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STRUCTURE-BORNE POWER TRANSMISSION FROM A LIGHTWEIGHT STAIR INTO A CONNECTED WALL

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Abstract

Measurement and prediction of the structure-borne powers from vibro-acoustic sources in buildings is an important task. It is in particular useful to evaluate the predominant components of excitation (forces and moments) since this can lead to simplifications in the representation by, for example, neglecting the least significant components. In the installed condition, direct measurement of the component powers is difficult or even impossible since it requires the installation of force transducers between the source and receiver. The registration of moments is particularly problematical. Using reciprocity principles, an indirect method is investigated that circumvents these difficulties. The reciprocal method was applied to investigate the power flow from a vibrating lightweight stair into a receiving wall through a single contact. The aim was to establish the predominant components of excitation and thus simplify the characterisation of the stair as structure-borne sound source by means of the contact free velocity and mobility.

INTRODUCTION

Prediction of the structure-borne sound transmission from vibro-acoustic sources in buildings is complicated because several contacts and up to six degrees of freedom can contribute to the overall emission. There is a need to establish a hierarchy of transmission paths and thence, by elimination of the least influential components, simplify calculation. The full transmission process can only be investigated accurately in the installed condition but the direct measurement of forces and moments at the contact(s) however is difficult or even impossible. In this paper a reciprocal method is investigated that circumvents the problem of registering force and moment directly. The method was applied to investigate the power flow from a vibrating lightweight stair into a receiving wall through a single contact.

RECIPROCAL METHOD

For a single contact point, there are up to six degrees of freedom (3 translational; 3 rotational), which can contribute on the excitation of a receiving structure; in this case, a wall which supports a lightweight stair system.

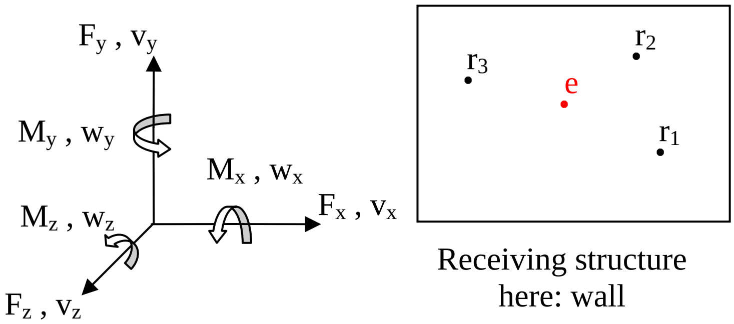

Figure 1 – coordinate system: e = excitation point; r = remote point

The structure-borne power, imparted to the wall by a force F_e and a moment M_e acting at the contact e is given by [1]:

$$P_{F_e} = \frac{1}{2}\mathrm{Re}\left\{F_e \cdot v_e^*\right\} \qquad P_{M_e} = \frac{1}{2}\mathrm{Re}\left\{M_e \cdot w_e^*\right\} \qquad (1)$$

All quantities in equation (1) are complex, the asterisk denotes complex conjugate. The component powers are obtained directly if the cross-spectrum of force and translational velocity, and moment and angular velocity, are known. Direct measurement of the cross-spectra requires the installation of transducers to register force or moment directly which is difficult or even impossible. Using reciprocity principles, these practical measurement difficulties can be circumvented.

Single component case

In the simplest case, the receiving structure (the wall) is excited by a perpendicular force $F_{e,z}$ only. Under action of this force the translational response velocity at the contact point e is $v_{e,z}$. Considering an arbitrary remote point r excited simultaneously the translational response velocity at this point is $v_{r,z}$. The power transmitted through the contact is:

$$P_{F_{e,z}} = \frac{1}{2}\mathrm{Re}\left\{F_{e,z} \cdot v_{e,z}^*\right\} = \frac{1}{2}\mathrm{Re}\left\{F_{e,z} \cdot v_{e,z}^* \cdot \frac{v_{r,z}}{v_{r,z}}\right\} = \frac{1}{2}\mathrm{Re}\left\{Y^{-1}_{v_{r,z}F_{e,z}} \cdot v_{e,z}^* \cdot v_{r,z}\right\} \qquad (2)$$

$Y_{v_{r,z}F_{e,z}}$ is the transfer mobility from the contact point e to the reference point r. It can be measured reciprocally by exciting the remote point and registering the velocity at the contact point, where $Y_{v_{r,z}F_{e,z}} = Y_{v_{e,z}F_{r,z}}$. Thus this arrangement converts the problem of direct force measurement to a simpler transfer mobility measurement and cross-spectra of velocities. Transfer mobilities can easily be measured using a calibrated hammer and a pair of matched accelerometers situated in equal distance around the contact point. The velocity at the contact point is given by the average of the accelerometer signals.

