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Operational Modal Analysis in the Presence of Unknown Arbitrary Loads Using Transmissibility Measurements

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Abstract

Operational modal analysis allows us to identify the dynamic behavior of a structure from the knowledge of the measured responses only, using the Spectral Densities of the outputs. These techniques have serious limitations because they can only be applied under the assumption that the forces are the result of a stochastic process, so being white noise. This is no longer necessary for the new proposed transmissibility-based approach to identify modal parameters. The unknown operational forces can be arbitrary (colored noise, swept sine, impact ...) as long as they are persistently exciting in the frequency band of interest. In general, the poles that are identified from transmissibility measurements do not correspond with the system's poles. However, by combining transmissibility measurements under different loading conditions, it is shown in this paper that model parameters can be identified. The procedure will be elaborated and illustrated by means of real measurement results performed on a plate structure.

INTRODUCTION

Experimental modal analysis allows us to identify the dynamic behavior of a structure from the knowledge of the applied forces and the measured responses, using the frequency response functions. In the case of structures in operational conditions we often do not know these forces. For instance, civil structures (e.g. bridges, buildings, off-shore platforms, etc.) in operating conditions, who are excited by unmeasurable ambient excitation sources (e.g. traffic, wind, waves, etc.). For these situations output-only techniques were developed using the responses only. These techniques have been widely and successfully used. The enormous advantage of this technique is that it provides a model under operating conditions, within true boundary conditions, and actual force levels. Current output only techniques have serious limitations because they can only be applied under the assumption that the forces are the result of a stochastic process, so being white noise. In many mechanical structures the loading forces are often more complex and harmonic components are present in the response. This is especially true, when measuring on mechanical structures containing rotating or reciprocating parts (e.g.cars, turbines, windmills), but also civil engineering structures may have responses superimposed by harmonic components.

Since OMA is an in-situ type measurement you do not measure the input forces to the structure, the art is then to distinguish real structural behavior from noise and excitation contributions. In order to separate the true structural modes from the forced excitation components, a number of techniques can be used as indicator. Some of them where presented in [6] and some will be discussed in this contribution.

This separation is no longer necessary for the proposed transmissibility-based approach. The unknown operational forces can be arbitrary (colored noise, swept sine, impact ...) as long as they are persistently exciting in the frequency band of interest. The transmissibility measurements that are obtained by taking the ratio of two response spectra do not dependent on the amplitude nor on the coloring of the unknown input forces. In general, the poles that are identified from transmissibility measurements do not correspond with the system's poles. However, by combining transmissibility measurements under different loading conditions, it is shown in this paper that model parameters can be identified. An experimental test validates the proposed technique.

Experimental and operational modal analysis

During the last decade modal analysis has become a key technology in structural dynamics analysis [5, 10, 9]. Experimental modal analysis (EMA) identifies a modal model, $[H(\omega)]$, from the measured forces applied to the test structure, $\{F(\omega)\}$, and the measured vibration responses $\{X(\omega)\}$,

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \tag{1}$$

with

$$[H(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{L_m\}^T}{i\omega - \lambda_m} + \frac{\{\phi_m\}^*\{L_m\}^H}{i\omega - \lambda_m^*} \tag{2}$$

and

$$\lambda_m = -\sigma_m + i\omega_{dm} \tag{3}$$

The modal model (2) expresses the dynamical behavior of the structure as a linear combination of N_m resonant modes. Each mode is defined by a damped resonant frequency, $f_{dm} = \omega_{dm}/2\pi$, a damping ratio, $\zeta_m = \sigma_m/|\lambda_m|$, a mode shape vector, $\{\phi_m\}$, and a modal participation vector, $\{L_m\}$. These modal parameters depend on the geometry, material properties and boundary conditions of the structure.

