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# QUANTIFYING THE CORRELATION BETWEEN TONAL NOISE SOURCES

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## Abstract

Many methods for tracking time varying tones have been proposed. Those noise and vibration processes often include both random and tonal components. Coherence analysis cannot be readily applied to the tonal components, since the coherence between two tones having the same frequency is unity. Thus, for example, two independent engines operating at the same speed will exhibit unity coherence at harmonics of the operating speed. For this reason, noise source identification using coherence analysis has typically ignored the near-unity coherence associated with narrow spectral lines. However, those spectral lines not only often times dominate the spectrum, and so it is reasonable to question whether there is, indeed, information in them that would allow one to use them to quantify the level of correlation between tonal noise sources. The answer to this question is the focus of this paper. It is affirmative, and based on the fact that two independent machines will never operate at exactly the same speed. Furthermore, the machine speed always entails minor speed fluctuations. By tracking the amplitude and frequency fluctuations associated with a given tonal component using an extended Kalman filter (EKF) one can extract amplitude and frequency time series that can quantify the level of correlation between two tones having the same nominal frequency. We present the structure of our EKF, investigate its sensitivity to critical EKF parameters, and offer novel measures for quantifying the correlation between two tones. These measures range from statistical correlations between amplitude/frequency methods have been evaluated in high SNR settings (>10dB). The EKF method presented in this work is shown to work well at much lower SNRs (e.g. -3 dB). This is important, since many machinery applications entail a notable amount of random noise/vibration.

## INTRODUCTION

In this work we address the influence of frequency jitter on the stochastic properties of sinusoids. Sinusoids are commonplace in applications associated with sound and vibration analysis of mechanical systems [1], [3]. Examples include their use in detection of defects in gears and bearings, and in fan noise, to name a few. The focus of this work is limited to the influence of frequency jitter on the power spectral density (PSD) for a single process, and on the coherence between two processes. Consider the following random process:

$$x(t) = \cos[\omega_o t + \lambda\varphi(t) + \varphi_o] \tag{1}$$

where the nominal frequency, $\omega_o$, is known, where the initial phase, $\varphi_o$, is a random variable that satisfies the following expected value condition

$$E(e^{j\varphi_o}) = E(e^{j2\varphi_o}) = 0 \tag{2}$$

and where

$$\varphi(t) = \int_0^t c(\alpha)\, d\alpha\,. \tag{3}$$

We are interested in not only spectral structure of (1), but also that of $\lambda\, c(t)$. The goal here is to determine to what extent the structure of $\lambda\, c(t)$, which we term 'jitter', can be used to quantify the level of correlation between two processes, each having the form (1) with the same nominal frequency $\omega_o$. The rationale for this goal is simple. If in (1) we have $\lambda = 0$, then there is no frequency jitter. Additionally, if the initial phase, $\varphi_o$ is unknown (as is the case when one collects repeated measurements of (1) randomly, as opposed to synchronous with a periodic trigger such as a top dead center pulse), the autocorrelation function (4) is simply a cosine function, and the PSD will include Dirac delta functions at $\pm\,\omega_o$. Furthermore, in the case of two such processes whose initial phases are mutually independent and each satisfies (2), then the coherence will equal zero at these frequencies. But often, for this independence to be satisfied, the record length must be sufficiently long so that the cumulative effect of a small amount of jitter is sufficiently large. This has been a long standing fundamental limitation of coherence analysis in relation to rotating mechanical systems. Very often it will happen that at harmonics of the machine frequency, there are large values of coherence between any two measurement processes associated with the machine. Consequently, this information is dismissed; the logic being that since all processes are associated with the same periodicity, then it is natural that this uninformative coherence behavior should occur. The goal of this work is to investigate to what extent one can use jitter information to quantify a meaningful coherence level. For example, if a gear pair entails one type of jitter, while the machine noise entails another, then one might conclude that the noise is not caused by the gear vibration; even though the nominal coherence is close to one.

