



## **VIBROACOUSTIC ENVIRONMENT PREDICTION ON DETAILED SPACECRAFT MODELS USING REDUCTION TECHNIQUES**

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### **Abstract**

In general, spacecraft and spacecraft equipment are subjected to random vibration requirements intended to cover the acoustic environment specified by the launcher authorities. The derivation of this requirement is often based on extrapolation from similar past projects or on empirical formulations. Meanwhile, the use of detailed Boundary and Finite Elements models to predict a more realistic environment is becoming more frequent. However, even for such cases where an advanced acoustic analysis is performed (BEM-FEM) the instruments and equipments are generally modelled as rigid masses or with spring-mass systems representing the main modes of the payload structure only. The fidelity of the dynamic interaction is thus very limited, driven by modelling time and computational effort. It has recently been demonstrated that, in spite of the computational effort, the use of complex and large combined spacecraft and instrument models offer advantages with respect to the usefulness of the analysis results.

In this paper, an example of a vibroacoustic environment prediction for the European Space Agency's HERSCHEL spacecraft, using the BEM-FEM coupled vibroacoustic approach will be outlined. The HERSCHEL spacecraft weighs about 3250 kg at launch and it is about 7 metres high. The spacecraft carries a 3.5 m aperture telescope and three scientific payloads operating at near absolute zero Kelvin. The complete mathematical model of the spacecraft structure has over 1.8 million degrees of freedom. The dimension of such mathematical model poses huge difficulties to the BEM-FEM computation.

To tackle this rather unusual large model, different reduction methods such as the classical Guyan and the Craig-Bampton condensation have been implemented in the coupled vibroacoustic analysis. A new technique called "redundant eigenvector dofs reduction" will also be presented. These techniques were initially tested on the cryogenic vessel of the HERSCHEL spacecraft before being applied to the full spacecraft model. Comparison of the results from the analyses using the reduction methods with those from the full models will be summarized and discussed.

## INTRODUCTION

The results from the fully coupled BEM-FEM simulations carried out on the HERSCHEL spacecraft from the European Space Agency will be presented in this paper. The main area of interest from this particular prediction was the dynamic response at the payload interfaces to the spacecraft and the responses of several fragile internal units found inside the payloads. To be able to capture the static response of these payloads in high accuracy, the mathematical finite element model was created with great details leading to a very high number of degrees of freedom in the complete system. In general, the BEM-FEM technique is employed to assess the vibroacoustic responses of standalone subsystems such as appendage, telescope and antenna found in a spacecraft. The matrix size of these subsystems usually do not exceed a couple of hundred thousand dofs; comparing this with over 1.8 million dofs found in the full HERSCHEL spacecraft, a new approach thus had to be derived in order to solve the problem in a timely manner and without the need of supercomputer. To overcome this problem, different dynamic model reduction techniques have been implemented in the actual coupled BEM-FEM simulations of the HERSCHEL spacecraft in order to reduce the overall size of the system matrix. In the commercial code “RAYON-3D” [1] employed for the analyses, the coupling between BEM and FEM is carried out in the modal space and this permits the application of model reduction to the concerned payloads or to the full spacecraft. It should be reminded that the payloads in this spacecraft are housed inside a vacuum vessel and are thus not exposed to a diffuse pressure field load.

Reduction technique is commonly used in the finite element analysis of spacecraft since often the payload mathematical model is delivered in condensed modal form by the contractors, and adequate representation of the structural dynamic characteristic is usually achieved with such condensation. The most popular reduction methods are Craig-Bampton [2] and Guyan [3]. However, each of these methods has its own advantages as well as limitations when it is used in a coupled BEM-FEM simulation. Another new reduction approach involving the removal of redundant eigenvector dofs directly from the modal base matrix of the spacecraft has also been tested. Similar to the Guyan technique, the selection of the so called “redundant” dofs is critical to the quality of the results to be obtained. The terminology “redundant” refers to the minimal contribution of the removed dofs to the overall response of the system. The adequacy of the three aforementioned reduction techniques in coupled BEM-FEM simulation was verified by performing analysis of a subsystem of the HERSCHEL spacecraft, and based on the experience gained it was decided that only two out of the three reduction techniques would be used for the final coupled BEM-FEM simulation of the full spacecraft.

