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COMPARISON OF A DIFFRACTING AND A NON-DIFFRACTING CYLINDRICAL MICROPHONE ARRAY

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Abstract

There are applications where, for practical reasons, microphone arrays must be integrated into/onto a structure. As an example there are efforts to instrument soldiers with sensors and vehicles with sensors in order to detect and localize noise events in the environment. In both of these cases the microphones in the array are unlikely to be equally spaced and in both cases the sound they measure will be diffracted around their support platform. It is postulated that the diffracting support platform is actually an advantage and in order to demonstrate this, the case of a microphone array mounted on a diffracting cylinder is used as an example. The cylinder was chosen as it has a relatively simple analytical solution for calculating the diffraction. Specifically this paper presents a theoretical comparison of the performance of two geometrically identical cylindrical microphone arrays with and without the diffracting cylinder present. An inverse method is used to compensate for the diffraction and allows the system to be conditioned for noise. Conditioning is particularly important at low frequencies where the array becomes small as compared to a wavelength. The performance of the array is evaluated using its white noise gain and its directivity index. It is shown that the diffracting array performs better than its non-diffracting counterpart at low frequencies and equivalently at high frequency. Experimental data measured in an anechoic chamber using a 12 channel diffracting and non-diffracting array are used to validate the theory.

INTRODUCTION

The motivation for this project derives from an effort to instrument small platforms such as helmets/headsets and small autonomous vehicles with microphone arrays. This allows noise events in the field to be identified and localized (see for example [1][9]). The use of these small platforms creates two practical issues. i) The microphone array is small as compared to a wavelength since high frequency noise attenuates as it propagates through the environment and (ii) the array is located on or very near to a diffracting body. This paper uses a diffracting cylinder as an example to investigate the consequences of these two issues and to show that at low frequencies the diffracting body can be an advantage over a geometrically identical array without the diffracting cylinder. The vast majority of the literature (see for example Brandstein and Ward [1]) assume sensors in a free-field environment. Arrays of microphones mounted on diffracting spheres have been investigated recently [2][4][7] but this work concentrates mainly on finding optimal beamforming weights and calculating the resulting directivity. This paper uses a generalized weighting scheme and specifically looks at the performance in the presence of noise for both diffracting and non-diffracting arrays. In addition, this paper validates the theory using experiments.

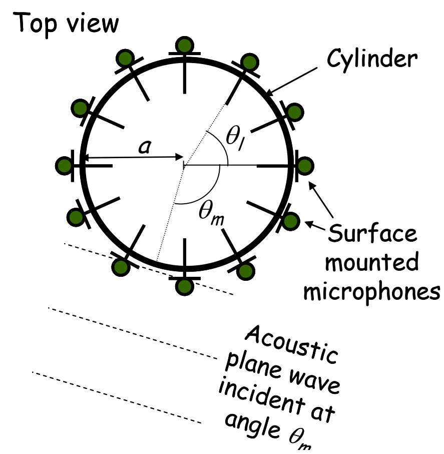

Figure 1: Array of microphones placed on a diffracting infinite cylinder

Figure 1 shows the general layout for the array studied in this paper. The cylinder is assumed to be infinitely long (i.e. into page) and the waves are assumed to come at angles perpendicular to the cylinder (effectively a 2-D problem).

The main tradeoff in designing small arrays is the relationship between directivity and sensitivity to sensor noise. If the array is small as compared to a wavelength, the directivity of the array can only be achieved at the expense of high sensitivity to sensor noise. Therefore, in order to make comparisons between diffracting and non-diffracting arrays, the sensitivity to sensor noise (known as the White Noise Gain [1][7]) and the directivity will be both used as measures of performance. It will be shown that under these circumstances the diffracting array out performs an equivalent non-diffracting array.

THEORY

By placing the array near or on a diffracting body the diffraction from the body must be accounted for in order to determine beamforming weights. What follows here is a generalized formulation used to compensate for diffraction but requires the diffraction characteristics be known. For simple shapes, such as the cylinder used here or the sphere used in [7], the diffraction can be calculated analytically. In other cases the diffraction can be measured experimentally in an anechoic chamber [3].

