



## **DAMAGE TRACKING IN VARIABLE ENVIRONMENTAL CONDITIONS**

David Chelidze<sup>1</sup> and Martin Lugmair<sup>1</sup>

<sup>1</sup>Department of Mechanical Engineering and Applied Mechanics  
University of Rhode Island, Kingston, RI 02881, USA  
[chelidze@egr.uri.edu](mailto:chelidze@egr.uri.edu) (e-mail address of lead author)

### **Abstract**

A new method for damage tracking and identification in variable environmental conditions is described. The method is based on dynamical systems perspective on damage evolution, where damage is viewed as evolving in a hierarchical dynamical system with distinct, two time-scales. Fast-time dynamics is related to structural vibrations that can be measured directly. Slow-time dynamics describes damage evolution driven by fast-time variables and causes drifts in structural system parameters. Fast-time vibration measurements are used to develop damage tracking feature vectors based on phase space warping concept. A smooth orthogonal decomposition (SOD) is used to identify linear subspace of the feature space related to the changes in environmental conditions. Damage tracking and identification is performed in a feature subspace that is not sensitive to the changes in environmental conditions. For this the SOD is applied to the feature vectors projected onto this subspace. The method is validated using an experimental system, where a vibrating cantilever beam is subjected to a two-dimensional damage process. The damage is introduced by perturbing the nonlinear potential field of the beam with a pair of electromagnets. The changing environmental condition is simulated by altering the lighting conditions that affect the optical motion sensor readings. The results show that the damage modes can be separated from the environmental changes.

### **INTRODUCTION**

Phase space warping is a concept that has been used for multidimensional damage identification [1]. In this work the system's dynamics is considered in its phase space. A hidden damage, slowly growing within a complex mechanical system, changes the dynamics and henceforth the phase space of the system. This phase space deformation is called phase space warping. In damage identification these changes in the phase space are determined to identify and track

the growing damage. One major advantage of the phase space warping based methodology is that no mathematical model for the system's dynamics or for the damage itself is needed, since the phase space is directly reconstructed from measurements.

Besides growing damage, changing environmental conditions also influence the system's dynamics. In order to avoid confusion between altering damage and changing environmental states the influences of the environment on phase space warping based damage tracking need to be known. The influences of changing environmental conditions together with techniques to reduce the environmental effects were investigated using both numerical simulations and experiments on a real system.

Sizable body of papers have been published focusing on the identification of modal parameters (vibration characteristics) in a dynamical system to detect damage. The vibration characteristics are the modal parameters *modal damping*, *frequencies* and *mode shapes*. These vibration-based damage detection methods refer to changes in the vibration characteristics of a dynamical system caused by an evolving damage.

Another vibration-based damage detection technique that is not based on the detection of modal parameters is based on *phase space warping*. Phase space warping describes how the phase space of a dynamical system changes during the growth of damage. These changes are then related to the damage in order to track the evolution of the damage and to predict the future states of the dynamical system. The concept of phase space warping is described in [2]. Its applications in the field of multidimensional damage detection and prediction are shown in [2], [3], [1] and [4].

## PHASE SPACE WARPING BASED DAMAGE IDENTIFICATION

The major steps of phase space warping based damage identification are as follows: First, measured fast-time scalar time series (an output-signal of the dynamical system) is arranged into data records  $\{x_n^j\}_{n=1}^N$ , where  $j = 1, \dots, N_r$  indicates the record number in time sequence, and converted into the points on the system's reconstructed phase space trajectory using *Delay coordinate embedding*  $\mathbf{y}_n^j \in \mathbb{R}^d$ :

$$\mathbf{y}_n^j = [x_n^j, x_{n+\tau}^j, \dots, x_{n+(d-1)\tau}^j]^T, \quad (1)$$

where, subscript  $T$  is used to denote the transpose of a vector or matrix,  $d$  is embedding dimension, and  $\tau$  is delay in number of time samples. Next, the alterations in the phase space due to the slow-time growth of damage need to be determined. For this purpose a *damage tracking matrix*  $\mathbf{Y}$  is built, which contains information about the changes in the phase space. Each component of  $\mathbf{Y}$  is defined as:

