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IMPROVED BROADBAND PERFORMANCE OF STRAIN-BASED CONTINUOUS RADIATION SENSORS

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Abstract

In the field of Acoustic Structural Active Control (ASAC) the use of continuous strain sensors (for example PVDF sensor strips) is often considered. These have the advantage of directly measuring radiation modes through a specific combination of structural modal amplitudes without requiring further signal processing by shaping the width of the sensor strip along its length. However, radiation modes are frequency dependent, more so in the frequency region where ASAC would be applied for typical panel sizes. One solution would be to use discrete sensors but this loses the advantage of using PVDF sensors. It is shown the main frequency dependence is largely due to i) displacement instead of velocity sensing, and ii) a frequency dependent offset term that can be corrected with a temporal filter. The accuracy of the measurement of the first radiation mode can be improved and the total power radiated better represented.

INTRODUCTION

The measurement of the amplitude of the radiation modes of a plate allows the far-field acoustic radiation to be evaluated directly on the plate surface [1]. The radiation modes form an orthogonal set and so their reduction by a control system guarantees the reduction of the acoustic radiation. It is often considered sufficient to control the first radiation mode only, leading to possible simple feedback control systems. The transformed radiation modes can be measured via specific combinations of structural modes [2] and the task becomes one of structural modal detection. PVDF (polyvinylidene fluoride) is often used as a continuous vibration sensor strip whose shape can be formed to measure the required combination of structural modes without further signal processing. At frequencies above a limit determined by the relation between the acoustic wavelength and the plate size, the radiation modes become frequency dependent. An alternative solution of using discrete sensors would lose the benefits of the PVDF sensor.

This paper shows that a frequency correction may be used to improve the performance of PVDF sensors.

TRANSFORMED RADIATION MODES

The Rayleigh integral allows the pressure, p, at a point $\mathbf{r}$ in an anechoic acoustic field to be calculated due to the vibration of a plate in an infinite baffle. In essence, it sums the contributions from all regions of the plate considering the transverse velocity, v, (plate plane perpendicular) and relative phase. Assuming time dependence $e^{j\omega t}$ for notational simplicity, the pressure is given by [1]

$$p(\mathbf{r}) = \int_S \frac{j\omega\rho_o v(\mathbf{r}_s)e^{-jkR}}{2\pi R}\mathrm{d}S \ . \tag{1}$$

ρ_o is the density of the air, v the velocity distribution of the plate surface, k is the acoustic wavenumber, and R is distance between each elemental plate radiator $\mathbf{r}_s$ and the point $\mathbf{r}$ in the half-space ($R = |\ \mathbf{r} - \mathbf{r}_s\ |$).

Fuller et al [1], among others, used acoustic radiation modes specified in terms of the velocity distribution of the plate. These are termed velocity distribution radiation filters (VDRF) here to distinguish them from the radiation modes used. In the laboratory it is feasible to implement VDRFs using, for example, scanning laser velocimetry to measure the velocity distribution. However, it is not practical for in-field application.

Taneka et al [2] derived *transformed* radiation modes that are defined in terms of structural modal amplitudes defined in displacement instead of the plate velocity distribution. Here the plate radiation is defined by the modal amplitudes in $\mathbf{a}$ and a transformation matrix $\mathbf{A}$, thus

$$\Pi = \mathbf{a}^{\mathrm{H}}\mathbf{A}\mathbf{a} \ . \tag{2}$$

$\mathbf{A}$ is calculated by

$$\mathbf{A} = \omega^2\mathbf{\Phi}^{\mathrm{T}}\mathbf{R}\mathbf{\Phi} \ , \tag{3}$$

where the elements of $\mathbf{R}$ are given

$$R_{ij} = \frac{\omega^2\rho_o S_i^2}{4\pi c}\frac{\sin kr_{ij}}{kr_{ij}} . \tag{4}$$