This section deals with the calculation of beamforming weights, the calculation of diffraction around a cylinder, the definition of Directivity Index and the definition of White Noise Gain.

Array Weights in a Diffracting Environment

Let $\boldsymbol{x}$ be the vector of sound pressures measured by L microphones and $\boldsymbol{b}$ be a vector of sound pressures arriving from M directions at frequency ω. $\boldsymbol{x}$ and $\boldsymbol{b}$ are related by $\boldsymbol{G}$, an $L\times M$ matrix of transfer functions, where $g_{l,m}(\omega)$ is the transfer function between the sound arriving from direction m and the l^{th} microphone. The matrix of transfer functions $\boldsymbol{G}$ describes both the propagation delays (or phases) between the microphones, and any diffraction that may occur due to a nearby object (such and a support structure). The number of waves M is chosen to be large enough such that the behavior of the array is accurately described.

$$\begin{aligned}
\boldsymbol{x} &= \begin{bmatrix} x_1(\omega) & \cdots & x_L(\omega) \end{bmatrix}^T \\
\boldsymbol{b} &= \begin{bmatrix} b_1(\omega) & \cdots & b_M(\omega) \end{bmatrix}^T \\
\boldsymbol{G} &= \begin{bmatrix} g_{1,1}(\omega) & \cdots & g_{1,M}(\omega) \\ \vdots & \ddots & \vdots \\ g_{L,1}(\omega) & \cdots & g_{L,M}(\omega) \end{bmatrix} \\
\boldsymbol{x} &= \boldsymbol{G}\boldsymbol{b}
\end{aligned} \tag{1}$$

In principle the pressures arriving from the M directions, $\boldsymbol{b}$, can be estimated using a pseudo-inverse $\boldsymbol{W}$ of the complex-valued matrix $\boldsymbol{G}$ and the microphone outputs.

$$\boldsymbol{b} \approx \boldsymbol{W}\boldsymbol{x} = \boldsymbol{G}^H [\boldsymbol{G}\boldsymbol{G}^H + \alpha \boldsymbol{I}]^{-1} \boldsymbol{x} \tag{2}$$

where 'H' denotes the Hermitian or conjugate transpose. In calculating the pseudo inverse of $\boldsymbol{G}$ it is necessary to invert the square matrix [$\boldsymbol{G}\boldsymbol{G}^{\boldsymbol{H}}$] (or equivalently invert the singular values of $\boldsymbol{G}$). If this matrix is poorly conditioned the inversion will become inaccurate in the presence of any uncertainty or sensor noise. For this reason it is necessary to condition the matrix before inversion using a normalization factor $\alpha\boldsymbol{I}$ where $\boldsymbol{I}$ is the identity matrix. The resulting matrix $\boldsymbol{W}$ has M rows of weights that are essentially beamformers in each of the M directions. The conditioning coefficient α can be chosen to achieve a desired white noise gain (WNG) as will be discussed further on in this paper. It should be noted that the square matrix [$\boldsymbol{G}\boldsymbol{G}^{\boldsymbol{H}}$] is the cross-spectral matrix of the microphone outputs assuming M incoherent noise sources of unit amplitude (i.e. diffuse noise field).

Calculation of Diffraction on a Cylinder

In order to compare a diffracting and a non-diffracting circular array of microphones it is necessary to calculate the diffraction from the cylinder and therefore calculate the matrix $\boldsymbol{G}$ describing the phase and amplitude relationships between the sensors. The diffraction from the cylinder is calculated by assuming hard walls and hence achieved by setting the normal particle velocity at the surface to zero. It is beyond the scope of this paper to describe in detail the calculation of the pressure on the surface of a diffracting cylinder and the interested reader may want to refer to standard texts [8] and recent papers [6] for more details. The results were validated by comparing the calculated diffraction patterns to those shown in Morse and Ingard [8].