Multi component case

The reciprocal method as described above can be expanded to the problem of multiple degrees of freedom and points of excitation. Consider the case of a perpendicular force F_z and two moments M_x and M_y at a single contact. The net active power is:

$$P = \frac{1}{2}\mathrm{Re}\left\{F_{e,z} \cdot v^*_{e,z} + M_{e,x} \cdot w^*_{e,x} + M_{e,y} \cdot w^*_{e,y}\right\} \tag{3}$$

The contribution of the remaining components $F_{e,x}$, $F_{e,y}$ and $M_{e,z}$ is assumed to be insignificant, which is reasonable for typical walls and ceilings in buildings [2]. It also is assumed that cross-coupling between components (for example, the excitation of velocity $v_{e,z}$ by moment $M_{e,x}$) can be neglected at central locations of the receiving structure.

Three remote points r_1, r_2, r_3 are required for estimating the three excitation components. The translational velocities at the remote points with the source in operation result from a superposition of all components including cross-transfer terms:

$$\begin{Bmatrix} v_{r1,z} \\ v_{r2,z} \\ v_{r3,z} \end{Bmatrix} = \begin{bmatrix} Y_{v_{r1,z}F_{e,z}} & Y_{v_{r1,z}M_{e,x}} & Y_{v_{r1,z}M_{e,y}} \\ Y_{v_{r2,z}F_{e,z}} & Y_{v_{r2,z}M_{e,x}} & Y_{v_{r2,z}M_{e,y}} \\ Y_{v_{r3,z}F_{e,z}} & Y_{v_{r3,z}M_{e,x}} & Y_{v_{r3,z}M_{e,y}} \end{bmatrix} \cdot \begin{Bmatrix} F_{e,z} \\ M_{e,x} \\ M_{e,y} \end{Bmatrix} \tag{4}$$

By virtue of reciprocity the moment cross-transfer mobility can be replaced by its associated force cross-transfer mobility e.g. $Y_{v_{r,z}M_{e,x}} = Y_{w_{e,x}F_{r,z}}$ which requires no moment excitation. The angular velocity required in the reciprocal measurement is obtained using the finite difference approximation e.g. a pair of matched accelerometers. In order to obtain the phase relationship between the excitation components, a reference point r_1 is used with complex velocity transfer functions φ between it and remote points r_2, r_3. The components are then obtained by inversion of the mobility matrix as complex values phase linked to r_1:

$$\begin{Bmatrix} F_{e,z} \\ M_{e,x} \\ M_{e,y} \end{Bmatrix} = \begin{bmatrix} Y_{v_{e,z}F_{r1,z}} & Y_{w_{e,x}F_{r1,z}} & Y_{w_{e,y}F_{r1,z}} \\ Y_{v_{e,z}F_{r2,z}} & Y_{w_{e,x}F_{r2,z}} & Y_{w_{e,y}F_{r2,z}} \\ Y_{v_{e,z}F_{r3,z}} & Y_{w_{e,x}F_{r3,z}} & Y_{w_{e,y}F_{r3,z}} \end{bmatrix}^{-1} \cdot \begin{Bmatrix} 1 \\ \varphi(v_{r1,z}, v_{r2,z}) \\ \varphi(v_{r1,z}, v_{r3,z}) \end{Bmatrix} \cdot \left| v_{r1,z} \right| \tag{5}$$

The velocities at the contact point are equally obtained as complex values phase linked to r_1 from autospectra and velocity transfer functions:

$$\begin{Bmatrix} v_{e,z} \\ w_{e,x} \\ w_{e,y} \end{Bmatrix} = \begin{Bmatrix} \varphi(v_{r1,z}, v_{e,z}) \\ \varphi(v_{r1,z}, w_{e,x}) \\ \varphi(v_{r1,z}, w_{e,y}) \end{Bmatrix} \cdot \left| v_{r1,z} \right| \tag{6}$$