## METHODOLOGIES

### Exterior problem and coupled BEM-FEM solution

The general form of the Helmholtz integral equation [4] used to describe the pressure field,  $p$ , in terms of Green’s function of any point,  $r$ , in a problem domain is given by

equation (1).

$$p(r) = \frac{1}{4\pi} \oint_S \left( G(r|r_q) \nabla_q p(r_q) - p(r_q) \nabla_q G(r|r_q) \right) dS + \int_V \rho(r_q) G(r|r_q) dV \quad (1)$$

By discretising the surface integrals with boundary elements and approximating the variables with shape functions  $[N]$ , the discrete form shown in equation (2) is obtained.

$$p(r) = \sum_{i=1} p_i \left( \sum_{s_i} \int \frac{\partial G(r|r_{q_i})}{\partial n_{q_i}} [N_i] dS \right) + \sum_{i=1} v_{n_i} \left( \sum_S \int G(r|r_{q_i}) j \rho \omega [N] dS \right) \quad (2)$$

Here,  $v_{ni}$  is the normal velocity of a discretised point  $i$  found on the surface. Equation (2) can be coupled with the equation of motion of the structure in the coupled BEM-FEM solution resulting in a linear system as stated in equation (3).

$$\begin{pmatrix} \phi^T ([k_s] + j\omega[b_s] - \omega^2[m_s])\phi & \phi^T C \\ \rho_f \omega^2 B & A \end{pmatrix} \begin{pmatrix} \zeta \\ p \end{pmatrix} = \begin{pmatrix} \phi^T F_s \\ F_f \end{pmatrix} \quad (3)$$

The system as described by equation (3) represents the interaction of a single plane wave with the structure and in order to form the solution of a diffuse field excitation, a random vibration solution approach [5] using auto and cross spectral density functions is generally used leading to the final solution being expressed as a power spectral density function.

As can be seen also, the system equation (3) is expressed in modal space of the structure and thus the solution time is highly influenced by the dimension of the modal space. Any kind of reduction in the size of the modal space will translate into gain in solution time. Basically the dimension of the modal base depends upon the reduction of the total number of dofs of the structural matrix. The reduction techniques described hereafter have been implemented at either payload level or spacecraft level.

### ***Guyan Reduction***

The first step in Guyan reduction is to express the physical displacement vector  $[u]$  in a structural equation of motion into the retained  $[u_a]$  set and the condensed  $[u_o]$  set such that  $[u] = [-K_{oo}^{-1}K_{oa}] [u_a]$ , leading to equation (4).

$$[T]^T \begin{bmatrix} M_{aa} & M_{ao} \\ M_{oa} & M_{oo} \end{bmatrix} [T] [\ddot{u}_a] + [T]^T \begin{bmatrix} K_{aa} & K_{ao} \\ K_{oa} & K_{oo} \end{bmatrix} [T] [u_a] = [T]^T \begin{bmatrix} F_a \\ F_o \end{bmatrix} \quad (4)$$

where  $[T] = \{I \mid [-K_{oo}^{-1}K_{oa}]\}$

The matrix size of the structure can be seen to contain now only the retained dofs set  $[u_a]$ . Guyan reduction is cost effective for a system of small dimension. In the context of coupled BEM-FEM simulation with Guyan reduction, the following points should be considered :

a) External loads can be applied to all the retained dofs.

- b) Vibroacoustic response from diffuse field load can be obtained on all retained dofs.
- c) The reduction process can be very expensive in terms of CPU time for large system
- d) Accuracy of the reduced system is sensitive to the selection of the condensed dofs.

The Guyan technique has been tested with one of the payload FE models of the HERSCHEL spacecraft. In summary, the payload matrix dimension was first statically reduced and then the corresponding reduced mass and reduced stiffness matrices were integrated into the spacecraft before the computation of the system modal base for the coupled vibroacoustic analysis. The results are presented later.

### ***Craig Bampton Reduction***

In Craig-Bampton reduction, the physical displacement is expressed as a combination of the static modes at the interface dofs  $[u_j]$  and the dynamic modes of the interior dofs  $[\xi]$  as given in equation (5).

$$\begin{bmatrix} I & 0 \\ \phi_{ij} & \phi_{jp} \end{bmatrix}^T [M] \begin{bmatrix} I & 0 \\ \phi_{ij} & \phi_{jp} \end{bmatrix} \begin{bmatrix} \ddot{u}_j \\ \ddot{\xi} \end{bmatrix} + \begin{bmatrix} I & 0 \\ \phi_{ij} & \phi_{jp} \end{bmatrix}^T [K] \begin{bmatrix} I & 0 \\ \phi_{ij} & \phi_{jp} \end{bmatrix} \begin{bmatrix} u_j \\ \xi \end{bmatrix} = \begin{bmatrix} I & 0 \\ \phi_{ij} & \phi_{jp} \end{bmatrix}^T \begin{bmatrix} F_b \\ F_i \end{bmatrix} \quad (5)$$

where  $[\phi_{ij}] = [-K_{ii}^{-1} K_{ij}]$  and  $[\phi_{jp}]$  is the constrained modes

The interface dofs are usually the dofs which are to be connected to another structure. Since all the interior dofs have been reduced to modal space, the size of the Craig-Bampton system thus comprises of only the interface dofs plus the number of included modes from the interior dofs.