Performance metrics

A good beamformer accurately measures the sound in the "look" direction while suppressing sound from all other directions. Unfortunately the ability to achieve this, especially at low frequencies, results in a high sensitivity to sensor noise and errors in the sensor weights. For this reason a balance must be struck between Directivity index and White Noise Gain.

Suppression of Noise in a Diffuse Field: Directivity Index

The directivity index (DI) measures the ability of a beamformed array to accurately measure a sound source in a particular "look" direction while suppressing a diffuse noise field [1],

$$DI = \frac{M\left|\mathbf{w}_m \mathbf{g}_m\right|^2}{(\mathbf{w}_m GG^H \mathbf{w}_m^H)} \tag{3}$$

Where $\boldsymbol{g_m}$ is a vector of sensor outputs due to a wave from the m^{th} direction (i.e. the m^{th} column of the matrix $\boldsymbol{G}$) and $\boldsymbol{w_m}$ is the weighting vector looking in the m^{th} direction (i.e. the m^{th} row of the matrix $\boldsymbol{W}$).

Suppression of Sensor Noise: White Noise Gain

The white noise gain (WNG) measures the ability of a beamformed array to accurately measure a sound source in a particular "look" direction while suppressing sensor noise i.e. uncorrelated noise at the sensors [1][7]. This can also be considered as a measure of the robustness of the beamformer to changes in sensor sensitivity or weights.

$$WNG = \frac{L\left|\mathbf{w}_m \mathbf{g}_m\right|^2}{(\mathbf{g}_m^H \mathbf{g}_m)(\mathbf{w}_m \mathbf{w}_m^H)} \tag{4}$$

Without a diffracting body, a wave of unit amplitude will be measured at all of the microphones with unit amplitude i.e. free field and $L/(g_m^H g_m)=1$ and indeed without a diffracting object this normalization term is not needed and is not found in the common literature. A white noise gain greater than one indicates a reduction in sensitivity to sensor noise at the output of the beamformer relative to the noise at the sensors themselves.

NUMERICAL RESULTS

Comparison of a Diffracting and Non-Diffracting Circular Array

This section compares the performance of a diffracting and non-diffracting array of similar geometry. A cylinder with radius 4 inches ($a=0.102m$), which is similar to the size of a helmet, is used along with an equally spaced 12 element microphone array. The sound field is created using 80 far-field sources ($M=80$).

As α is increased in the matrix [$\boldsymbol{GG^H+\alpha I}$], equation 2 becomes better conditioned and the sensitivity to sensor noise gets lower i.e. the WNG improves. Unfortunately this also leads to a reduction in the directivity of the array. Therefore to compare the diffracting and non-diffracting arrays the value of α has to be chosen to achieve some specific aim i.e. to either create a specific WNG or specific directivity.

Directivity Index with Constant White Noise Gain

In order to make a comparison between the diffracting and non-diffracting array the conditioning term α is chosen to maintain a constant white noise gain of 0dB. The directivities of the two beamformers can then be compared with equivalent sensitivity to sensor noise. Figure 3 shows the directivities of the two arrays at 200Hz ($ka=0.37$) where it can be seen that the directivity of diffracting array is superior to that of the non-diffracting array. The Directivity Index calculated using equation 3 is 6.00dB for the diffracting and 5.28dB for the non-diffracting array.

Fixed White Noise Gain and Directivity

Another comparison is to fix the directivity index (6dB) and white noise gain (0dB) and to look for *ka* equivalent values. For example with the diffracting array set to *ka=0.37* the equivalent non-diffracting array needs to be 43% larger to achieve the same performance (Figure 2).