$$[\mathbf{Y}]_{ij} = \langle \hat{\mathbf{e}}_R(\mathbf{y}_n^j) \rangle_{\mathbf{y}_n^j \in \mathcal{B}_i} \quad (2)$$

where, the brackets  $\langle \cdot \rangle$  represent the expected value the *estimated tracking function*  $\mathbf{e}_R$  in the partition  $\mathcal{B}_i$  of the reconstructed phase space for the data record  $j$ , and

$$\hat{\mathbf{e}}_R(\mathbf{y}_n^j) = \mathbf{y}_{n+1}^j - \hat{\mathbf{y}}_{n+1}^1, \quad (3)$$

where  $\hat{\mathbf{y}}_{n+1}^1$  is the estimated evolution of  $\mathbf{y}_n$  in the reconstructed phase space that is based on the reference or first data record ( $j = 1$ ). In stationary operating and environmental conditions, the damage related information (e.g. damage modes) can finally be extracted by applying *smooth orthogonal decomposition* to this damage tracking matrix, which is obtained as a solution to the following generalized eigenvalue problem:

$$[\mathbf{Y}^T \mathbf{Y}] \varphi = \lambda [(\mathbf{D}\mathbf{Y})^T \mathbf{D}\mathbf{Y}] \varphi, \quad (4)$$

where,  $\mathbf{D}$  is a differential operator. The Smooth orthogonal decomposition can be viewed as a constrained version of the proper orthogonal decomposition (which in the time discrete case is called singular value decomposition), and in the time discrete case can be solved using *generalized singular value decomposition* of  $\mathbf{Y}$  and  $\mathbf{D}\mathbf{Y}$  matrices. Smooth orthogonal decomposition can identify the active damage modes (smooth orthogonal modes) and gives information about the time history of the damage evolution (smooth orthogonal coordinates).

Since this paper focuses on the environmental aspect a detailed description of phase space warping based damage identification is not given here. Please refer to [?], for instance, for a complete description of this technique.

## VARIABLE ENVIRONMENTAL CONDITIONS

One of the main advantages of the phase space warping based damage identification is that it does not require the knowledge of particular physics of damage or the system model. However, this also causes the main problem for the method since the SOD identifies any source of nonstationarity such as damage, even when the source is the change in environmental or operating conditions. The matrix  $\mathbf{Y}$  and henceforth the SOC and SOMs obtained from the SOD contain information about the damage as well as the changing environmental/operating conditions. Thus, the objective is to manipulate the damage tracking matrix  $\mathbf{Y}$  in such a way that when the SOD is applied only the SOM and the SOC of the actual damage evolution are obtained.

The main hypothesis of this paper is that the environmental or operating conditions would cause changes to the damage tracking matrix that are confined to separate subspace from the changes caused by damage variables. Therefore, by identifying this subspace one should be able to remove the influence of the environmental conditions from the damage identification procedure.

If the modes corresponding to the changing environmental/operating conditions all occupy the same subspace regardless of type of these changes and if this subspace is different from the subspace spanned by the modes related to an evolving damage a distinction between environmental/operating and damage modes will be possible. In this case the modes of the environmental/operating conditions could be removed from the matrix  $\mathbf{Y}$  as follows.

To remove the influences of environmental/operating conditions one can project the row vectors of  $\mathbf{Y}$  onto a hyperplane perpendicular to the environmental/operating modes. This projection can be calculated using this equation

$$\hat{\mathbf{Y}} = \mathbf{Y} - (\mathbf{Y}\tilde{\mathbf{a}})\tilde{\mathbf{a}}^T \quad (5)$$

where  $\tilde{\mathbf{a}}$  stands for the normalized basis vector that spans the environmental/operating-subspace. This procedure will only work if each changing environmental/operating condition spans unique subspace in the feature space of the matrix  $\mathbf{Y}$ . In other words, this projection only works if the modes for different alterations of the same environmental/operating condition all point approximately in the same direction (the direction of  $\tilde{\mathbf{a}}$ ), i.e. they live in the same one-dimensional subspace (in this case, a straight line).