The component powers accordingly are:

$$\begin{aligned} P_{F_{e,z}} &= \frac{1}{2}\mathrm{Re}\left\{F_{e,z} \cdot \left|v_{r1,z}\right| \cdot \varphi^*(v_{r1,z}, v_{e,z})\right\} \\ P_{M_{e,x}} &= \frac{1}{2}\mathrm{Re}\left\{M_{e,x} \cdot \left|v_{r1,z}\right| \cdot \varphi^*(v_{r1,z}, w_{e,x})\right\} \\ P_{M_{e,y}} &= \frac{1}{2}\mathrm{Re}\left\{M_{e,y} \cdot \left|v_{r1,z}\right| \cdot \varphi^*(v_{r1,z}, w_{e,y})\right\} \end{aligned} \tag{7}$$

Note that the contact velocities are measured as “sum” of all excitation components and thus involve cross-coupling between components. For this reason it is in principle not possible to segregate and quantify the relative contribution of the components due to the pure and cross mobility terms.

EXPERIMENTAL RESULTS

Validation by a shaker experiment

The reciprocal method was applied to investigate the structure-borne sound transmission from a shaker source driven with random noise attached to a staircase wall through a force transducer, to obtain the force directly (Figure 2). To avoid moment excitation, a piano wire formed the contact with the wall.

Despite the fact that the shaker represents a pure force source three components $F_{e,z}$, $M_{e,x}$ and $M_{e,y}$ were assumed to contribute to the excitation of the wall.

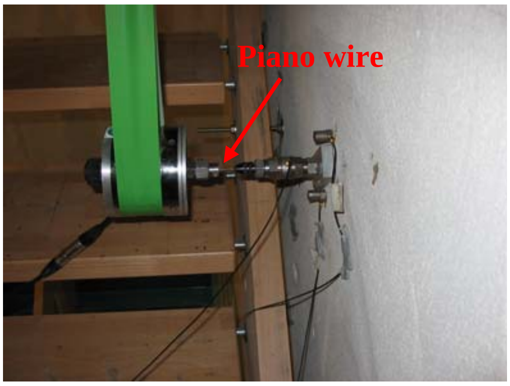

Figure 2 – shaker experiment: set-up

In Figure 3 the force spectra obtained directly and reciprocally are shown as narrow band and 3rd octave band values. The agreement is satisfactory up to about 1.5 kHz. The discrepancy at higher frequencies is due to a longitudinal resonance of the piano wire at approximately 800 Hz resulting in a force maximum and a strong decrease to higher frequencies along with insufficient signal/noise ratio in the reciprocal measurement. In the frequency range below 1.5 kHz discrepancies at certain frequencies result partly from the measurement of the transfer mobility which is inaccurate when the excitation or response coincides with nodal points.

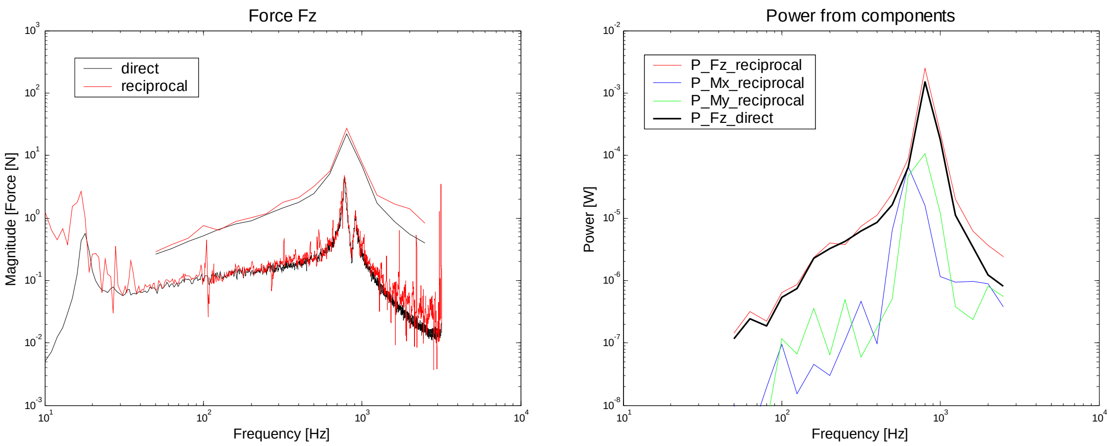

Figure 3 – shaker attached to wall: force (left) and component powers (right) measured directly and reciprocally

