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RELATING FAR FIELD NOISE SPECTRA OF COLD AND HOT JETS

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Abstract

A general representation scheme for solutions of Lilley's equation, along with a particular expression for the source terms, is used in order to establish a relationship between the far field spectra of hot and cold jets, which are used as reference. The method is such that no precise knowledge of the actual Green's function is required, but only coefficients which are introduced in the computation of its space derivatives, and which can be expressed in terms of Mach number, temperature ratio, polar angle and frequency. High- and low-frequency limiting forms for plug-flow coefficients are used, along with an expression providing a smooth transition. The use of such coefficients is justified since effects of mean velocity and temperature gradients are explicitly accounted for in the formulation. The model predicts that three different components are present in the far field of hot jets, which is supported by the comparison with experimental data for a polar angle of ninety degrees.

INTRODUCTION

The prediction of jet noise depends both on reliable models and on turbulence measurements, which supply the required input data. Lilley's equation [1], which considers adequately mean flow effects in the generation and propagation of sound, provides a relatively simple and comprehensive model (although much more complex than Lighhill's analogy in what regards solution), being used in most computations (see e.g., [2, 3]). A general scheme for representing solutions of Lilley's equation was developed by Musafir [4, 5], the quantities to be modelled or measured, however, depending on the representation for the source terms adopted, many different ones existing (see e.g [6]). The present paper uses this general scheme along with a particular representation of the source function, in order to relate the far field spectra of cold and hot jets.

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THE MODEL: EQUATION AND SOLUTION

By considering 'generalised' external sources of 'volume' (due to the addition of mass or heat) and momentum, given respectively by q and $\boldsymbol{f}^*$, the inhomogeneous linear wave equation describing sound generation and propagation in a parallel mean flow with uniform mean pressure is given, if $\boldsymbol{f}^*$ is replaced by $\boldsymbol{f} - \nabla\cdot\boldsymbol{T}$, so that an external force distribution in dipole form, $-\nabla\cdot\boldsymbol{T}$, corresponding to the stress distribution $\boldsymbol{T}$, is explicitly accounted for in addition to the pure external force field $\boldsymbol{f}$, by

$$\frac{\mathrm{D}_0^3\pi}{\mathrm{D}t^3} - \left(\frac{\mathrm{D}_0}{\mathrm{D}t}\nabla\cdot - 2\nabla U\cdot\frac{\partial}{\partial x_1}\right)\left(c_0^2\nabla\pi\right) = \frac{\mathrm{D}_0^2}{\mathrm{D}t^2}q - \left(\frac{\mathrm{D}_0}{\mathrm{D}t}\nabla\cdot - 2\nabla U\cdot\frac{\partial}{\partial x_1}\right)\left(\boldsymbol{f} - \nabla\cdot\boldsymbol{T}\right) \quad (1)$$

where π represents the acoustic variable, c_0^2 is the local mean value of the squared sound speed c^2, $U = U(x_2, x_3)$ is the mean flow velocity, which is assumed to be purely axial and incompressible, and $\mathrm{D}_0/\mathrm{D}t = \partial/\partial t + U\,\partial/\partial x_1$.

By using the auxiliary problem,

$$\frac{\partial^2\pi}{\partial t^2} - \nabla\cdot\left[(1-M\alpha)^{-2}c_0^2\nabla\pi\right] = \nabla\cdot\left[(1-M\alpha)^{-2}\left(\nabla\cdot\boldsymbol{T} - \boldsymbol{f}\right)\right] + (1-M\alpha)^{-1}\frac{\partial}{\partial t}q \quad (2)$$

which can be obtained from equation (1) if $\partial/\partial x_1$ is replaced by $-(\alpha/c_\infty)\partial/\partial t$, where c_∞ is the far field value of c_0 and α is treated as a 'constant' (to be identified, in the far field solution at $\boldsymbol{x}$, with $x_1/|\boldsymbol{x}| = \cos\theta$, where θ is the angle with the x_1 axis) and some rearrangement is performed, it is possible to show that the solution of equation (2) and hence, the far field solution of equation (1), can be represented, if both U and c_0 are assumed to depend only on a single (transverse) coordinate, x_t, as [4, 5]