More recently, system identification techniques were developed to identify the modal model from the structure under its operational conditions using output-only data [8, 1]. These techniques, referred to as operational modal analysis (OMA) or output-only modal analysis, take advantage of the ambient excitation. Frequency-domain output-only estimators start from power spectral densities, a quantity that can be derived from output-only measurements. For stationary stochastic processes the spectral density matrix, $[S_X(\omega)]$, of the outputs is given by

$$[S_X(\omega)] = [H(\omega)][S_F(\omega)][H(\omega)]^H \tag{4}$$

here $[S_F(\omega)]$ contains the cross power spectra of the (unknown) input forces. Under the assumption that the forces are white noise sequences, $[S_F(\omega)]$ can be considered to be a constant matrix with respect to the frequencies. It can be shown that by substituting the expression for an element of the matrix $[H(\omega)]$ equation (2) in the spectral densities of the outputs $[S_X(\omega)]$ equation(4) evaluated at the frequency ω can be modally decomposed as follows:

$$\begin{aligned}[S_X(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{K_m\}^T}{i\omega - \lambda_m} &+ \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega - \lambda_m^*} \\ &- \frac{\{\phi_m\}\{K_m\}^T}{i\omega + \lambda_m} - \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega + \lambda_m^*}\end{aligned} \tag{5}$$

with $\{K_m\}$ the operational participation vectors, which depend on the modal participation vector, $\{L_m\}$, and the power spectrum matrix of the unknown operational forces.

The problem as was mentioned above rises when the ambient forces have a colored nature, as often is in practice. The latter introduces some additional terms Θ_{exc} into the modal decomposition of the power spectra that are dependent on the coloring of the unknown input forces.

$$\begin{aligned}[S_X(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{K_m\}^T}{i\omega - \lambda_m} &+ \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega - \lambda_m^*} \\ &- \frac{\{\phi_m\}\{K_m\}^T}{i\omega + \lambda_m} - \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega + \lambda_m^*} + [\Theta_{exc}(\omega)]\end{aligned} \tag{6}$$

For this reason, additional peaks can appear into the power spectra of the responses that are not related to resonance of the system with the already mentioned difficulties. An overview of possible techniques - for the discrimination of true physical modes from peaks related to the excitation - is given in [7] (Lisowski, 2001)

Transmissibilities

In this paper attention will be paid to the use of transmissibilities to derive modal parameters. In general, it is not possible to identify modal parameters from transmissibility measurements. Transmissibilities are obtained by taking the ratio of two response spectra, i.e. $T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)}$. By assuming a single force that is located in, say, the input degree of freedom (DOF) k, it is readily verified that the transmissibility reduces to

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{H_{ik}(\omega)F_k(\omega)}{H_{jk}(\omega)F_k(\omega)} = \frac{N_{ik}(\omega)}{N_{jk}(\omega)} \triangleq T_{ij}^k(\omega) \tag{7}$$

with $N_{ik}(\omega)$ and $N_{jk}(\omega)$ the numerator polynomials occurring in the transfer-function models $H_{ik} = \frac{N_{ik}(\omega)}{D(\omega)}$ and $H_{jk} = \frac{N_{jk}(\omega)}{D(\omega)}$. Note that the common-denominator polynomial, $D(\omega)$, which roots are the system's poles, λ_m, disappears by taking the ratio of the two response spectra. Consequently, the poles of the transmissibility function (7) equal the zeroes of transfer function $H_{jk}(\omega)$, i.e. the roots of the numerator polynomial $N_{jk}(\omega)$. So, in general, the peaks in the magnitude of a transmissibility function do not at all coincide with the resonances of the system. In next section it will be shown that by combining transmissibility measurements under different loading conditions it is possible to identify the modal parameters (i.e., resonant frequencies, damping ratios and mode shape vectors).