## TOOLS FROM FM THEORY

From (3), the instantaneous frequency is $\lambda c(t)$. If it can be assumed that $\lambda c(t)$ is a strictly stationary random process, then (1) is a wide sense stationary (*wss*) zero mean random process, and its autocorrelation function is [2]:

$$R_{xx}(\tau) = \frac{1}{2}\Re e[e^{j\omega_o \tau} E(e^{j\lambda\,\varphi(t)})] \tag{4}$$

We now define the following complex-valued random processes, and give their corresponding autocorrelation functions:

$$z(t) \overset{\Delta}{=} e^{j\omega_o t + \varphi_o} \quad , \quad R_{zz}(\tau) = e^{j\omega_o \tau} \tag{5}$$

$$w(t) \overset{\Delta}{=} e^{j\lambda\varphi(t)} \quad , \quad R_{ww}(t) = E(e^{j\lambda\varphi(\tau)}). \tag{6}$$

Comparing (4) to (5) and (6), we see that the PSD of $x(t)$ is proportional to the real part of the convolution of the PSDs of the analytic processes defined in (5) and (6). Since the PSD for (5) includes only Dirac delta functions at $\pm\,\omega_o$, the effect of frequency jitter is to center the PSD for (6) at these locations. We now address the PSD for (6), for Markov jitter. Suppose that the frequency jitter $\lambda c(t) \overset{\Delta}{=} q(t)$ is a Gaussian Markov process; that is,

$$R_{qq}(\tau;\lambda) = \lambda^2\,\sigma_c^2\,e^{-\beta|\tau|}. \tag{7}$$

This jitter process has a ± bandwidth (BW) of $2\beta$. In view of the term 'jitter', we will require that $2\beta << \omega_o$. It follows that

$$\sigma_{\varphi(t)}^2 = \frac{2\sigma_c^2\,|t|}{\beta}[1 - e^{-\beta|t|}]. \tag{8}$$

Since $q(t)$ is a zero mean Gaussian random variable, then so is the phase $\varphi(t)$. Consequently, the autocorrelation function in (6), which is also the characteristic function for $\varphi(t)$ (with respect to the variable $\lambda$), is

$$R_{ww}(\tau) = E(e^{j\lambda\varphi(\tau)}) = e^{-\frac{1}{2}\lambda^2\,\sigma_{\varphi(\tau)}^2} = e^{-\frac{\lambda^2\,\sigma_c^2}{\beta}|\tau|(1-e^{-\beta|\tau|})}. \tag{9}$$

In FM theory the variable $\lambda$ is termed the modulation index. We used it here only to arrive at (9) via characteristic functions. We will henceforth set $\lambda$ to one. Then the strength of the frequency jitter (i.e. the range of frequencies traversed) is controlled by $\sigma_c^2$, and the jitter speed is controlled by $\beta$. Define the variable $\gamma \overset{\Delta}{=} \sigma_c / \beta$ and note that this variable is dimensionless, as is $\beta\tau$. Then we can express (9) as

$$R_{ww}(\tau) = e^{-\gamma\,\beta|\tau|(1-e^{-\beta|\tau|})}. \tag{10}$$

The behaviour of (10) is illustrated below.