Each element of $\mathbf{A}$ is therefore

$$A_{ij} = \omega^2\Phi_i^{\mathrm{T}}\mathbf{R}\Phi_j \ , \tag{5}$$

where Φ_i is a column of the modal matrix. Using eigen-decomposition $\mathbf{A}$ is replaced by a matrix of eigenvectors $\mathbf{Q_t}$ and a diagonal matrix $\mathbf{\Lambda_t}$:

$$\Pi = \mathbf{a}^{\mathrm{H}}\mathbf{Q}_t\mathbf{\Lambda}_t\mathbf{Q}_t^{\mathrm{T}}\mathbf{a} = \mathbf{u}^{\mathrm{H}}\mathbf{\Lambda}_t\mathbf{u} \ . \tag{6}$$

The matrix $\mathbf{u}$ contains the transformed radiation modes which are orthogonal and with amplitudes defined in $\mathbf{\Lambda_t}$. Thus each row of $\mathbf{Q_t}$ defines a radiation mode in terms of the amplitudes of the structural modes. While radiation modes are defined purely by the plate geometry, as the structural modes depend on the physical plate properties and the boundary conditions then the definition of the radiation modes by $\mathbf{Q_t}$ will change accordingly.

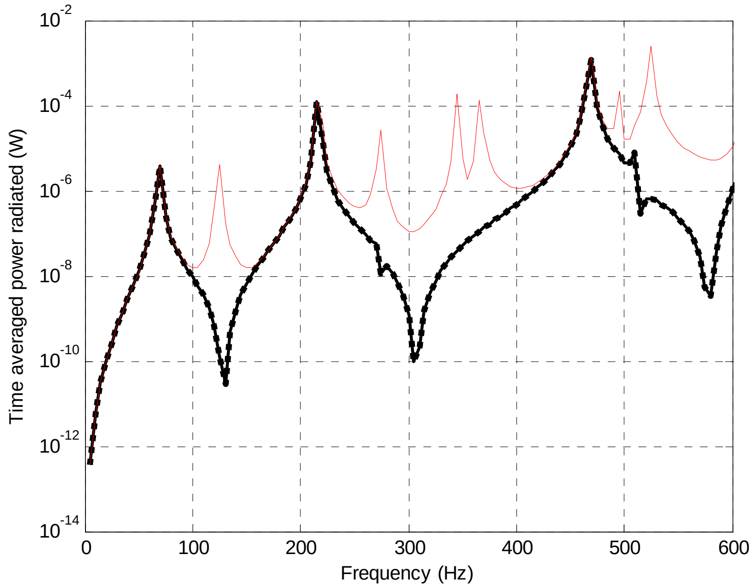

Figure 1 – The acoustic power radiated by the plate considered (Figure 5) when driven by the actuator and measured by the Rayleigh integral (red.) The value of the frequency-global first radiation mode is shown in black.

In the above the ω is the frequency at which the radiation modes are calculated, and for this reason it is termed here the *design* frequency. Radiation modes are frequency dependent but below a lower frequency limit they are considered constant [1]. In the literature the limit is commonly quoted as ka<<1 where k is the acoustic wavenumber and a the principal plate dimension.

It is unfortunate that in the application of ASAC typical ranges of operational frequency and typical panel sizes mean that the value of ka is about equal to or greater than unity. To illustrate this, consider a higher operating frequency limit of 100Hz, this means that the principal dimension of the plate must be significantly less than 0.55m (for example, 0.1m if "<<" is considered to be 20%, it is commonly taken as one order of magnitude less). Alternatively consider a typical panel size of 1m, and then the highest operating frequency must be significantly less than 55Hz. Thus it is unrealistic to simply consider that using constant radiation modes will provide the best results. Using discrete sensors this can be addressed [3] but the great advantage of continuous sensors: that they provide an electrical signal directly proportional to the radiation mode with further treatment, is lost.