Craig-Bampton reduction can be used to reduce system of size from a few hundred thousand dofs to a few hundred dofs without much lost in accuracy. However, when it is used in coupled BEM-FEM simulation, one should also consider that :

- a) Structures condensed by this technique will not have any physical existence in the coupled BEM-FEM simulation except for the interface dofs. External loads therefore cannot be applied to the condensed part.
- b) Vibroacoustic response from a diffuse field load can only be obtained at the interface dofs. Recovery of the responses for the interior dofs from the interface dofs responses using transformation matrices would make no sense since all the phase and cross spectral density information are not available.
- c) Accuracy of the system is sensitive to the number of included interior modes

This technique has been applied to several payloads of the HERSCHEL spacecraft. The reduced payload models were integrated into the spacecraft before the spacecraft modal base was computed for the coupled BEM-FEM analysis.

### ***Redundant Eigenvector Dofs Reduction***

Here, the eigenvalue problem of the spacecraft system is solved unaltered as  $([\bar{K}] - \lambda[\bar{M}])\bar{\phi} = 0$ . In other words, the payloads are integrated as physical entities without any prior reduction in size or to modal space. Reduction is only carried out on the extracted eigenvector of the system  $[\bar{\phi}]$  by simply partitioning the eigenvector matrix into the retained eigenvector dofs set  $[\bar{\phi}_a]$  and the redundant eigenvector dofs

set  $[\bar{\phi}_o]$  as  $[\bar{\phi}] = [\bar{\phi}_a | \bar{\phi}_o]$ . The retained dof set  $[\bar{\phi}_a]$ , together with the mass normalized generalized stiffness  $[\hat{k}]$  derived from the eigenvalue system, i.e.,  $[\hat{k}] = [\bar{\phi}]^T [\bar{K}] [\bar{\phi}] = [\omega^2]$ , are subsequently supplied as the spacecraft modal base in the coupled computation.

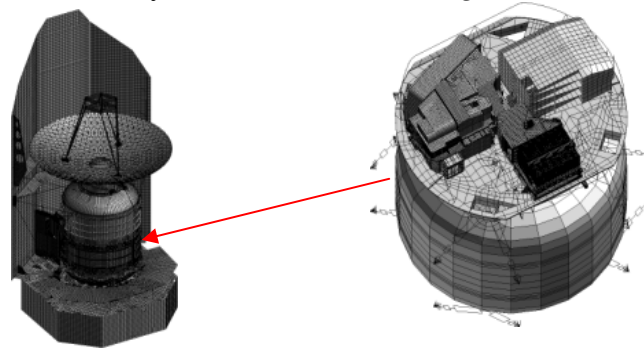
The rationale behind this reduction method lies in the fact that for a finely meshed structure, as is often used for static analysis, the contribution of some of the eigenvector dofs (especially those with small magnitude) is negligible to the overall dynamic response of the physical structure. The positive aspect of this method is the saving in computational time on creating a more mathematically correct (e.g. Guyan) reduced system and this saving can be substantial. In use with coupled BEM-FEM simulation, the issues as stated hereafter should be taken into account :

- a) External loads can be applied to all the retained dofs.
- b) Vibroacoustic response from diffuse field load can be obtained on all retained dofs
- c) Accuracy of the reduced system is sensitive to the selection of the redundant dofs.
- d) The mathematical correctness of this approach should be considered carefully.

## THE HERSCHEL SPACECRAFT AND ANALYSIS PARAMETERS

The HERSCHEL spacecraft weighs about 3250 kg at launch and it is about 7 metres in height. The spacecraft carries a 3.5 m aperture telescope and three scientific payloads operating at near absolute zero K. The payloads themselves are located on the optical bench which is itself encapsulated by a vacuum cryo-vessel.