$$\pi = \int\int_{-\infty}^{\infty} C^{-2}\left[(\nabla\nabla)_0 G:\boldsymbol{T} + 2\cos\theta\, C^{-1}\nabla M\cdot\boldsymbol{T}\cdot\nabla^* G - \left(T_{\mathrm{tt}}c_0^{-2}\nabla c_0^2 - \boldsymbol{f}\right)\cdot\nabla G + \frac{\partial G}{\partial t}qC\right]\mathrm{d}V_y\,\mathrm{d}\tau \quad (3)$$

where $G = G(\boldsymbol{x}, t|\,\boldsymbol{y}, \tau)$ is the Green's function corresponding to the wave operator in equation (2); C is the Doppler factor $1 - M\cos\theta$, where $M = U/c_\infty$; $(\nabla\nabla)_0G$ stands for $\nabla\nabla G$ with the second transverse derivative of G, $\partial^2/\partial y_\mathrm{t}^2$ (which can be written in terms of other derivatives of G), replaced by the expression applying when $\nabla M = 0 = \nabla c_0$; ∇^* is the gradient operator with the transverse component replaced by zero; T_{tt} is the component of $\boldsymbol{T}$ with both indexes in the transverse direction. This representation has the advantage that cancelling introduced by the second transverse derivative of G is already accounted for.

A compact exact expression for the equivalent source terms was derived by Goldstein [7] using $\pi = (p/p_\infty)^{1/\gamma} - 1$, where p is pressure, p_∞ is a reference value and γ is the specific heat ratio. The obtained equivalent sources correspond to $\boldsymbol{T}$ =

$(1 + \pi')\boldsymbol{uu}$, $\boldsymbol{f} = -(c^2)'\nabla\pi'$ and $q = 0$, where $\boldsymbol{u}$ is the velocity fluctuation vector and primes are used to denote fluctuations of other variables. The expression for $\boldsymbol{f}$ poses the difficulty that the quantities involved are not as easily measured as velocity fluctuations, no information on the required correlations being available. This term, however, although most likely unimportant for cold jets, cannot possibly be neglected for hot jets, when temperature fluctuation become large. Other recent formulations also derived by Goldstein [3, 8] have the feature that temperature effects in the source term are expressed, at least in part, by the momentum source term $\boldsymbol{f}^* = -c_0^2\nabla\cdot(\rho\boldsymbol{uu})$, where ρ is density, which can be expanded, as pointed out in [6], as $\nabla c_0^2\cdot\rho\boldsymbol{uu} - \nabla\cdot\left(\rho c_0^2\boldsymbol{uu}\right)$, corresponding, respectively, to the $\boldsymbol{f}$ and $\boldsymbol{T}$ source terms in equation (1). The corresponding q, however, is non-zero, its expression depending also on the mean flow. A simpler equivalent expression was attained by Musafir [6], with p' as dependent variable (as in [8]), in which $\boldsymbol{f}^*$ is unchanged and $q = (\gamma-1)\boldsymbol{u}\cdot\nabla p' - \nabla\cdot[\rho\boldsymbol{u}(c^2)']$. The first part of this q is possibly negligible, both for cold and hot jets, at least for low Mach numbers, and will be discarded. As for the second part, which describes a volume dipole source, although it is surely affected by temperature, it will also be, for the moment, discarded, on the assumption — to be revised latter — that, since its efficiency is roughly like that of a momentum term in dipole form (usually called ‘quadrupole’), it will be less relevant than the momentum term $\boldsymbol{f} = \nabla c_0^2\cdot\rho\boldsymbol{uu}$, which has a dipole-like efficiency. With these hypotheses, the whole field can be calculated from the knowledge of the correlations of $u_i u_j$ and of mean flow properties, provided the relevant Green’s function is known. By inserting the corresponding forms of $\boldsymbol{T}$ and $\boldsymbol{f}$ in equation (3) and rearranging, one obtains, for $q = 0$

$$p' = \int_{-\infty}^{\infty}\int_{\infty}\rho c_0^2 C^{-2}\left[(\nabla\nabla)_0 G : \boldsymbol{uu} + \nabla^* G\cdot\boldsymbol{uu}\cdot\left(2C^{-1}\cos\theta\nabla M + \nabla c_0^2/c_0^2\right)\right]dV_y\,d\tau\,. \quad (4)$$

The above expression shows that the source components that will be explicitly affected by the mean shear are the very same that will be affected by the mean temperature gradient, contrary to what may be suggested by equation (3).