Figure 1 shows the total acoustic radiation evaluated by the Rayleigh integral of the simply supported baffled plate considered here (see Figure 5) when excited by the actuator. The value of the *frequency-global first radiation mode* is also shown. This is the first radiation mode evaluated at each frequency at which it is applied and so does not contain the errors of ignoring the frequency dependence. This is then used as a reference for the radiation mode sensor design considered here. Thus, if a control system is implemented using the first radiation mode, then the best performance possible will be for a sensor whose output adheres as closely as possible to the value of the frequency-global first radiation mode at all frequencies.

The first radiation mode for the plate is calculated at a four different design frequencies, f_{res} : 10Hz (0.09), 30Hz (0.28), 100Hz (0.93) and 500Hz (4.6). The corresponding value of *ka* is given in parenthesis after each frequency, only at 10Hz and possibly at 30Hz can *ka* be considered to be significantly less than unity. The radiation mode is then used to represent the radiation of the plate over the frequency range 5Hz to 600Hz, as shown in Figure 2 using the frequency-global first radiation mode as a reference.

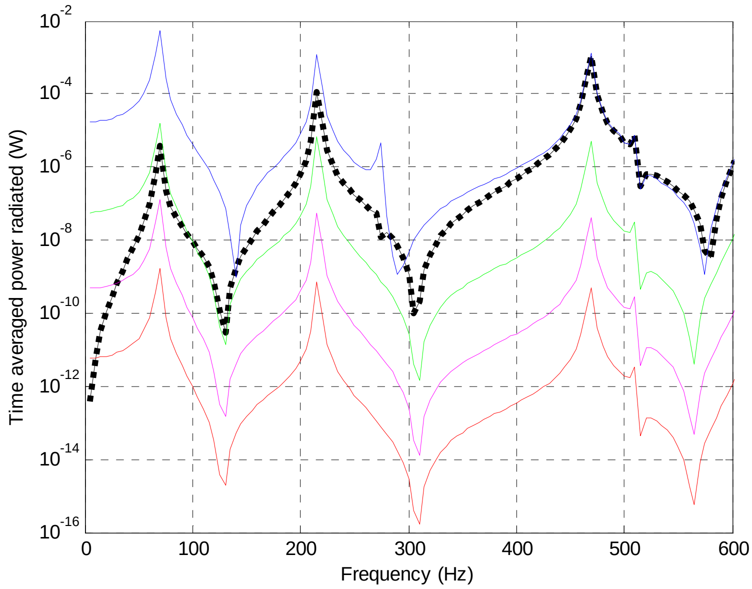

Figure 2 – The acoustic power radiated by the plate driven by the actuator and measured by the first transformed radiation mode with design frequencies (f_{res}) of 10Hz (red), 30Hz (magenta), 100Hz (green) and 500Hz (blue). The frequency-global first radiation mode is shown in black for comparison.

It can be seen that at the frequency corresponding to the design frequency of each radiation mode the estimated radiated power corresponds well with the frequency-global first radiation mode, except with f_{res} equal to 500Hz but agreement can be seen in general in around this frequency. It is also seen that there is very little discernable difference in between the “shape” of the responses, except again with f_{res} equal to 500Hz. Thus it appears that the main difference is a frequency dependent offset, and it is thus proposed that a correction by a temporal filter with a suitable response could be used to correct this. It is also noted that the error in the responses of the radiation filters at frequencies lower than f_{res} is due to the fact that the transformed modes are defined in terms of the plate displacement, and the displacement response has a constant residue term below the resonant frequency.

Figure 3 shows the response of the transformed radiation modes with the same four design frequencies corrected by a factor of ω^2. This is equivalent to applying a 2nd order high pass filter characteristic over the frequency region. Essentially, velocity in place of displacement is now being sensed and the output is similar to that achieved using a

VDRF. Each response is normalised to the total power radiation corresponding to f_{res} in each case. While the frequency dependence appears to be less, each designed radiation filter is still only accurate in the region of its design frequency.