The evaluation inevitably gives results for the two moments considered. The respective powers are shown in Figure 3 (right) as 3rd octave band values. The directly and reciprocally measured force induced powers are almost identical. As expected, the moment induced powers are typically well below the force induced power. However, in the frequency range around 600 Hz the moments appear influential. This is likely the result of cross-coupling of the components e.g. the force produces a high angular velocity. The total power imparted to the wall, including cross coupling, can be written as:

$$
\begin{aligned}
P_{total} = & \frac{1}{2}\left|F_{e,z}\right|^2 \cdot \mathrm{Re}\left\{Y_{v_{e,z}F_{e,z}}\right\} + \frac{1}{2}\left|M_{e,x}\right|^2 \cdot \mathrm{Re}\left\{Y_{w_{e,x}M_{e,x}}\right\} + \frac{1}{2}\left|M_{e,y}\right|^2 \cdot \mathrm{Re}\left\{Y_{w_{e,y}M_{e,y}}\right\} \\
& + \mathrm{Re}\left\{F^{*}_{e,z} \cdot M_{e,x}\right\} \cdot \mathrm{Re}\left\{Y_{w_{e,x}F_{e,z}}\right\} + \mathrm{Re}\left\{F^{*}_{e,z} \cdot M_{e,y}\right\} \cdot \mathrm{Re}\left\{Y_{w_{e,y}F_{e,z}}\right\} \\
& + \frac{1}{2}\mathrm{Re}\left\{M^{*}_{e,x} \cdot M_{e,y} \cdot Y_{w_{e,x}M_{e,y}}\right\} + \frac{1}{2}\mathrm{Re}\left\{M^{*}_{e,y} \cdot M_{e,x} \cdot Y_{w_{e,y}M_{e,x}}\right\}
\end{aligned}
\quad (8)
$$

The first row terms give the pure force and moment induced power, the second row terms give the power due to cross-coupling between force and angular velocity. The third row is the power due to cross-coupling between moment and angular velocity at right angle to the moment, and is most likely negligible. The second row terms were calculated from the reciprocally obtained components and the directly measured point cross mobilities from excitation with the shaker. It was found that the second row terms are almost equal to the moment induced powers shown in Figure 2. From this it could be assumed that the apparently high moment induced powers are actually resulting from cross-coupling effects.

In general, the reciprocal measurement of the force induced power as the dominant excitation component is reliable. It is indicated that the reciprocal measurement of the moment induced powers is more sensitive to experimental error at least when moment excitation is not predominant.

Case study: power flow from a vibrating stair

The reciprocal method was applied to investigate the power flow from a vibrating lightweight stair with a single rigid wall contact at a central wall location. Background of this study is the characterization of stairs as structure-borne sound sources allowing a prediction of the sound transmission into a receiving room e.g. the normalized impact sound pressure level. The stair was excited by the shaker attached to a central step and driven with random noise. The experimental set-up is shown in Figure 4, the results in Figure 5.