The mathematical model of the spacecraft without the payloads has about 678000 dofs. The three payloads namely HIFI, PACS and SPIRE have respectively 870000 dofs, 290000 dofs and 24000 dofs. The total number of dofs thus attains 1.86 millions. The spacecraft in such configuration has a relatively high modal density of over 1460 modes for frequency up to 350 Hz. A view of the HERSCHEL spacecraft and its payloads inside the cryo-vessel is shown in *Figure 1*.



*Figure 1 – HERSCHEL spacecraft and its payloads*

The transmission media of the diffuse field load considered was air at room temperature; the properties being: sound speed = 340.0 m/s and density = 1.27 kg/m<sup>3</sup>. The actual diffuse field load was simulated by the superposition of 26 plane waves uniformly distributed around the structure. For the acoustic sound pressure level, unity input (1.0 Pa<sup>2</sup>/Hz) was taken for the verification test case. As for the full spacecraft analysis, the Ariane 5 launch vehicle sound pressure levels as shown in *Figure 2* were used.

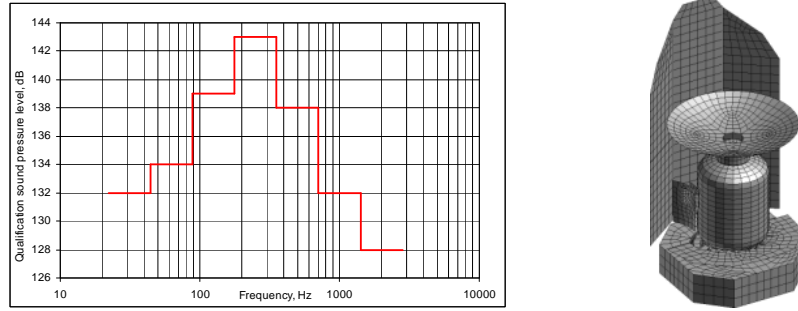


Figure 2 – Applied SPL and HERSCHEL spacecraft coupling fluid surface

As can be seen also in Figure 2, the boundary element fluid surface, with which coupling between the pressure waves and the structure takes place, has been meshed with a nominal element size of approximately  $\lambda/3$  to  $\lambda/4$ , where  $\lambda$  is the wavelength. For analysis up to 350 Hz, the element size translates to 0.24 m ~ 0.32 m in physical term. In total, there were 3276 grid and 2757 elements to describe the coupling surfaces. The vibroacoustic solution was discretely computed in the frequency range between 24 Hz and 346 Hz, at every 2 Hz. Structural viscous damping of 2% was applied in all the computations.

## VERIFICATION CASE – THE CRYO-VESSEL

The cyro-vessel of the HERSCHEL spacecraft together with its payloads were extracted and used in the verification of the reduction techniques. In this particular study, only the payload model PACS has been reduced with either one of the three techniques; the payload HIFI was simply replaced by a lumped mass whilst the payload SPIRE remained unchanged in physical form.

The same set of dofs from PACS was retained in both the Guyan and the Redundant Eigenvector Dofs reduction processes. In fact the model size of PACS was reduced from 290000 dofs to about 123000 dofs in both cases. As for the reduction using the Craig-Bampton technique, the resulting model size of PACS has only 18 physical dofs and 192 generalised dofs. The results from the verification case are shown in Figure 3 and Figure 4 using the acceleration PSD obtained respectively at the payload interfaces and payload interiors of PACS. As can be seen, the Guyan technique was found to give slightly inferior results at the payload interfaces among the three methods employed. Moreover, when the critical payload interior responses were compared, the inferiority was found to increase further. This observation could be attributed to the rather high degree of reduction which was applied to the model.