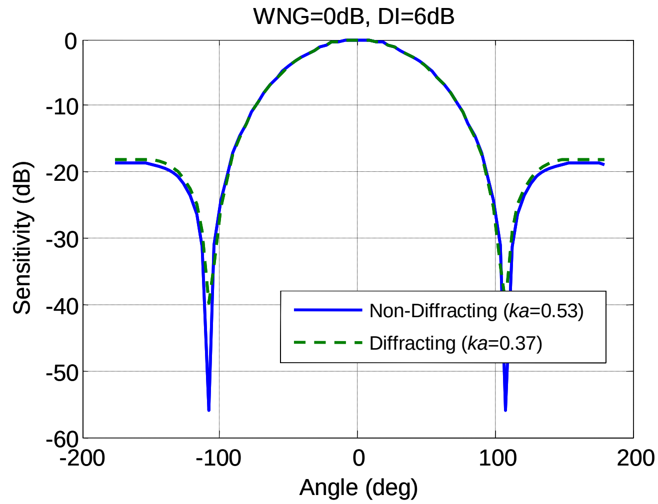

Figure 2: Comparison of diffracting and non-diffracting arrays with 0dB WNG and 6dB DI. ka has to be 43% higher for the non-diffracting array

White Noise Gain vs Directivity for variations in α

To demonstrate the tradeoff between white noise gain and directivity, these two variables can be plotted over a very large range of conditioning terms α. This is done at four non-dimensional frequencies *ka=0.1, 0.3, 1 and 3* which corresponds to 53.7Hz, 161Hz, 537Hz and 1612Hz for the array considered and is plotted in Figure 4. Along with these curves is plotted the maximum WNG possible which is 12 or 10.8dB. The best performance occurs in the top right-hand corner of the graph where both indexes are high. A number of observations can be made from these curves.

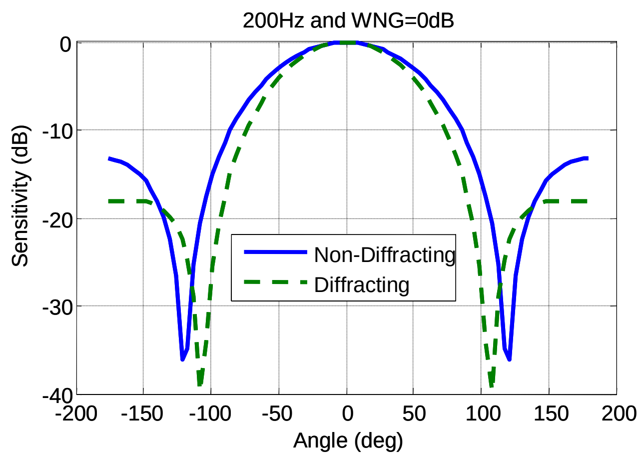

Figure 3: The directivity (or sensitivity) of a diffracting and a non-diffracting array with 0dB WNG

1. There is a tradeoff between WNG and DI so that increasing the performance of one reduces the performance of the other.
2. The tradeoff is more severe at low frequencies. In Figure 4 (a), in order to achieve the maximum WNG the array has no-directivity at all (0dB)
3. In all cases the diffracting array out performs the non-diffracting array
4. At high frequencies, when the wavelength becomes comparable to the array size, high WNG and high DI can be achieved simultaneously (d).

Variation in Number of Elements in the Array at Low Frequencies

Interestingly, the sharp knee in the curves shown in Figure 4 (a) & (b) are due to the physics of the sound around a cylinder (or a circle) and not due to the number of elements in the array. Therefore if the WNG vs DI is plotted for different array numbers (see Figure 5) then the knee in the curve can be see to occur in a similar location. The maximum achievable WNG increases with the number of sensors but if a reasonable WNG (say 0dB) is required, then there is not a vast difference between the performance of the different arrays (DI=5.2dB for six element array and 5.8dB for 20 element array). However, at high frequencies the performance of the array in both DI and WNG increases linearly with the number of sensors.