Since equation (4) considers explicitly the effects of the mean velocity and temperature gradients, it is acceptable to approximate G by the corresponding ‘plug flow-homogeneous medium’ one, discussed in [9] (see also [10]). In this case, the space derivatives of G are given essentially by directivity factors, which accounts for mean flow effects, multiplied by $(1/c_\infty)\partial/\partial t$, in addition to the appropriate retarded time and to the amplitude factor $(4\pi|\boldsymbol{x}|)^{-1}$. This property can be used to express the mean squared value of p' (or the corresponding frequency filtered form) in simple terms.—

If density fluctuations in ρc_0^2 are neglected and the sources are assumed to move with a Mach number M_c (which introduces the corresponding convection Doppler factor $C_c = 1 - M_c\cos\theta$, $P \equiv |\tilde{p}|(4\pi|\boldsymbol{x}|)$, where $\tilde{p}$ is the Fourier transform of p' is given, considering a circular jet and the source coordinates $\boldsymbol{y} = (y_1, r, \phi)$, by

$$P = \int_{\infty} \rho_0 c_0^2 C^{-2} \left| -k^2 C_c^{-2} \boldsymbol{F} : \widetilde{\boldsymbol{uu}} + \mathrm{i}k C_c^{-1} \left(g_1 \widetilde{\boldsymbol{u}_r \boldsymbol{u}_1} + g_\phi \widetilde{\boldsymbol{u}_r \boldsymbol{u}_\phi} \right) \left(2C^{-1} \cos\theta \frac{\mathrm{d}M}{\mathrm{d}r} + \frac{1}{c_0^2} \frac{\mathrm{d}c_0^2}{\mathrm{d}r} \right) \right| \mathrm{d}V_y \quad (5)$$

where $k = \omega/c_\infty$, ~ denotes the Fourier transform, the matrix $\boldsymbol{F}$ contains the flow factors F_{ij} stemming from $(\nabla\nabla)_0 G$ and g_1 and g_ϕ are those stemming from $\nabla^* G$.

The needed spectral function is actually the normalized Fourier transform of the mean squared value of p' which, if C and C_{c} are assumed to be identical, can be calculated from the volume integration of

$$H(\omega)\left\{ k^4 D + k^2 C^{-2} \left(g_1^2 A + \langle g_\phi^2 \rangle B \right) \left[4C^{-2} \cos^2\theta \left(\frac{\mathrm{d}M}{\mathrm{d}r} \right)^2 + 4C^{-1} \frac{\cos\theta}{c_0^2} \frac{\mathrm{d}c_0^2}{\mathrm{d}r} \frac{\mathrm{d}M}{\mathrm{d}r} + \left(\frac{1}{c_0^2} \frac{\mathrm{d}c_0^2}{\mathrm{d}r} \right)^2 \right] \right\} \quad (6)$$

where $H(\omega)$ represents the spectral distribution of the Fourier transform of the volume integral (over the source coordinate difference, $\Delta \boldsymbol{y}$) of the fourth order two-point correlation $\overline{u_1^2(\boldsymbol{y},t) u_1^2(\boldsymbol{y}+\Delta\boldsymbol{y}, t+\Delta t)}$, self- or cross-correlations involving other components being assumed to differ by a numerical factor; D stands for the non-dimensional directivity of the terms associated with $(\nabla\nabla)_0$, being a function of angle θ, nominal jet Mach number M_J, frequency ω and temperature ratio $R = c_\infty^2 / c_J^2$, where c_J^2 is the nominal jet temperature; A and B are coefficients describing the relative importance of the (Fourier transformed) autocorrelations terms associated with g_1 and g_ϕ and $\langle\ \rangle$ denotes the average over the far field azimuthal angle.

