldsymbol{A}_v\left(\boldsymbol{m};\boldsymbol{\alpha}^{\mathbf{U}},\boldsymbol{\alpha}^{\mathbf{L}}\right)$ occurs when the tensions or bending moments are zero, we obtain

$$\begin{aligned}
&\boldsymbol{M}\ddot{\boldsymbol{w}}^k + \boldsymbol{C}\dot{\boldsymbol{w}}^k + \boldsymbol{T}_u\boldsymbol{t}^{\boldsymbol{k}} - \boldsymbol{T}_v\boldsymbol{m}^{\boldsymbol{k}} = \boldsymbol{f}(t),\\
&\boldsymbol{T}_u^T\boldsymbol{w}^k - \boldsymbol{A}_u\left(\boldsymbol{t}^{k-1};\boldsymbol{\alpha}\right)\boldsymbol{t}^k = \boldsymbol{0},\\
&\boldsymbol{T}_v^T\boldsymbol{w}^k + \boldsymbol{A}_v\left(\boldsymbol{m}^{k-1};\boldsymbol{\alpha}^{\mathbf{U}},\boldsymbol{\alpha}^{\mathbf{L}}\right)\boldsymbol{m}^k = \boldsymbol{0}.
\end{aligned} \tag{14}$$

The second and third equations of Eq.(14) can be solved for $\boldsymbol{t}^k$, $\boldsymbol{m}^k$ to obtain

$$\boldsymbol{t}^k = \boldsymbol{A}_u^{-1}\left(\boldsymbol{t}^{k-1};\boldsymbol{\alpha}\right)\boldsymbol{T}_u^T\boldsymbol{w}^k, \quad \boldsymbol{m}^k = -\boldsymbol{A}_v^{-1}\left(\boldsymbol{m}^{k-1};\boldsymbol{\alpha}^{\mathbf{U}},\boldsymbol{\alpha}^{\mathbf{L}}\right)\boldsymbol{T}_v^T\boldsymbol{w}^k. \tag{15}$$

Substituting the above equations into the first of Eq.(14), we obtain

$$\begin{aligned}
\boldsymbol{M}\ddot{\boldsymbol{w}}^k + \boldsymbol{C}\dot{\boldsymbol{w}}^k &+ \boldsymbol{T}_u\boldsymbol{A}_u^{-1}\left(\boldsymbol{t}^{k-1};\boldsymbol{\alpha}\right)\boldsymbol{T}_u^T\boldsymbol{w}^{\boldsymbol{k}}\\
&+ \boldsymbol{T}_v\boldsymbol{A}_v^{-1}\left(\boldsymbol{m}^{k-1};\boldsymbol{\alpha}^{\mathrm{U}},\boldsymbol{\alpha}^{\mathrm{L}}\right)\boldsymbol{T}_v^T\boldsymbol{w}^{\boldsymbol{k}} = \boldsymbol{f}(t).
\end{aligned} \tag{16}$$

Note that the above equation is linear in $\boldsymbol{w}^k$.

We consider to solve Eq.(16) by the harmonic balance method. Since the excitation is periodic one whose fundamental frequency is ω, the vector $\boldsymbol{f}(t)$ can be written in the Fourier series of the form

$$\boldsymbol{f}(t) = \boldsymbol{f}_0 + \sum_{j=1}^{\infty}\left\{\boldsymbol{f}_{\mathrm{c}j}\cos(j\omega t) + \boldsymbol{f}_{\mathrm{s}j}\sin(j\omega t)\right\}. \tag{17}$$

In order to obtain the steady-state solution to the above excitation, we express $\boldsymbol{w}^k$ in the Fourier series of the form

$$\boldsymbol{w}^k = \boldsymbol{w}_0^k + \sum_{j=1}^{N_f}\left\{\boldsymbol{w}_{\mathrm{c}j}^k\cos(j\omega t) + \boldsymbol{w}_{\mathrm{s}j}^k\sin(j\omega t)\right\}, \tag{18}$$

where $\boldsymbol{w}_0^k$, $\boldsymbol{w}_{\mathrm{c}j}^k$, $\boldsymbol{w}_{\mathrm{s}j}^k$ are unknown vectors and N_f the order of the truncation of the Fourier series.

We substitute Eqs.(17) and (18) into Eq.(16). Following the principle of the harmonic balance, we equate the constant terms and coefficients of the terms $\cos(j\omega t)$, $\sin(j\omega t)$ of the both sides. Then, we obtain an equation of the form

$$\boldsymbol{A}^k \boldsymbol{z}^k = \boldsymbol{b}, \tag{19}$$

where $\boldsymbol{z}^k$ is an unknown vector obtained by arranging the coefficient vectors in Eq.(18), $\boldsymbol{A}^k$ a known matrix determined from the coefficient matrices in Eq.(16), $\boldsymbol{b}$ a known vector obtained by arranging the coefficient vectors in Eq.(17). Concrete expressions of these vectors and matrix are omitted here. Solving Eq.(19), we can have $\boldsymbol{z}^k$, and hence $\boldsymbol{w}^k$. Substituting the obtained $\boldsymbol{w}^k$ into Eq.(15), we can have $\boldsymbol{t}^k$, $\boldsymbol{m}^k$. This completes the kth iteration.

Using the obtained solution for the kth iteration, we can obtain the solution for the $(k+1)$th iteration in the same way as shown above. We repeat this procedure until the solution converges.