Theoretical resuls

Making use of the modal model (2) between input DOF, k, and, say, output DOF, i,

$$H_{ik}(\omega) = \sum_{m=1}^{N_m} \frac{\phi_{im} L_{km}}{i\omega - \lambda_m} + \frac{\phi_{im}^* L_{km}^*}{i\omega - \lambda_m^*} \tag{8}$$

one concludes that the limit value of the transmissibility function (7) for $i\omega$ going to the system's poles, λ_m, converges to

$$\lim_{i\omega \to \lambda_m} T_{ij}^k(\omega) = \frac{\phi_{im} L_{km}}{\phi_{jm} L_{km}} = \frac{\phi_{im}}{\phi_{jm}} \tag{9}$$

and is independent of the (unknown) force at input DOF k. Consequently, the substraction of two transmissibility functions with the same output DOFs, (i, j), but with different input DOFs, (k, l) satisfies

$$\lim_{i\omega \to \lambda_m} \Big(T_{ij}^k(\omega) - T_{ij}^l(\omega) \Big) = \frac{\phi_{im}}{\phi_{jm}} - \frac{\phi_{im}}{\phi_{jm}} = 0 \tag{10}$$

To sum up, the system's poles, λ_m, are zeroes of the rational function $\Delta T_{ij}^{kl}(\omega) \triangleq T_{ij}^k(\omega) - T_{ij}^l(\omega)$, and, consequently, poles of its inverse, i.e.

$$\Delta^{-1} T_{ij}^{kl}(\omega) \triangleq \frac{1}{\Delta T_{ij}^{kl}(\omega)} = \frac{1}{T_{ij}^k(\omega) - T_{ij}^l(\omega)} \tag{11}$$

As was shown in [4] $\Delta^{-1} T_{ij}^{kl}(\omega)$ can contain more poles than the system's poles only. Hence, in general, only a subset of the poles of $\Delta^{-1} T_{ij}^{kl}(\omega)$ will correspond to the real system's poles.

Measurement setup

The measurements were performed using a steel plate structure freely supported by elastic tape to a small frame. A small loudspeaker at the back of the plate was amplified to excite the plate with a user-defined broadband acoustic excitation signal. This allows us to have no fixed connections between the structure and excitation device and eliminate all possible structure-exciter interaction. Velocity responses were measured subsequently in 9 points with a Polytec PSV-300 scanning vibrometer. This way, again no physical contact was required with the plate during the vibration measurements.

Figure 1: The measurement setup

Experimental Results

In the first test a stationary output-only data set was obtained while the structure was excited by means of a periodic chirp signal. This signal is characterized by a uniform spectra in the excited frequency range. This data was obtained with the speaker in the upper right corner of the plate. A reference response was chosen (left lower corner) and a frequency-domain Maximuum Likelihood estimator (adapted for output-only data [3], Guillaume et al., 1999) was used for the determination of an output only modal based on the calculated cross power spectra between each response and the reference response.

Next transmissibility measurements were obtained by taking the ratio between each response and the reference response. In a second measurement the same transmissibility measurements were obtained with the speaker located in the lower right corner. In a third measurement the speaker was located in the lower left corner. One notice that the amplitudes peaks of the transmissibilities do not correspond with the resonant frequencies (Figure 3). However, all transmissibilities cross each other at the resonant frequencies of the beam, which is in agreement with the theoretical results. The ML frequency-domain estimator was applied to the $\Delta^{-1}T_{ij}^{kl}(\omega)$ functions and all poles could be identified. It can be observed that only a subset of the amplitudes peaks in $\Delta^{-1}T_{ij}^{kl}(\omega)$ coincide with the resonant peaks of the considered plate (Figure 3). The non-system-poles could easily be eliminated by a proposed SVD approach [4].