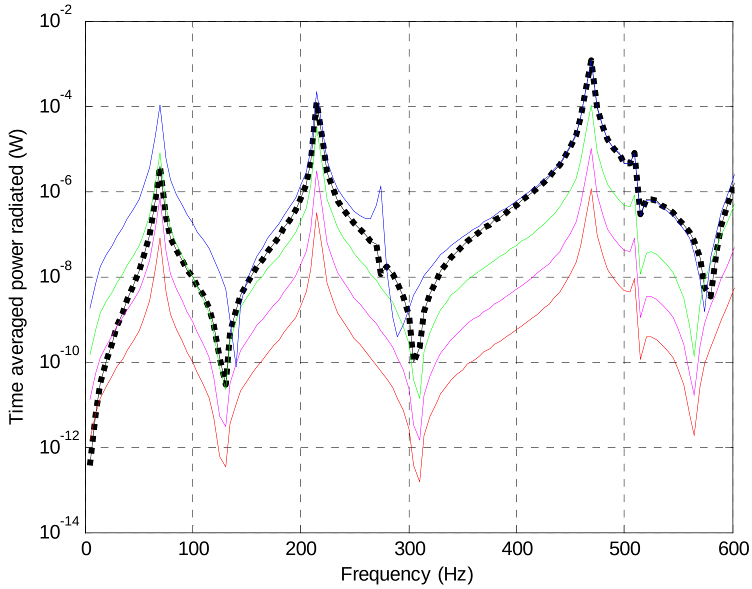

Figure 3 – The acoustic power radiated by the plate driven by the actuator and measured by the first transformed radiation mode with the four design frequencies that have been corrected by a factor of ω^2 and normalized to the frequency-global first radiation mode at each design frequency. Key as for Figure 2.

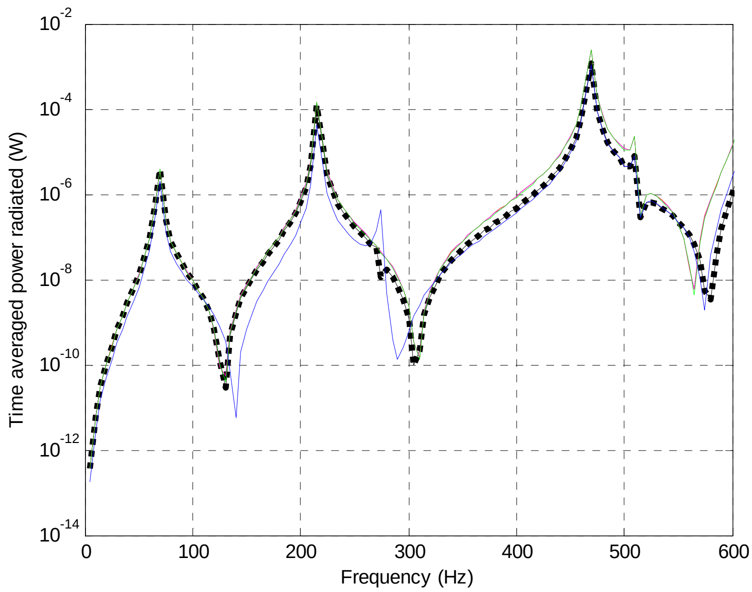

Figure 4 – The acoustic power radiated by the plate driven by the actuator and measured by the first transformed radiation mode with the four design frequencies that have been corrected by a factor of ω^4 and normalized to the frequency-global first radiation mode at each design frequency. Key as for Figure 2.

Following this trend the responses of the transformed radiation modes with the same four design frequencies are now corrected by a factor of ω^4 and are shown in Figure 4. It is seen that with the exception of f_{res} = 500Hz that each of the frequency corrected responses follows the true value of the first radiation mode almost exactly up to about 300Hz. Above this frequency a good approximation is still achieved. This frequency correction would require a 4[th] order filter characteristic.