PROPOSITION OF A DETECTION TECHNIQUE

Now we consider to develop a detection technique of breathing cracks in a frame structure. We assume that dimensions, Young's modulus, density, damping coefficients are known.

We take N measurement points on the frame structure, and measure steady-state responses of deflections, tensions and bending moments to the periodic excitation of the form of Eq.(17). These quantities can be easily measured by displacement sensors, strain gages.

We denote the measured deflection, tension and bending moment at the ith measurement points by v_i^{m}, T_i^{m} and M_i^{m}, and introduce vectors $\boldsymbol{z}_i^{\mathrm{m}}$ defined by

$$\boldsymbol{z}_i^{\mathrm{m}} = \left\{ v_i^{\mathrm{m}} \quad T_i^{\mathrm{m}} \quad M_i^{\mathrm{m}} \right\}^{\mathrm{T}} \quad (i = 1, 2, \cdots, N). \tag{20}$$

Since the measured data $\boldsymbol{z}_i^{\mathrm{m}}$ are steady-state responses, they can be expressed in the Fourier series of the form

$$\boldsymbol{z}_i^{\mathrm{m}} = \boldsymbol{Z}_{0i}^{\mathrm{m}} + \sum_{n=1} \Big(\boldsymbol{Z}_{ni}^{\mathrm{m}} \cos n\omega t + \boldsymbol{Z}_{ni}^{\mathrm{m}\prime} \sin n\omega t \Big). \tag{21}$$

The coefficient vectors in the above equation are known quantities obtained from $\boldsymbol{z}_i^{\mathrm{m}}$.

Next we calculate the response of the frame structure by the method presented in the previous chapter. In the calculation, we take as nodes the measurement points. In the following, we express the obtained responses corresponding to the experimental data $\boldsymbol{Z}_{0i}^{\mathrm{m}}$, $\boldsymbol{Z}_{ni}^{\mathrm{m}}$ and $\boldsymbol{Z}_{ni}^{\mathrm{m}\prime}$ by $\boldsymbol{Z}_{0i}$, $\boldsymbol{Z}_{ni}$ and $\boldsymbol{Z}'_{ni}$, respectively. Using these quantities, we introduce the quantity J defined by

$$\begin{aligned} J = \frac{1}{2} \sum_{i=1}^{N} \Big\{ & \big(\boldsymbol{Z}_{0i} - \boldsymbol{Z}_{0i}^{\mathrm{m}}\big)^{\mathrm{T}} \big(\boldsymbol{Z}_{0i} - \boldsymbol{Z}_{0i}^{\mathrm{m}}\big) + \sum_{n} \big(\boldsymbol{Z}_{ni} - \boldsymbol{Z}_{ni}^{\mathrm{m}}\big)^{\mathrm{T}} \big(\boldsymbol{Z}_{ni} - \boldsymbol{Z}_{ni}^{\mathrm{m}}\big) \\ & + \sum_{n} \big(\boldsymbol{Z}'_{ni} - \boldsymbol{Z}_{ni}^{\mathrm{m}}\big)^{\mathrm{T}} \big(\boldsymbol{Z}'_{ni} - \boldsymbol{Z}_{ni}^{\mathrm{m}}\big) \Big\} \end{aligned} \tag{22}$$

The quantity J is the error between the analyzed and measured responses. Next we consider to determine $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^{\mathrm{U}}$ and $\boldsymbol{\alpha}^{\mathrm{L}}$ which minimize J. As mentioned above, $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^{\mathrm{U}}$ and $\boldsymbol{\alpha}^{\mathrm{L}}$ must satisfy Eq.(9). Thus, the problem of detection of breathing cracks is reduced to the problem of finding $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^{\mathrm{U}}$ and $\boldsymbol{\alpha}^{\mathrm{L}}$ which minimize the objective function J subjected to the constraints Eq.(9). Such problems can be solved by, for example, the gradient projection method. In the elements corresponding to non-zero components of the obtained $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^{\mathrm{U}}$, $\boldsymbol{\alpha}^{\mathrm{L}}$, breathing cracks exist

In the above technique, if we take for analysis the same number of nodes as that of the measurement points and if the number of nodes is small, there may be cases in which the cracks are not detected correctly due to the problem of accuracy in the analysis. Even if the detection can be done correctly, it is impossible to specify smaller regions where cracks exist than the element size. Hence, in general, the number of nodes in the analysis is taken larger than that of the measurement points. In this case, if we try to determine the increase rates of the flexibility of all elements, computational cost becomes high. Thus, we first impose the condition that α, α^{U}, α^{L} of some elements are the same, and apply the above detection technique. Then, in the region in which α, α^{U}, α^{L} are not zero, we relax the condition and increase the number of elements in which α^{U}, α^{L} are treated independently. In addition, we take only α, α^{U}, α^{L} in this region as unknowns. Repeating this procedure appropriate times enables us to detect cracks finely suppressing the increase of computational cost.

SUMMARY

In this paper, a technique for detection of breathing cracks in a planar frame structure was presented. First, we derived the governing equations of a planar frame structure with breathing cracks, using the finite element method based on the mixed variational principle. In the equations, we modeled the crack by a uniform increment of the element flexibility, and introduced as the crack parameter the increment ratio of the flexibility. Then, we presented a method for analysis of the steady-state vibration of the frame structure. Based on the presented analysis method, we proposed a method to determine the crack parameters.

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