Equation (5) could then be applied to any matrix  $\mathbf{Y}$  in order to remove the modes introduced by the changing environmental/operating conditions and if the SOD is used for the matrix  $\hat{\mathbf{Y}}$  it will give the damage-related SOC and SOMs only.

## EXPERIMENTAL VALIDATION

### Description of the Experiments

A vibrating cantilever beam is used to carry out the experimental investigations. This beam is harmonically excited by an electromagnetic shaker and performs chaotic vibrations above a two-well electromagnetic force field. Laser displacement sensors measure the deflection of the beam. An evolving damage is simulated by changing the voltage of the electromagnets (which causes perturbations to the potential of the beam and simulated fatigue damage). Altering environmental conditions are introduced through the variation of the lighting conditions that affect the vibration sensor readings. The light source is a halogen bulb. It is positioned such that one had approximately 10 000 lx illuminance at the surface of the beam when the nominal voltage of 120 V is applied to the bulb.

The lighting is altered by changing the voltage of the bulb via a DC-controlled dimmer. The range of the control voltage is 0 V to 10 V DC, where 0 V control voltage corresponds to 0 V AC lamp voltage and 10 V control voltage corresponds to 120 V AC lamp voltage.

It should be emphasized that the relationship between control voltage and light output of the bulb is highly nonlinear. In the following experiments the DC voltage signals used as dimmer input are shown. It is sufficient to know that 0 V means 0 lx illuminance and 10 V means approximately 10 000 lx illuminance. The exact change of the illuminance in between is not very important for the performed investigations.

The output characteristics of the displacement sensors are 3 mm/V. The sensor signals are filtered by a low pass filter with a cut off frequency of 50 Hz and sampled by an A/D converter with a sampling frequency of 160 Hz and a resolution of 16 bit. The amplitude and frequency of the shaker are chosen such that one has nominally chaotic vibrations of the beam at the beginning of each experiment.

### Results of the Experiments

During these experiments an evolving two-dimensional damage and changing lighting are simulated. The two-dimensional damage is introduced by changing the voltages of the two electromagnets. This alters the potential of the beam and simulates fatigue damage.

The experiments are performed using the lighting signals shown in figure 1. The light-

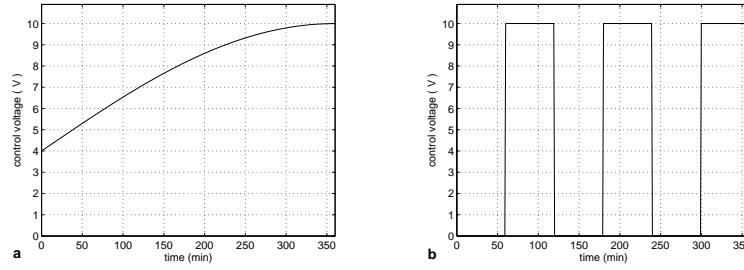


Figure 1: Control voltage signals used during the experiments.

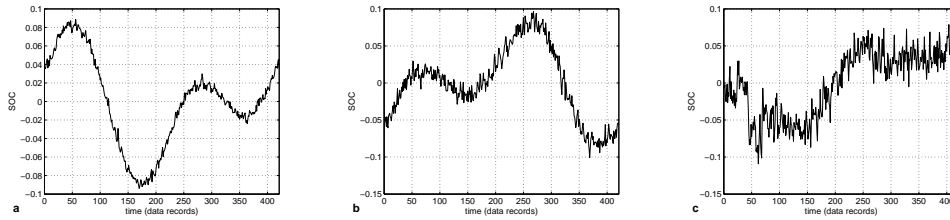


Figure 2: The first three SOC of an experiment where a two-dimensional damage was simulated and the lighting was changed according to the signal shown in figure 1 a. The third SOC represents the change of the lighting.

ing signal of figure 1 a is used in that experiment whose resulting SOC's are depicted in figure 2. The corresponding singular values are shown in figure 4. The changing lighting can be detected: The singular values indicate that three dominant modes are present: two damage modes and the lighting mode. The third SOC (figure 2 c) represents the rising signal of the used lighting progression.

The lighting signal of figure 1 b is used in the next experiment. The corresponding SOC's can be seen in figure 3. The third SOC represents the changing lighting. The step signal can clearly be observed and also three dominant singular values are present (see figure 5).