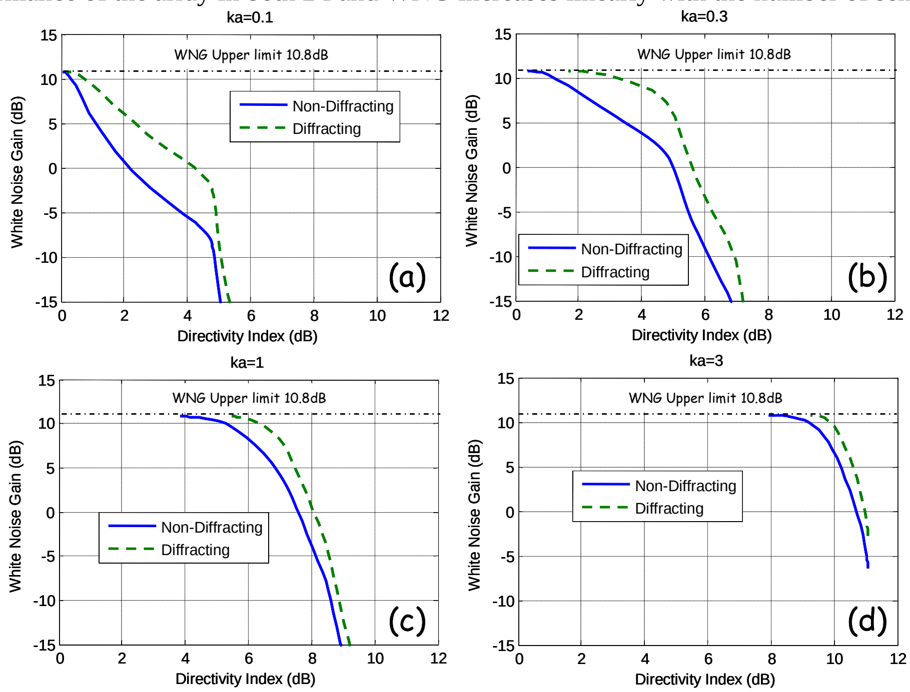

Figure 4: The white noise gain plotted against Directivity Index for a large range of conditioning terms. Four different frequencies are considered

EXPERIMENTAL RESULTS

Experimental Setup

In order to validate the numerical results an experiment was conducted to measure the matrix of transfer functions ***G*** for both a diffracting and a non-diffracting array. The setup (Figure 6) consisted of a 12-microphone array mounted on a wire mesh cylinder (for the non-diffracting array) placed over a stiff tube packed with acoustic foam (for the diffracting array). A speaker playing white noise was used as a

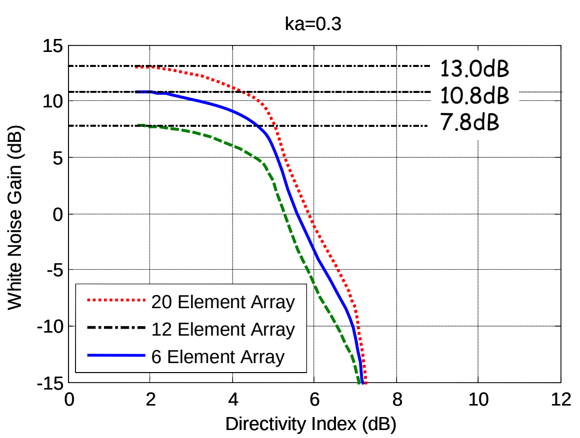

Figure 5: WNG vs DI over a range of conditioning values (a) for three different arrays (all diffracting arrays)

disturbance and a reference microphone was used to remove the dynamics of the speaker. By using a wire mesh structure to mount the microphones, the radius of the array did not change from test to test and for testing purposes the array used was the same radius as mentioned in the "Numerical Results" section, or a = 4 inches. The entire array is mounted on a turntable and placed in an anechoic chamber. The array was approximately 6 feet from the speaker while the reference microphone is approximately 2 feet from the speaker. The transfer functions between the reference microphone and the 12 array microphones were measured for 15^0 increments over a wide range of frequencies. Therefore $\boldsymbol{G}$ is a 12 by 24 matrix.

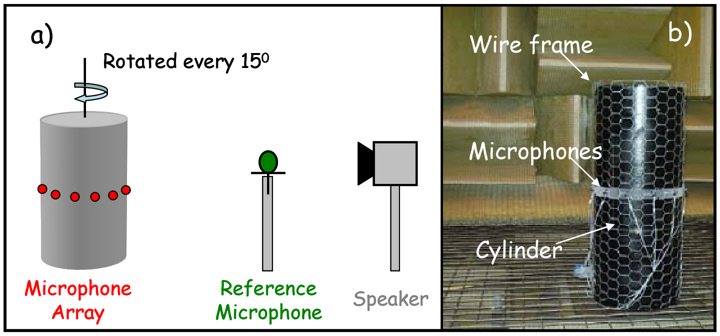

Figure 6: Anechoic chamber experimental setup of a free field 12-microphone array mounted on a wire mesh cylinder, a reference microphone, and a speaker playing white noise.