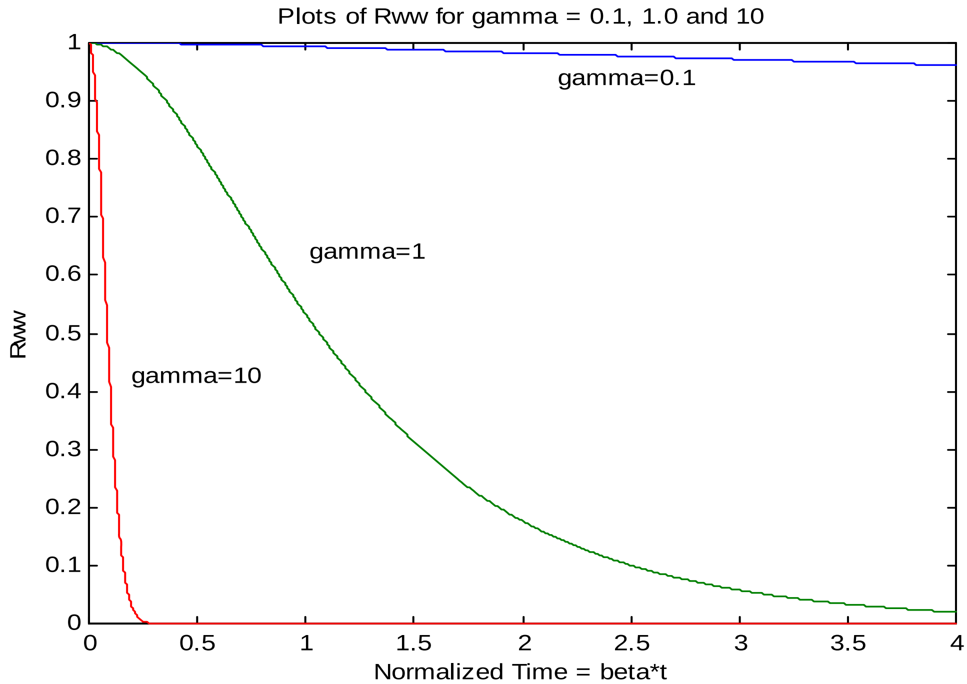


*Figure 1. Plots of (10) for* $\gamma = 0.1, 1.0$ *and* $10$.

From Figure 1, we see that as γ increases, the autocorrelation function (10) approaches that of white noise, in relation to the normalized time, $\beta\tau$. Hence, the behaviour of the corresponding PSDs will become increasingly broad.

## THE INFLUENCE OF JITTER ON PSDs AND COHERENCE

We now demonstrate the influence of these variables on the spectral and coherence properties of time-varying sinusoids modelled by (1).

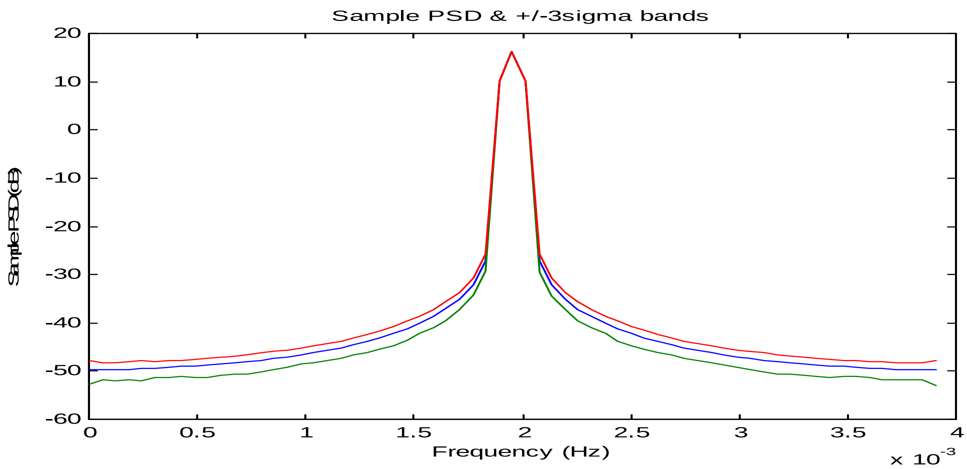


*Figure 2. The PSD estimate and* $\pm 3\,\sigma$ *bounds for (1) with* $\gamma = 1; \beta = 0.999$.

Figure 2 above shows the PSD estimate of (1), as well as uncertainty bounds. Clearly, there is very little uncertainty in the region of $\omega_o$ . Figure 3 below shows the coherence estimate for two sinusoids, each modelled by (1) and having the PSD shown in Figure 2. As expected, the coherence sample mean is less than one. But more importantly, the uncertainty is much more evident than that of the PSD. For an increase in jitter strength, we noted that the PSD estimate becomes broader, but that the uncertainty is still very low. In contrast, Figure 4 shows that not only is the mean coherence low, as expected, but that the uncertainty is high.