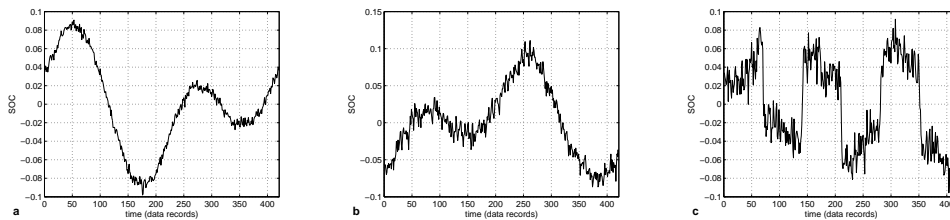


Figure 3: The first three SOC of an experiment where a two-dimensional damage was simulated and the lighting was changed according to the signal shown in figure 1 b. The third SOC represents the change of the lighting.

Further experiments are performed with other lighting changes. However, due to the high noise level present in the output signals of the sensors it is difficult to detect these lighting

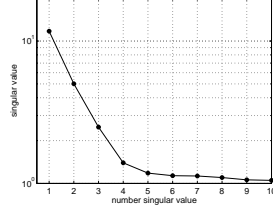


Figure 4: Singular values corresponding to figure 2. The third singular value indicates a present lighting mode.

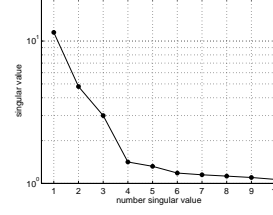


Figure 5: Singular values corresponding to figure 3. Three dominant modes are present: two damage modes and the lighting mode.

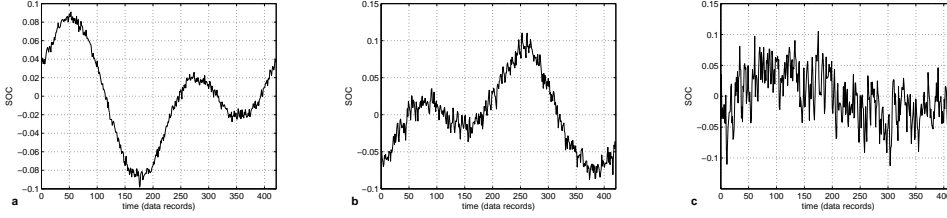


Figure 6: SOC after removing the influences of changing lighting by applying equation (5). (The original SOC can be seen in figure 3.)

modes clearly. Therefore, only the results of above described two experiments are used to investigate if the energy content of the modes corresponding to the changing lighting can be reduced. There are not very strong influences of the changing lighting on the damage modes and damage phase spaces respectively. However, to avoid confusion between growing damage and environmental changes it is nonetheless desirable to reduce the influences of changing environment. This issue is considered in the following section.

### Reduction of the Influences of Changing Lighting

The results of the previous experiments are used in this section to explore if the lighting influences can be reduced.

The correlation between the vectors  $\tilde{\mathbf{w}}_{\text{light}}$  corresponding to the executed experiments is examined by calculating the dot product between each other. The correlation between  $\tilde{\mathbf{w}}_{\text{light}}$  corresponding to figure 3 and that one that corresponds to the experiment of figure 2 is 0.806. This means the correlation is good and one of these vectors can be used as a general basis vector that spans the subspace of the lighting. The projection of equation (5) can then be performed using this basis vector to remove the environmental influences.

First, the vector  $\tilde{\mathbf{w}}_{\text{light}}$  and the tracking matrix corresponding to the experiment of figure 3 are used as basis vector  $\tilde{\mathbf{a}}$  and matrix  $\mathbf{Y}$  in equation (5). Figure 6 shows the new SOC obtained from the SOD of  $\tilde{\mathbf{Y}}$  and figure 8 depicts the related new singular values. It can be seen that, as for the results of the simulations, it is also possible to remove the lighting