DESIGN OF A PVDF RADIATION SENSOR

A radiation mode sensor can be constructed using PVDF sensor strips. When applied to the surface of a plate of dimensions (a,b), thickness h_p and with a sensor axis y_s, the electrical charge output of a sensor strip may be approximated from [4] as

$$q(t) \approx \frac{h_p}{2}\int_0^a \int_{y_s-cS(x)}^{y_s+cS(x)} \left(e_{31}^0 \frac{\partial^2 W}{\partial x^2} + e_{32}^0 \frac{\partial^2 W}{\partial y^2} \right) \mathrm{d}y\mathrm{d}x \quad , \tag{7}$$

where W is the displacement field, e_{31}^0 and e_{32}^0 are PVDF material constants, and $S(x)$ describes the variable width of the sensor strip along its length which is scaled for convenience by c. The design method follows that of Tanaka et al [2] and only brief details are given here. Limiting the frequency range of interest to 600Hz the value of the normalised amplitudes of each structural mode contributing to the first radiation mode calculated at the four design frequencies are given in Table 1. Up to about 300Hz the relative contributions of each structural mode are constant.

Mode (m,n)	Freq (Hz)	Modal amplitude for 1st radiation mode at design frequency ($\alpha_{m,n}$)			
		10 Hz	30 Hz	100 Hz	500 Hz
(1,1)	68.46	1.00e+000	1.00e+000	1.00e+000	1.00e+000
(3,1)	216.23	3.34e-001	3.34e-001	3.31e-001	2.42e-001
(1,3)	468.38	-3.35e-001	-3.35e-001	-3.35e-001	-3.35e-001
(5,1)	511.74	2.01e-001	2.01e-001	1.99e-001	1.43e-001
(3,3)	616.14	-1.12e-001	-1.12e-001	-1.11e-001	-8.12e-002

Table 1 – The amplitudes of the structural modes (normalised with respect to mode (1,1) in each case) required for the first transformed radiation mode of the plate considered at four design frequencies. Only the structural modes having normalised amplitude of greater than 0.005 are included.

The radiation sensor is applied to the plate considered as two identical sensor strips, as shown in Figure 5. The placement of the strips in the y-direction provides detection of the odd modes only in this direction on a discrete basis. The position of the sensor strips is given by resolving for y_s in the following equation

$$\frac{\phi_{1,1}(y_s)}{\phi_{1,3}(y_s)} = \frac{\sin\left(\dfrac{\pi\, y_s}{b}\right)}{\sin\left(\dfrac{3\pi\, y_s}{b}\right)} = \frac{\alpha_{1,1}}{\alpha_{1,3}} \quad . \tag{8}$$

Normally the number of strips would be equal to the highest modal order in this direction, but in this case two suffice due to a favourable value of $\alpha_{3,3}$ [5]. The lowest value of y_s is 0.3035 b . This is the distance of each sensor strip axis from the plate edge. The width of the sensor strip along its length, $S(x)$, is given by

$$S(x) = \frac{\alpha_{1,1}}{1^4}\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\alpha_{3,1}}{3^4}\frac{\partial^2 \phi_3}{\partial x^2} + \frac{\alpha_{5,1}}{5^4}\frac{\partial^2 \phi_5}{\partial x^2} \quad \text{where} \quad \phi_n = \sin\left(\frac{n\pi x}{a}\right). \tag{9}$$

$\alpha_{m,n}$ is the normalised value from Table 1. The fourth power in the denominator of the modal order is to compensate for the fact that the PVDF sensor detects the *second spatial derivative* of displacement and also that the structural wavenumber is proportional to the modal order squared.