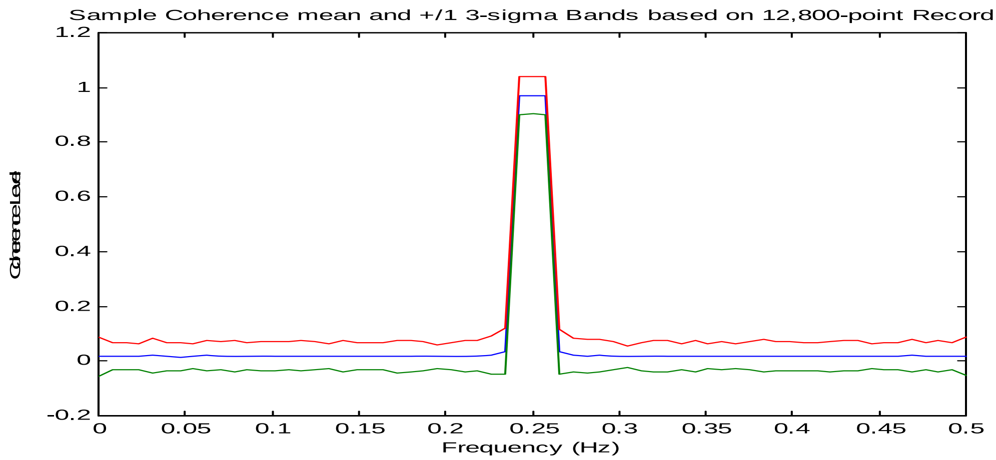


*Figure 3. Coherence estimate for two independent sinusoids with the PSD shown in Figure 1.*

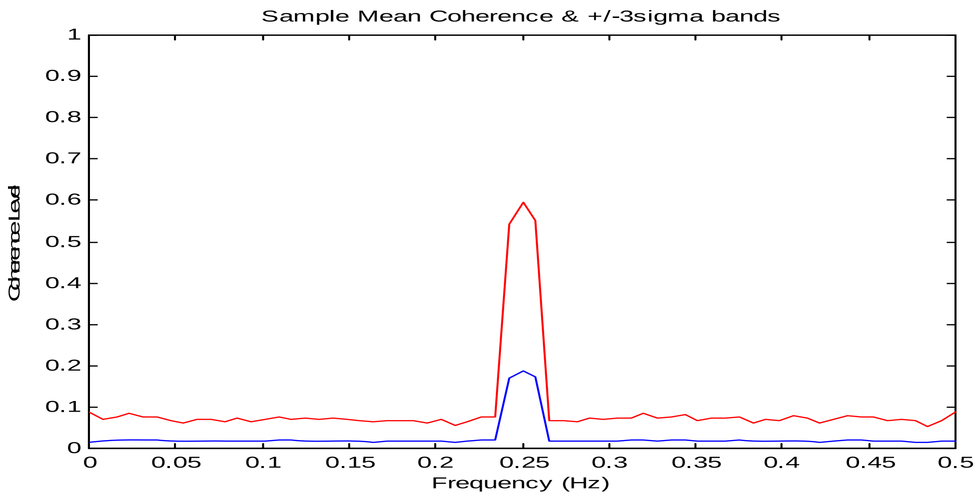


*Figure 4. Mean and* $3\,\sigma$ *bound for two t.v. sinusoids, each having an AR(1) trend about F=0.25 Hz. The AR(1) process has standard deviation* $10^{-4}$ *and drift parameter* $\alpha = 0.999$.

The influence of frequency jitter strength and speed is illustrated in Figure 5. It is far more dependent on strength than speed. For a strength of even 1% it is almost zero, even for $\alpha = 0.9999 \cong 1$.