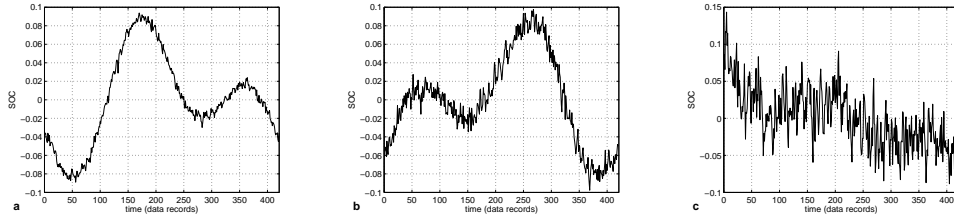


Figure 7: SOC after removing the influences of changing lighting by applying equation (5). (The original SOC can be seen in figure 2.)

modes. The lighting SOC disappeared and also the related singular value.

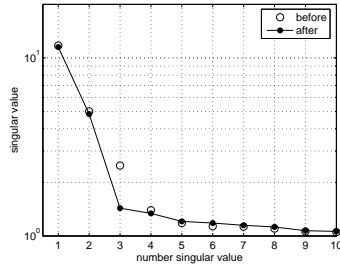


Figure 8: Singular values corresponding to figure 6. The third singular value decreased significantly.

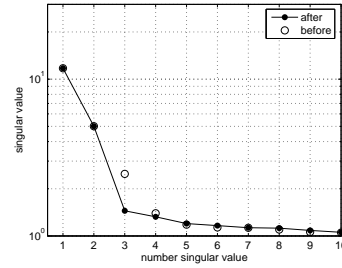


Figure 9: Singular values corresponding to figure 7. The energy content of the third mode is much smaller than before.

Now, the vector  $\tilde{\mathbf{w}}_{\text{light}}$  corresponding to figure 3 is used again as basis vector and projected (according to equation (5)) onto the tracking matrix  $\mathbf{Y}$  that corresponds to the experiment of figure 2. It becomes clear that this basis vector also works for the results of this experiment. The resulting new SOC and singular values can be seen in figure 7 and figure 9. The third singular value decreased significantly, which means the energy content of the lighting mode was strongly reduced.

These calculations demonstrate that the obtained basis vector corresponding to the influences of the lighting can be applied to reduce unwanted influences on the tracking matrix  $\mathbf{Y}$ . The basis vector works for different tracking matrices  $\mathbf{Y}$ .

## SUMMARY AND DISCUSSION

Influences of altering environmental conditions were investigated using a real experimental system. The mechanical system consisted of a cantilever beam vibrating above a two-well electromagnetic potential. The beam was periodically excited by an electromagnetic shaker and performed (due to the two-well potential) chaotic vibrations. The deflection of the beam was measured by two laser displacement sensors. A two-dimensional damage was simulated as changes in the voltages of the two electromagnets used to create the two-well potential. These changes in the strength of the magnetic fields simulated fatigue damage since they changed the stiffness of the beam. The varying environment was simulated as changing lighting condition. Different lighting conditions were used and several time series collected. After

applying phase space warping it was studied how the lighting affects the damage tracking process.

The results made clear that the influences of changing lighting on phase space warping based damage identification are extremely small. A high noise level and voltage drifts of the sensor signals made it difficult to extract the lighting modes. However, it could also be shown on the real system that it is possible to reduce the influences of changing surroundings on phase space warping.

## References

- [1] Chelidze, D., 2004. "Identifying multidimensional damage in a hierarchical dynamical system," *Journal of Nonlinear Dynamics*, **37** (4), pp. 307–322.
- [2] Chelidze, D., Cusumano, J. P., and Chatterjee, A., 2002. "Dynamical systems approach to damage evolution tracking, part 1: Description and experimental application," *Journal of Vibration & Acoustics*, **124** (2), pp. 250–257.
- [3] Chelidze, D., Cusumano, J. P., and Chatterjee, A., 2002. "Dynamical systems approach to damage evolution tracking, part 2: Model-based validation and physical interpretation," *Journal of Vibration & Acoustics*, **124** (2), pp. 258–264.
- [4] Chelidze, D., and Liu, M., 2004. "Dynamical systems approach to fatigue damage identification," *Journal of Sound and Vibration*, **281** (3-5), pp. 887–904.