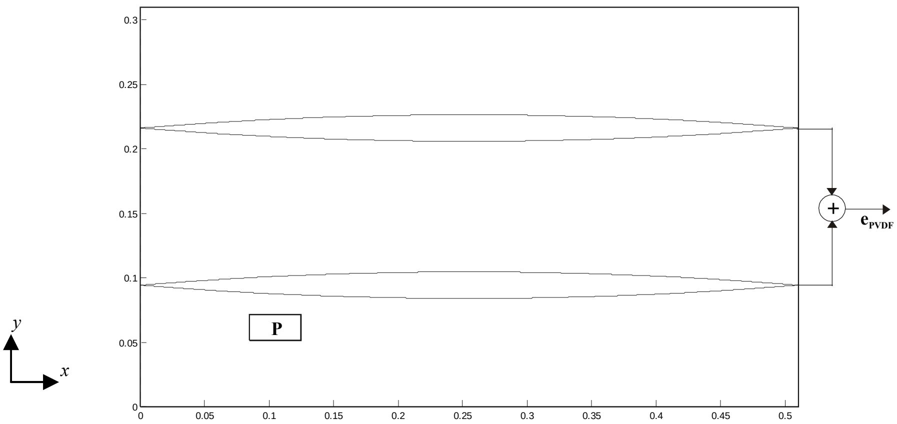

Figure 5 – The plate studied of dimensions 0.51×0.31m, the position of the actuator (P) and the two PVDF strips. The first mode radiation sensor output, e_{PVDF} , is the sum of the two sensor strip signals.

SENSOR PERFORMANCE USING NUMERICAL MODEL

A numerical model was used to predict the response of the radiation sensor designed above. The displacement of the plate with frequency excited by a piezoelectric actuator was solved using a modal model, and then the response of the PVDF sensor strips were calculated by the integration of the second spatial derivative over the sensor area [5].

The output of the ω^4 frequency corrected PVDF radiation sensor was modelled for a sensor that does not have the unwanted sensitivity to bending across the sensor axis (y-direction), and also for a fully realistic sensor. The results are shown in Figure 6 and total power radiation by Rayleigh is also included. It is seen that the frequency correction produces a sensor which can measure the true value of the first radiation mode up to about 80Hz. The response then deviates but shows improved accuracy around the peaks at approximately 215Hz and 470Hz although it does over estimate the radiation in between. The response would be better if the sensor had no cross-axis sensitivity. In practice it is expected that the transfer of strain to the PVDF in the cross-axis direction would be less, but this can only be verified experimentally.

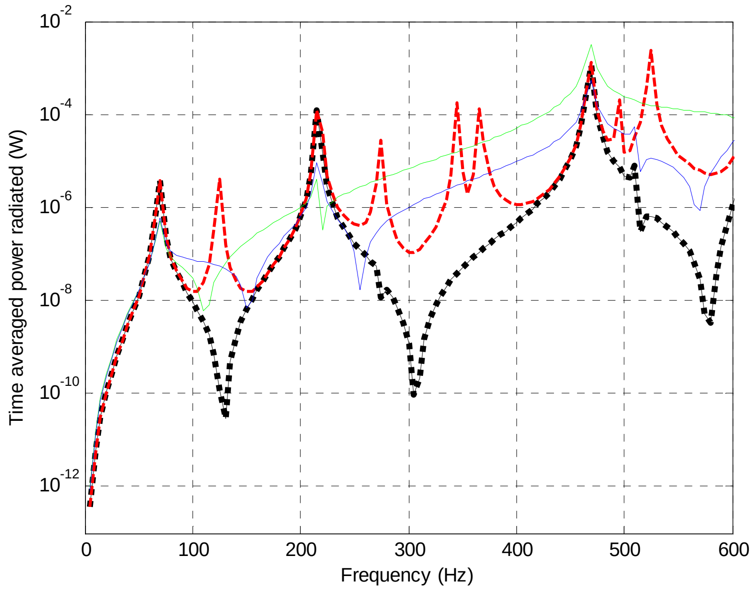

Figure 6 –The value of the PVDF radiation sensor output with the ω^4 frequency correction (green) and for a sensor that does not have the unwanted cross-axis sensitivity (blue). Both are normalised with the frequency-global first radiation mode (black) at 80Hz. The total power radiated by the Rayleigh integral is in red.

SUMMARY

An ω^4 frequency correction has been applied to the first radiation mode calculated at a single design frequency and applied to measure broadband radiation. In this way the frequency dependence of radiation modes has, in part, been overcome. The corrected response is accurate from 5Hz to 80Hz and shows a better general estimate of the true radiation mode at higher frequencies than without the correction. In practice using PVDF sensors the predicted response worsens in part due to the cross-sensitivity of the sensor.

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