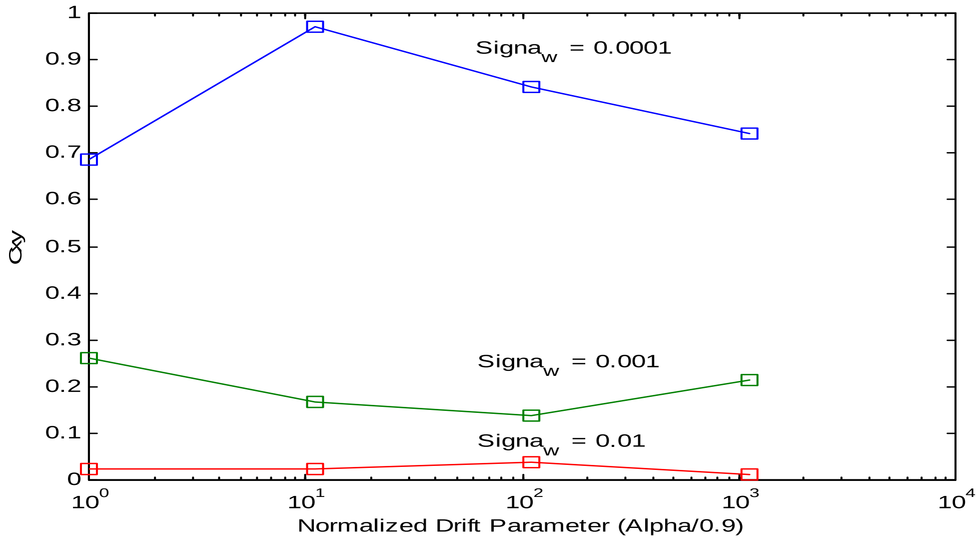


*Figure 5.* Estimated (squared) coherence, $C_{x\,y}$ (mean of n=50 simulations for each point) as a function of jitter strength, $\sigma_\omega$, and normalized drift parameter (speed), $10^4 \alpha / 0.9$.

The strong influence of jitter on the level of uncertainty in coherence estimates provides insight into just what has been so often observed in experimental settings. It also adds weight to a decision to ignore coherence at tone frequencies.

## TRACKING A SINUSOID HAVING FREQUENCY JITTER

The above results provide some insight into the influence of jitter strength and speed on PSD and coherence estimates. In practice, however, the jitter process (1) must be estimated. Given the aforementioned uncertainty, one might expect that reliable estimation might be nontrivial. In this section we present one method for tracking frequency jitter. Consider a signal that is a pure sinusoid; that is:

$$s_t = A\sin(\Omega t \quad + \quad \vartheta) \qquad (11)$$

It is well known that (11) may be expressed as:

$$s_{t+1} = 2\cos(\Omega)\, s_t - s_{t-1} \qquad (12)$$

From (12) and the assumption that the frequency uncertainty can be modeled as an AR(1) process, we have

$$\Omega_t = \overline{\Omega} \quad + \quad \delta\Omega_t \quad ; \quad \delta\Omega_{t+1} = \beta\,\delta\Omega_t + e\Omega_{t+1} \qquad (13)$$

We are now in a position to develop a state space representation for a measurement process given by:

$$z_t = s_t \quad + \quad v_t \quad ; \quad v_t \sim WN(\sigma_v^2)\,. \tag{14}$$

Define the state process $x_t = [s_t \ , s_{t-1} \ , \delta\,\Omega_t]^{tr}$ . If we assume that the sinusoid frequency is *slowly time-varying*, to the extent that $\Omega_t \cong \Omega_{t-1}$ then (12) may be expressed as:

$$s_{t+1} = 2\cos(\Omega_t)\, s_1 - s_{t-1} \quad \cong \quad g(x_t) = 2\cos(\overline{\Omega} + x_{3,t}) \bullet x_{1,t} - x_{2,,t} \tag{15}$$

Clearly, (15) is a nonlinear function of the state $x_t$, which is unknown. But the extended Kalman filter can be used to obtain an estimate of it.

**Example.** Figure 5 below shows a sample of a time-varying sinusoid plus white noise. The overall SNR is -3 dB. Figure 6 shows that even in a relatively low SNR setting, it is possible to achieve frequency jitter estimation with relatively low (< 10%) uncertainty.