Figure 4 – vibrating stair: set-up

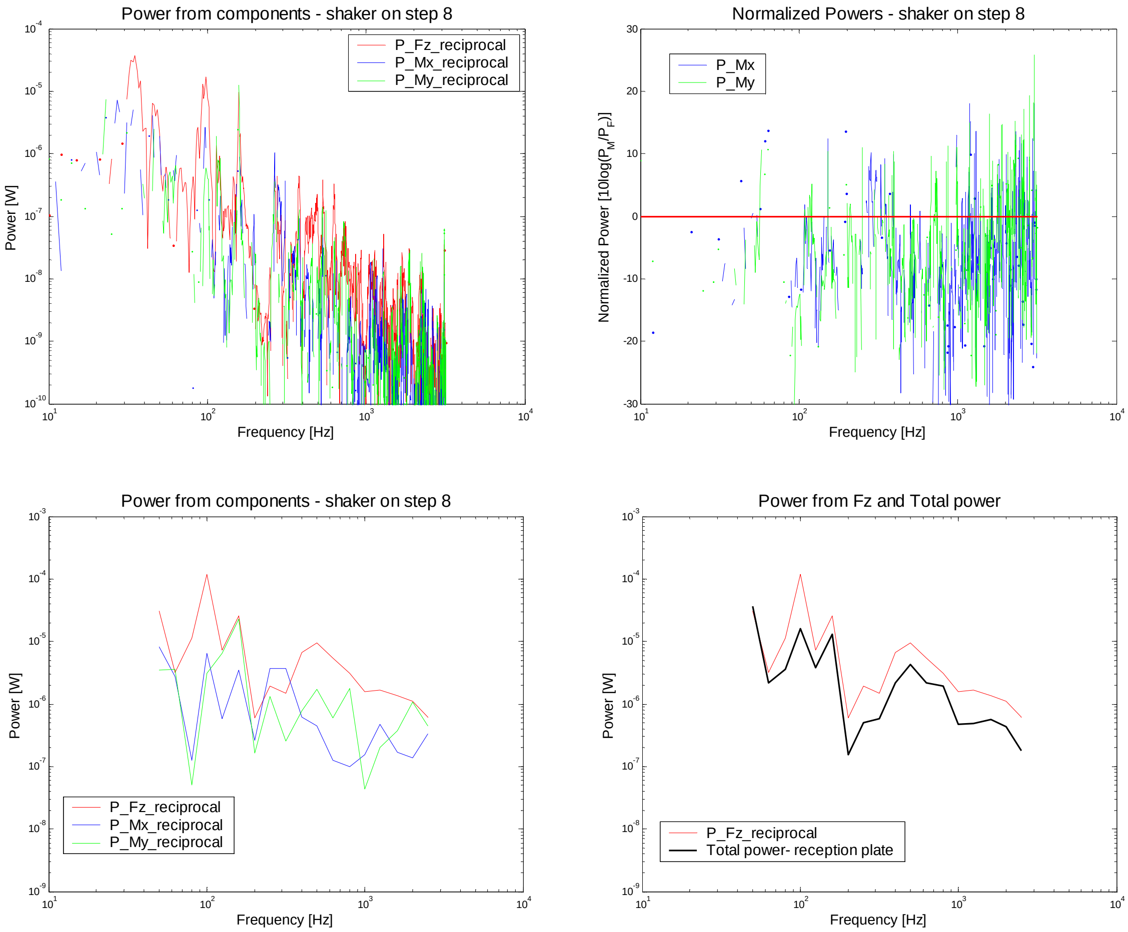

Figure 5 – vibrating stair: narrow band component powers (upper left); normalized component powers (upper right); 3rd octave band component powers (lower left); comparison of force induced power to reception plate power (lower right)

The force induced power is generally dominant within the considered frequency range. This can be seen clearly from the normalized narrow band powers and the component powers in 3rd octave bands. The force curve is generally continuous which indicates that the wall is primarily energised by the perpendicular force. In contrast, the moment induced power curves show discontinuities indicating negative power flow e.g. experimental error. The moment induced power increases with frequency as expected from theory but also contributes to the excitation in the frequency range below 1 kHz which is of prime concern for the impact sound transmission from stairs.

As validation for the results the total power was additionally evaluated from the reception plate approach e.g. measurement of the spatial average velocity and the total loss factor:

$$P = \omega \cdot m \cdot \eta \cdot \tilde{\bar{v}}^2 \tag{9}$$

As shown in the lower right corner of Figure 5 the force induced power is similar to the total power from (9) but generally higher about 3 dB. The same discrepancy occurred for the shaker attached directly to the staircase wall. It is thus assumed that the underestimation of the total power by the reception plate method is related to the "properties" of the wall which differs from an ideal reception plate as investigated in [4] e.g. in the boundary condition.

However the total power is reasonably approximated by force induced power alone and thus neglect of the moment contributions seems to be acceptable within engineering accuracy.

SUMMARY

Reciprocal measurement methods can be used in assessing the relative contribution of several components of excitation from vibrating sources when operating in the installed condition. Problems of directly registering forces and moments are therefore avoided and dimensionally incompatible components can be compared on a power basis.

In the case considered, it has been demonstrated that lightweight timber stair systems, which are attached and supported by a wall separating dwellings, excite the wall predominantly by perpendicular forces. Obviously the normal excitation of the stair causes strong bending vibration of the stringer normal to the wall. Moments can assume importance if the cross mobility of the wall at the contact point is significant, but, in general moments can be neglected. This result points to simplifications in characterising such stair systems as structure-borne sound sources. Only one component of the free velocity [3] needs to be considered, which corresponds to the perpendicular force. This, with the component source mobility, provides sufficient source data for prediction of the installed power by mobility methods.

It also has been demonstrated that a reception plate method can be employed to characterise the stair system on a power basis.

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