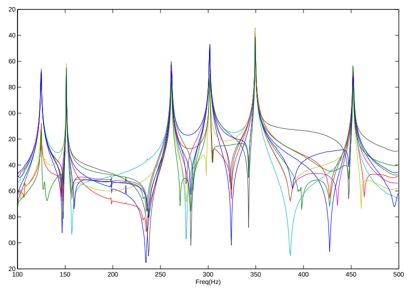

Figure 2: Cross power spectra

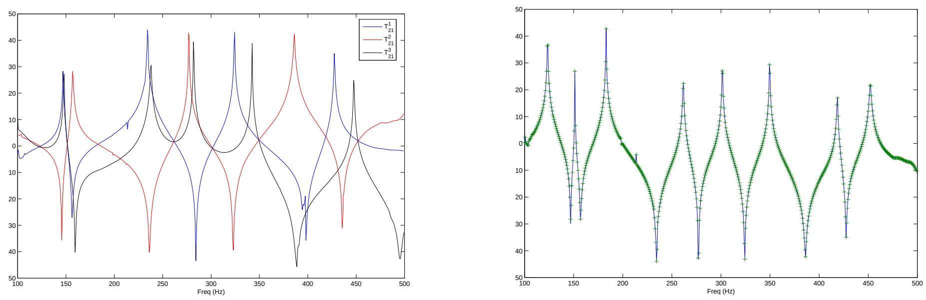

Figure 3: Transmissibility between the upper left response and the reference response in the case of the 3 loudspeaker locations; Estimation results of the $\Delta^{-1}T_{ij}^{kl}(\omega)$ functions. Cross line: data. Solid line: estimated model.

All modes were identified and did in general show excellent relationship with each other (Table 1). Care must be taken when the structure is excited by a non uniform force spectrum. In a new experiment the structure was excited by a signal with a force spectrum equal to the respons spectrum of a 1DOF-system with a system pole equal to $\lambda_m = -10 + i\omega 280$.

$\zeta(S)[\%]$	$\zeta(\Delta^{-1}T)[\%]$	$f_d(S)$ [Hz]	$f_d(\Delta^{-1}T)$ [Hz]
0.56	0.58	124.72	124.17
0.27	0.29	151.32	151.32
0.27	0.29	261.52	261.64
0.24	0.27	301.67	301.49
0.86	0.89	349.47	349.59
0.54	0.56	452.16	452.59

Table 1: Comparison of the estimated damping ratios and damped natural frequencies obtained from spectral densities and the transmissibility-based approach.

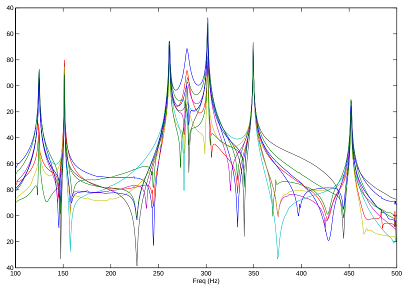

Figure 4: Cross power spectra

With this excitation signal an additional peak appears in the power spectra, see Figure 4. By now using a power spectrum driven identification method this mode is wrongly identified as a structural resonance with a resonance frequency of 280Hz and a damping ratio of 4.8%.

The transmissibility measurements do not dependent on the nature of the unknown input forces and therefore the same transmissibility results were obtained as before.

In above situation a good indicator to identify this mode as forced vibration mode would be a visual validation of the mode shapes as was discussed in [6]. This would alow us to identify the additional mode as an Operating Deflection Shape and we would easily eliminate this wrongly identified pole. This is only the case when a harmonic component is far away from a structural resonance. In this case the deflection pattern is a combination of several excited modes and the forces acing on the structure. However when a harmonic component is close to an isolated structural response, the deflection pattern will resemble the mode shape and thus can be mistaken for being a mode shape. Another forced vibration indicator often used is the damping ratio. A forced excitation can often be regarded as a lightly damped vibration, thus identified poles with extremely low damping ratios can be separated and identified as non-structural modes. In above example this is far from true, at the contrary. The damping ratio especially is a good harmonic indicator in cases of structures with heavily damped modes.

SUMMARY

It has been shown in this paper that correct systems poles can be identified starting from transmissibility measurements only. The theoretical results are verified by means of experimental data. A comparison has been made with classical output-only techniques who often require the operational forces to be white noise. This is not necessary for the proposed transmissibility-based approach. The unknown operational forces can be arbitrary (colored noise, swept sine, impact, ...) as long as they are persistently exciting in the frequency band of interest. The latter method reduce the danger to identify peaks related to the excitation as true physical modes.

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