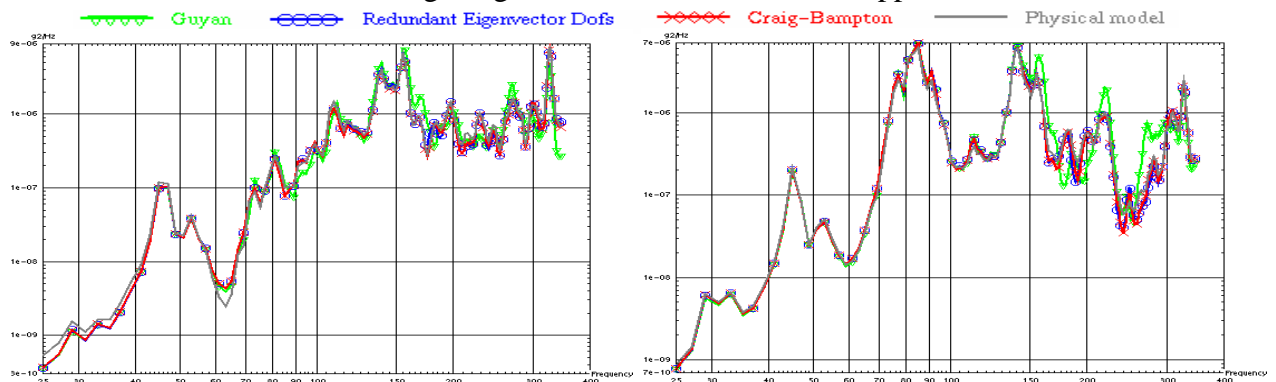


Figure 3 – Acceleration PSD at payload interfaces to spacecraft

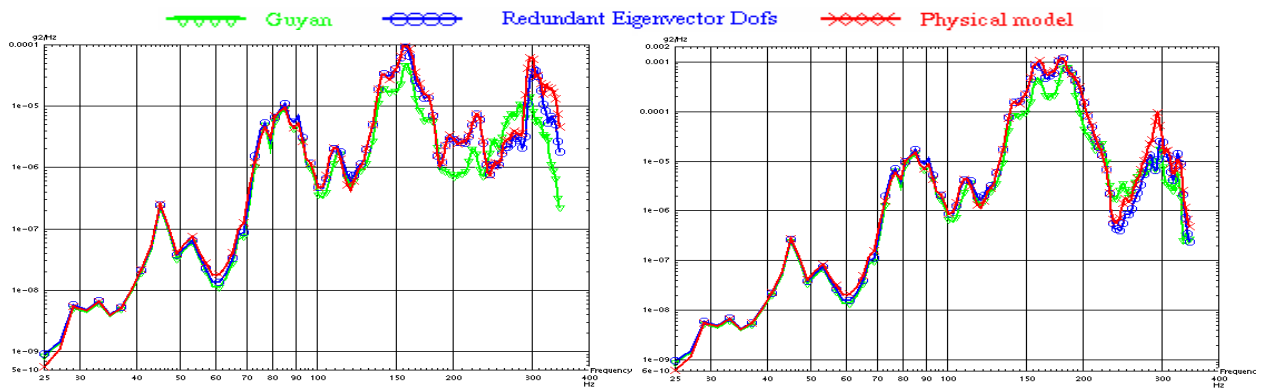


Figure 4 – Acceleration PSD at payload interior points

Thus, considering that the Guyan technique is rather computationally inefficient for large model as well as the inferior results found, it has been decided not to pursue with the use of Guyan reduction in the application case. It should be reminded that with the Craig-Bampton technique, interior response of the payload cannot be obtained at all in the coupled BEM-FEM simulation.

## APPLICATION CASE – THE FULL SPACECRAFT

In this coupled BEM-FEM case study, the full spacecraft model was used. Three versions of the full spacecraft were examined :

- i) Full spacecraft model without any model reduction being applied. The matrix size of the structural modal base was about 1863444 rows by 1462 columns. The latter represents the number of computed modes used in the coupled analysis.
- ii) Full spacecraft model with Craig-Bampton condensed payloads. The three payloads HIFI, PACS and SPIRE were respectively condensed to 24 physical / 96 generalised dofs, 18 physical / 192 generalised dofs, and 30 physical / 54 generalised dofs before they were integrated into the spacecraft and then the structural modal base computed. The final matrix size was 678588 rows by 1461 columns.
- iii) Full spacecraft model with the Redundant Eigenvector Dofs technique being applied to the full modal base as described in (i). The reduction was carried out on the eigenvectors dofs related to the payloads HIFI and PACS only. In the end, about 777000 dofs out of 870000 dofs were removed for HIFI and about 216000 dofs out of 290000 dofs from PACS. The final matrix size of the modal base being 871176 by 1462.