The radial derivatives of mean velocity and temperature may also affect the k-dependence in the above expression. It is known that dU/dr scales roughly like ω, the proportionality coefficient, β, being around 4.5 [11]. Also, there is no experimental evidence of a 'dipole-like' behaviour (i.e., a k^2 dependence for low frequencies) for cold jets, what supports this scaling. As for $c_0^{-2}\,\mathrm{d}c_0^2/\mathrm{d}r$, it can be approximated by $\varepsilon\delta/d$, where $\delta = 2(R-1)/(R+1)$, d is the jet diameter and ε is a proportionality factor which, most likely, depends on the axial coordinate y_1. This dependence, however, is probably more complex than the corresponding one in dM/dr — if it were identical, $c_0^{-2}\,\mathrm{d}c_0^2/\mathrm{d}r$ would be also locally proportional to k and the resulting form of expression (6) would depend on frequency only through H, D and g_ϕ, since, then, all terms would be multiplied by the same k^4 factor. It is expected, however, that dipole-like noise, introducing a k^2 dependence, be also present in hot jets, as predicted in [2]. In order to account for this feature, and to verify the adequacy of the hypothesis, it will be assumed that ε can be written as $\varepsilon = (\varepsilon_1 \beta k d + \varepsilon_2)/M_J$ where ε_1 and ε_2 are constants; the factor $1/M_J$ is included from the reasoning that a certain similarity between $c_0^{-2}\,\mathrm{d}c_0^2/\mathrm{d}r$ and $M^{-1}\mathrm{d}M/\mathrm{d}r$ is to be expected. Thus, the ε_1-part gives origin to a quadrupole-like term, while the ε_2 one, to dipole noise. Both components, however, are associated with the 'dipole' flow factor $\langle g_\phi^2 \rangle$.

With these hypotheses, expression (6) is written as

$$H(\omega)k^4\{D + C^{-2}\left(g_1^2 A + < g_\phi^2 > B\right)[4C^{-2}\cos^2\theta\,\beta^2 + (\delta/M_J)^2\left(\varepsilon_1^2\beta^2 + \varepsilon_2^2 (kd)^{-2} + 2\varepsilon_1\varepsilon_2\beta(kd)^{-1}\right) + 4C^{-1}\cos\theta(\delta/M_J)\left(\varepsilon_1\beta + \varepsilon_2 (kd)^{-1}\right)]\} \qquad (7)$$

which permits, if $H(\omega)$ is not affected by heating and the abandoned source terms are negligible, determining far field spectra of hot jets from those of cold jets, provided D, A, B, ε_1 and ε_2 are modelled.

The model shows that hot jets spectra are affected by three types of terms, which represent: 1) quadrupole noise stemming from the same terms as in the cold jet case, but modified by the temperature dependent flow factors, and which will be referred to as 'old' quadrupoles; 2) dipole-like noise, stemming from the mean temperature gradient and 3) 'new' quadrupole noise, also originating form the mean temperature gradient, but modified exclusively by 'dipole' flow factors. There is also some coupling between the terms. Here, it is convenient to re-examine the neglect of the volume source term $q \cong -\nabla\cdot[\rho\boldsymbol{u}(c^2)']$. Since it can be shown that it scales similarly as the 'new' quadrupole term, it should be included in the model. This can be done, in a somewhat crude way, by increasing the value of ε_1 (and assuming, implicitly, that the spectra of the self- and cross-correlation functions involving the components $\rho u_i(c^2)'$ are similar to $H(\omega)$). However, no interference between this additional ε_1 contribution and that of the ε_2 term would exist, what can be accounted for, e.g., by removing the factor 2 in the corresponding coupling term(s), in order to consider that only one half of the 'new' quadrupole-like noise would interfere with dipole noise (or with the mean shear dependent part of the 'old' quadrupole one).

The simpler case is for $\theta = 90^o$, when, for the cold jet (R =1) D can be taken as equal to unity for all values of ω and M. If the quantities in expression (7) are taken as being representative of the whole jet, and the mentioned reduced interference is considered, the relationship between the corresponding power spectral density, W, of hot and cold jets at $\theta = 90^o$ is given, since g_1 will always include the factor cos θ, by

$$W_{90,hot} = W_{90,cold}\left\{D + < g_\phi^2 > B(\delta/M_J)^2\left(\varepsilon_1^2\beta^2 + \varepsilon_2^2 (kd)^{-2} + \varepsilon_1\varepsilon_2\beta(kd)^{-1}\right)\right\}. \qquad (8)$$

COMPARISON WITH EXPERIMENT

The model was tested against recent experimental data obtained at NASA Glenn Research Center [13], which consists of sets of 1/3 octave spectra for a d = 5.08 cm jet, for different Mach number ($0.35 \le M_J < 1.5$) and temperature ratios ($0.8 < R < 3.0$).