Experimental Results

With the $\boldsymbol{G}$ matrix measured, the same methodology used in the "Numerical Results" section was used to analyze the experimental results. To demonstrate that the sensitivity and white noise gain of a diffracting array is dominant over a free field array, Figure 7 shows the directivity index and white noise gain plotted for many different conditioning terms α. The four plots shown in Figure 7 represent the same four *ka* values used in Figure 4, and the agreement between the theoretical calculations and experimental results is once again apparent. For a *ka* value of 0.1, or approximately 54 Hz in the example given here, the roll-off of the values for higher directivity indices and lower white noise gains can be attributed to measurement noise dominating the measurements i.e. the noise is effectively "conditioning" the matrix.

CONCLUSIONS

This paper compares the performance on a non-diffracting and a diffracting circular array of microphones and shows that at low frequencies where the array is small compared to a wavelength the diffracting cylinder outperforms the non-diffracting cylinder. These results were demonstrated both theoretically and experimentally with an extremely high degree of agreement. In addition it was shown that at low frequencies there is only a small advantage to increasing the number of sensors in the array as the performance is dominated by the physics of the array size i.e. the size of the singular values of the matrix $\boldsymbol{G}$. It should be noted that this phenomena is identical to the reciprocal problem of radiation from structure at low frequencies. Under these conditions it has been shown that there are only a few efficient ways (termed radiation modes [5]) in which sound can radiate from the structure irrespective of the complexity of the source.

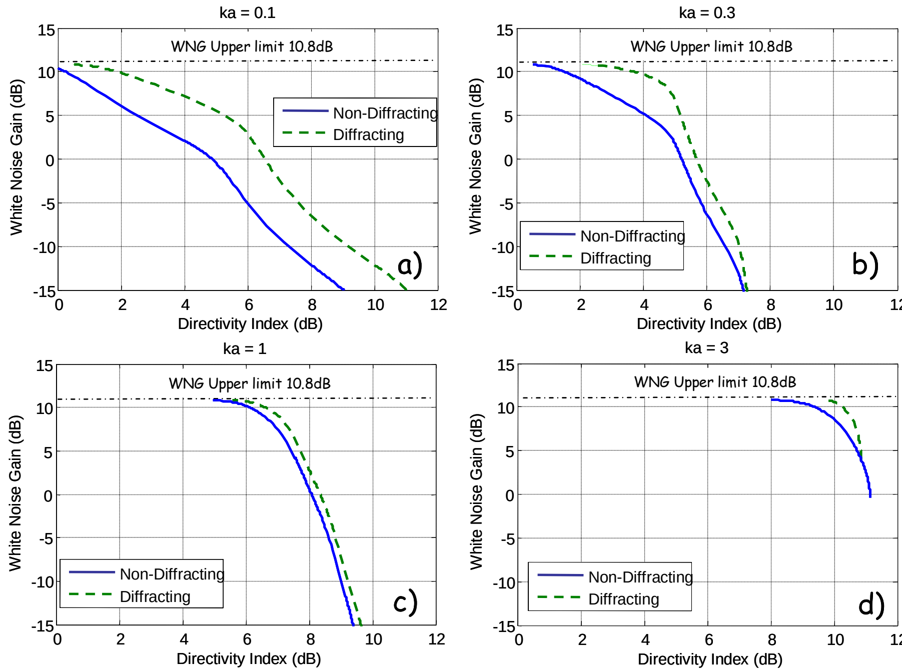

Figure 7: Experimental results showing the white noise gain plotted against Directivity Index for a large range of conditioning terms. The same four frequencies from figure 4 are considered.

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