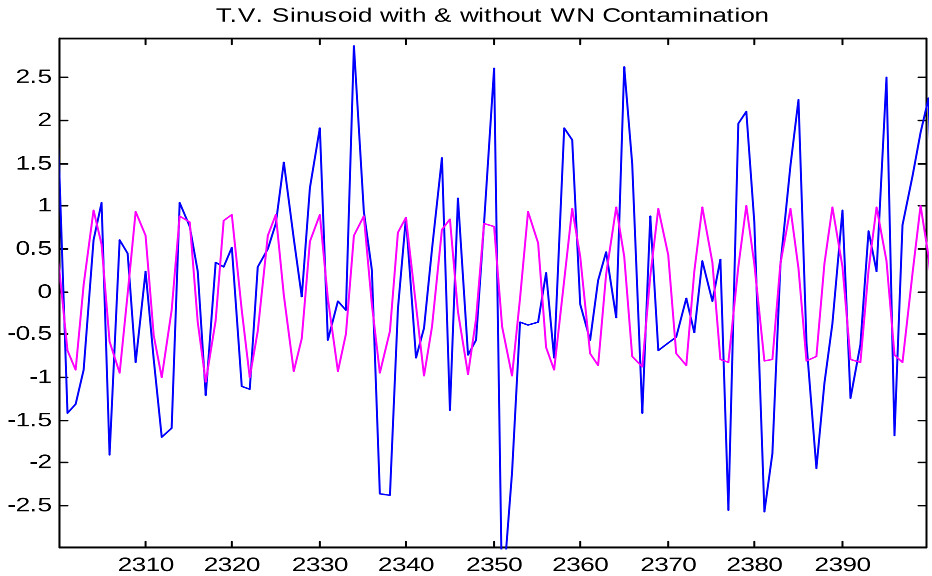


**Figure 5.** Overlaid plots of a time-varying sinusoid with (blue) and without (pink) white noise contamination.

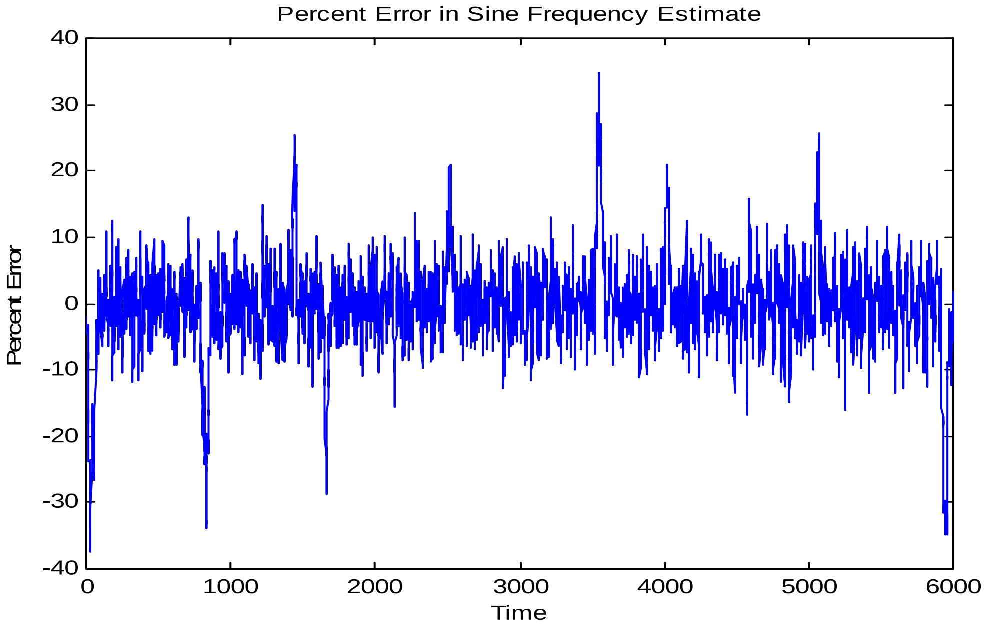


**Figure 6.** Percent error in the estimate of the time-varying frequency.

## COHERENCE BETWEEN TWO NOISE-CORRUPTED T.V. SINES

We conclude this work by briefly discussing the utility of jitter tracking information for the case of a time-varying sinusoid that is corrupted by additive noise. Suppose that both channels include the same sinusoid, but that the noise processes are independent. It is well known that the coherence at the sine frequency will drop as the SNR drops. However, if the frequency jitter could be perfectly recovered, then the jitter coherence would equal one. We applied the above EKF method to estimate the jitter for each channel, and subsequently obtained a coherence estimate of 0.92 in an SNR environment (0 dB) wherein the estimated coherence between the channels was less than 0.2. Even though this low value is to be expected, the jitter information allowed us to determine that, essentially, the same sine was on both channels.

## REFERENCES

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