At this stage, only the case $\theta = 90^o$ was used for comparison. For this value of the polar angle, D depends on temperature and frequency. In the low frequency limit, it depends also on the relative proportion of azimuthal modes 0 and 2 emitted, as is implied by the Fij derived by Dowling et al. [9]. Since the dependence on this quantity is not very pronounced, it will be considered that both modes are equally important, which can be shown to lead to the low frequency expression, D_{LF}, given by equation (9a), where $R_1 = (R + 1)/2$ is taken to represent the temperature ratio concerning the reference 'plug flow-homogeneous medium'; the high frequency form, D_{HF} (taken from equation (5.13) in [10]), is given by (9b):

$$D_{\mathrm{LF}} = \tfrac{1}{2} + \left(R_1 / (1 + R_1)\right)^2 ; \qquad D_{\mathrm{HF}} = 1 / R_1^3 . \qquad \text{(9a, b)}$$

The corresponding expressions for $< g_\phi^2 >$ (= ½$< g_2^2 + g_3^2 >$) are

$$< g_\phi^2 >_{\mathrm{LF}} = 2 / (1 + R_1)^2 ; \qquad < g_\phi^2 >_{\mathrm{HF}} = 1 / \left(2 R_1^2\right). \qquad \text{(10a, b)}$$

In order to provide a smooth transition, it will be assumed that the general expressions for D and $< g_\phi^2 >$ can be approximated by

$$X = \frac{X_{\mathrm{LF}} + a(kd)^n X_{\mathrm{HF}}}{1 + a(kd)^n} , \qquad (11)$$

where X stands for the desired quantity and a and n are constants.

Modelling of the velocity correlations [12] suggest for B a value around 0.75, which will be considered here; β was taken as 4.5 [11]. Somewhat arbitrarily, n = 2 and a = 1/4 (so that the transition depends on $(kd/2)^2$) were chosen. In order to reduce the number of adjustable constants, best fit with $\varepsilon_2 = 1 - \varepsilon_1$ was sought.

Fig. 1 shows measured sound pressure level data for both cold and hot (R = 2.9) situations, for different Mach numbers, along with predicted results for ε_1 = 0.35. Also shown are the contributions of 'old' and 'new' quadrupole noise, which are obtained, respectively, by setting δ and ε_2 to zero in equation (8). The contribution of dipole noise — the coupling with 'new quadrupoles' included — is the remaining one. Strouhal number St is calculated as $St = kd/(2\pi M_J)$. The two lower frequency points for the cold jet at M_J = 0.35, absent from the original data, were estimated from the normalised spectra for higher Mach numbers.

The agreement is excellent at high frequencies (except for the lower M_J case), results showing that without the 'new quadrupoles' the high frequency behaviour could not be reproduced. They also show that, for low frequencies, this contribution seems to be superfluous, although in most cases (i.e., excepting the higher M_J situation) dipole noise is essential to prediction. In fact, the dipole distinctive behaviour at low frequencies — a rise in the spectrum — is seen to depend on M_J in a stronger way than assumed in the model, since it is more pronounced at M_J = 0.35 and practically inexistent at M_J = 0.9. In the experimental data, this behaviour is clearly present for all temperature ratios larger than unity for the lower Mach numbers (M_J = 0.35 and 0.4), for R > 2 for M_J = 05, and, for M_J = 0.6 and 0.7, only for the higher measured R, (i.e., R = 2.9). For higher M_J, it is practically not seen anymore. Another important aspect is that the existence, sometimes, of a second hump in the hot jet data cannot be attributed to dipole noise, being instead, a feature of the temperature dependent flow factors, mostly of the quadrupole one: for θ = 90°, D imposes a significant reduction in high frequencies and a small increase in the lower ones, as shown in equation (9); the variation of $< g_{\phi}^{2} >$ with temperature, on the other hand, is much less pronounced.