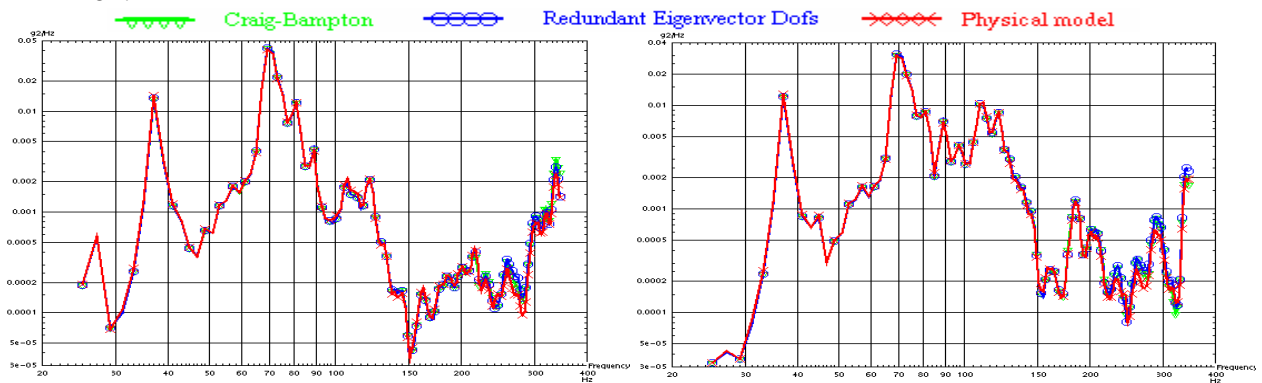


Figure 5 – Acceleration PSD at payload interfaces to spacecraft

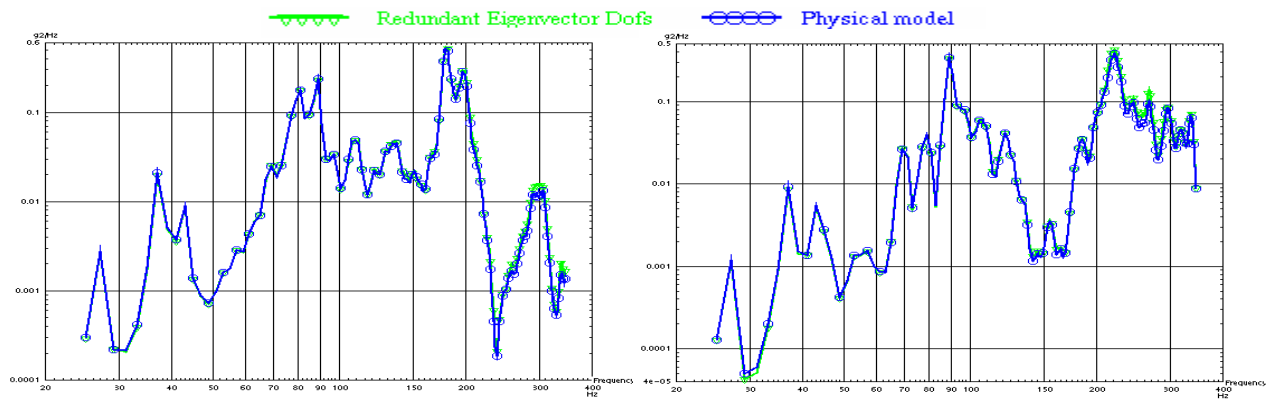


Figure 6 – Acceleration PSD at payload interior points

The results at full spacecraft level are presented in *Figure 5* and *Figure 6*. It can be seen that at the interfaces, both the Craig-Bampton and the Redundant Eigenvector Dofs techniques show very good agreement with the results obtained with the 1.8 million dofs full physical model. Furthermore, judging from the interior responses, the Redundant Eigenvector Dofs technique has been found to be rather accurate.

## CONCLUSION

Different dynamic model reduction techniques have been successfully implemented into a coupled BEM-FEM acoustic analysis for the prediction of spacecraft responses in a diffuse field environment. The Guyan reduction was found to be the least accurate and the most expensive in terms of CPU time. Beside the inability to obtain interior response of the condensed structure, the Craig-Bampton reduction has the advantage of giving the smallest matrix dimension without compromising in accuracy. The Redundant Eigenvector Dofs reduction was shown to be the best among the three techniques tested in view of its accuracy and the possibility to obtain interior responses. This latter technique deserves a more thorough examination despite its lack of mathematical rigour.

## ACKNOWLEDGEMENT

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## REFERENCES

- [1] ESI Group, “RAYON Version 2005”, 2005
- [2] R.R.Craig and M.C.C.Bampton, “Coupling of Substructures for Dynamic Analysis”, AIAA Journal, Vol.6, No.7, 1968
- [3] Guyan,R.J., “Reduction of stiffness and mass matrices”, AIAA Journal, Vol.3,No.2, 1965
- [4] R.D.Ciskowski & C.A.Brebbia, “Boundary element method in acoustics”, Springer
- [5] J.S.Bendat & A.G.Piersol, “Random data analysis & measurement procedures”, Wiley