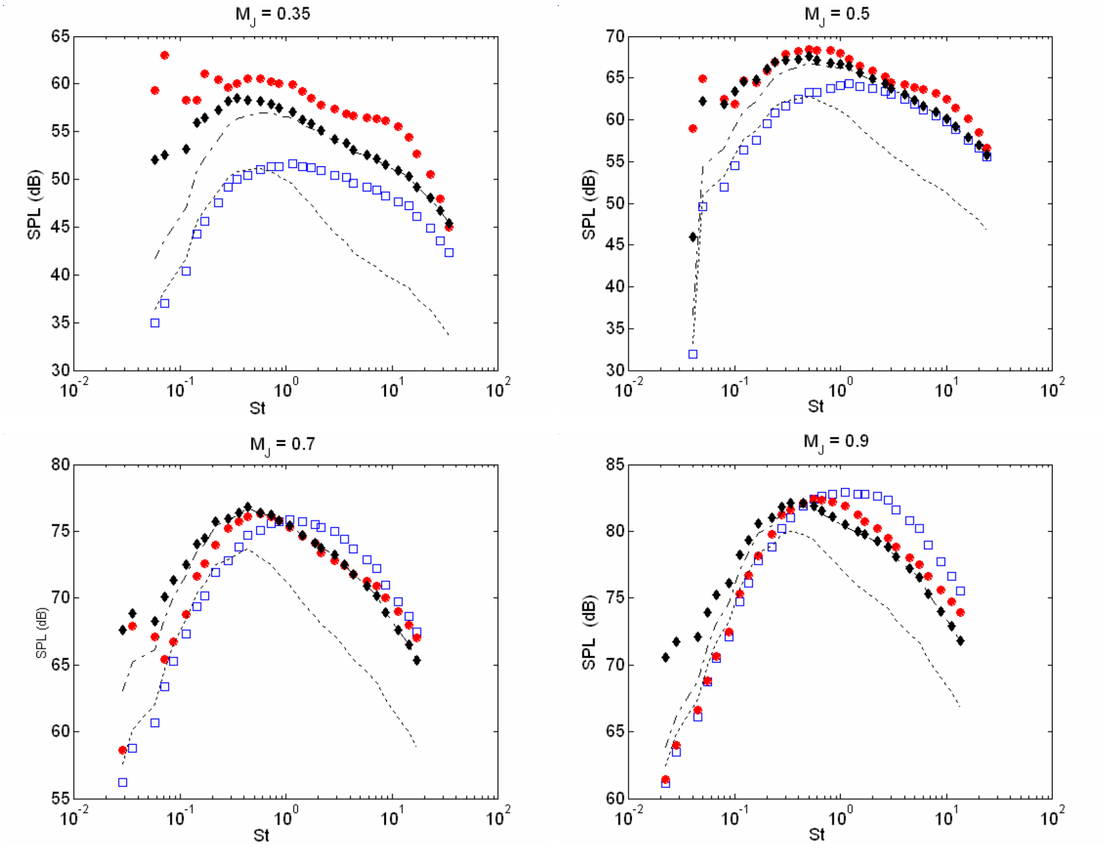

Figure 1.Jet noise third octave band sound pressure level for $\theta = 90^o$.
Measured data [13]*: □ cold, • hot (R = 2.9); Predicted (eq. (8)): ♦ $\varepsilon_1 = 0.35$, $\varepsilon_2 = 0.65$,*
···· exclusively 'old' quadrupoles ($\delta = 0$), –·–· 'old' +' new' quadrupoles ($\varepsilon_2 = 0$).

By altering the value of ε_2 (and, eventually, of ε_1) good agreement can be obtained for all cases, sometimes with a little the excess of "new quadrupole noise" at low frequencies. It was verified that, in general, reducing the dipole-'new quadrupole' coupling increases the low frequency agreement.

An aspect that deserves attention is that the 'new quadrupole' noise — be it originated from the mean temperature gradient term or from the volume dipole term (or from both) — may seem to be a feature of expressions of Lilley's equation closely related to Goldstein's formulation of 2002 [8], as is the case of the one used here. In all earlier formulations [2, 4, 7], the 'temperature' term is expressed exclusively as a momentum source term, which is usually expected to generate only dipole-like noise. The contradiction, however, is only apparent since, in these cases, the 'temperature' source term is always expressed in what might be called 'nearly higher-order multipole form', being of the type $\boldsymbol{f} = \psi\nabla\xi$, which can be written as $\boldsymbol{f} = \nabla(\psi\xi) - \xi\nabla\psi$. At higher frequencies, the higher order component, which corresponds to the equivalent stress distribution $T_{ij} = -(\psi\xi)\delta_{ij}$, takes over, and the consequence is that more quadrupole noise than would be classically expected (see, e.g., [2]) is present in the far field. At lower frequencies, the temperature term does have the expected dipole-like behaviour, which is clearly discernible, except for higher Mach numbers.

CONCLUSION

A simple model relating the spectra of cold and hot jets was developed, having been tested against experimental data for $\theta = 90^{o}$. Although results are highly dependent on the modelling of the mean temperature profile, the comparison is quite promising. In the present stage, the model, which attempts to describe the jet by considering a single ´typical' point source, is but a crude approximation which, however, seems to have captured the important features of sound generation in hot jets. It is expected that more detailed computations based on this approach, but considering the contribution of the different jet regions and detailed information on temperature profiles, may lead to significant improvement.

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